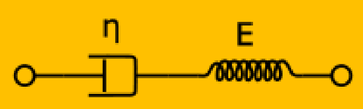
Role of viscoelasticity in mantle convection models

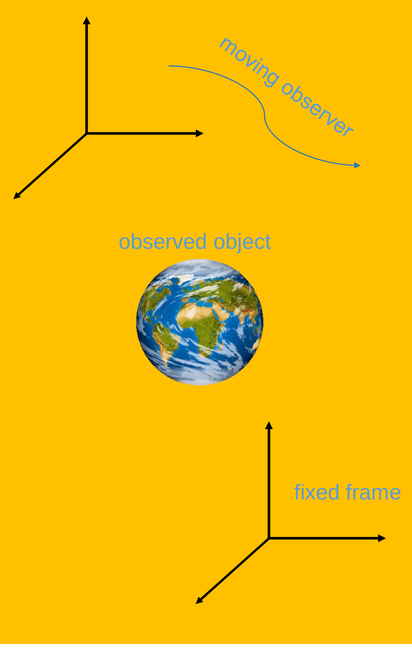
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1D Analog



- Maxwell element - spring and dashpot in series
- Forces equal: $F = F_S = F_D$, Deformation adds up: $\Delta = \Delta_S + \Delta_D$
- Constitutive relation: $\frac{\dot{\boldsymbol{\sigma}}}{E} + \frac{\boldsymbol{\sigma}}{\eta} = 2\mathbb{D}$, $\mathbb{D}$... Strain rate
- Physical equations must be written in terms of geometrical objects - i.e. scalars and co/contra-variant tensors of first or higher order.
- Objectivity - material time derivative of a tensor is geometrically not a tensor! ($\dot{\mathbb{A}}^* \neq \mathbb{Q}\dot{\mathbb{A}}\mathbb{Q}^t$)
- Only special (objective) stress rate may thus be chosen in the constitutive law

<- model derivation ->



3D Analog

Geodynamical texts often use $\boldsymbol{\sigma}_{ij} = 2\eta\dot{\varepsilon}_{ij}$ for viscous rheology - that is incorrect. There are four possible strain tensors: eulerian/lagrangian, full/linearized

$$\varepsilon^{\mathrm{lin}} := \frac{1}{2}\left(\nabla u + (\nabla u)^{\mathrm{T}}\right); \qquad \xi^{\mathrm{lin}} := \frac{1}{2}\left(\mathrm{Grad}U + (\mathrm{Grad}U)^{\mathrm{T}}\right)$$
$$\varepsilon := \frac{1}{2}(\mathbb{I} - \mathbb{F}^{-\mathrm{T}}\mathbb{F}^{-1}); \qquad \xi := \frac{1}{2}\left(\mathbb{F}^{\mathrm{T}}\mathbb{F} - \mathbb{I}\right)$$

which have the following material time derivatives (neither one is equal to the strain rate - which is what originally figures in the definition of viscous rheology)

$$\dot{\varepsilon}^{\mathrm{lin}} = \mathbb{D} - \frac{1}{2}((\nabla u)\mathbb{L} + \mathbb{L}^{\mathrm{T}}(\nabla u)^{\mathrm{T}}) \qquad \dot{\varepsilon} = \mathbb{D} - (\varepsilon\mathbb{L} + \mathbb{L}^{\mathrm{T}}\varepsilon)$$
$$\dot{\xi}^{\mathrm{lin}} = \mathbb{D} - \frac{1}{2}((\nabla u)^{\mathrm{T}}\mathbb{L}^{\mathrm{T}} + \mathbb{L}(\nabla u)) \qquad \dot{\xi} = \mathbb{F}^{\mathrm{T}}\,\mathbb{D}\,\mathbb{F}$$

Maxwell rheology is based on the following idea: Elastic and viscous deformations add together, but stress is the same for both elastic and viscous parts of the deformation

$$\tau = -p + \boldsymbol{\sigma}; \quad \mathbb{D} = \mathbb{D}^{\mathrm{vis}} + \mathbb{D}^{\mathrm{el}}$$
$$\boldsymbol{\sigma} = 2\eta\mathbb{D}^{\mathrm{vis}} = 2G\varepsilon^{\mathrm{el}}$$

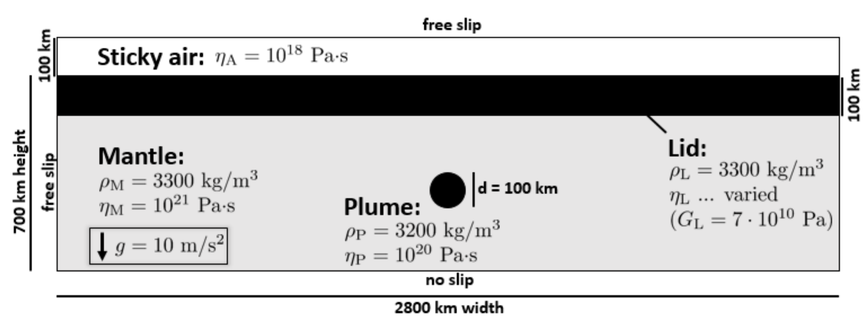
The choice of ε or ξ or $\varepsilon^{\mathrm{lin}}$ or ξ^{lin} gives us the relation $\varepsilon^{\mathrm{el}} \to \mathbb{D}^{\mathrm{el}}$ and thus specifies the rate used in our rheology (only for ε we get an objective rate - lower convected one). In this way the stress rate is directly an outcome of the method of derivation of the constitutive equation (contrary to 1D analogs)

Maxwellian viscoelasticity in StagYY

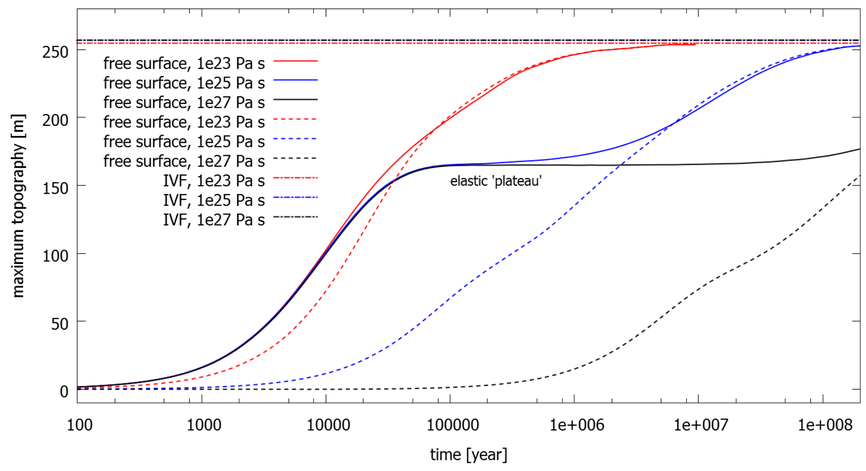
$$\boldsymbol{\sigma} + t_M \overset{\nabla}{\boldsymbol{\sigma}} = 2\eta\mathbb{D}, \qquad \overset{\nabla}{\boldsymbol{\sigma}} := \frac{\partial\boldsymbol{\sigma}}{\partial t} + \vec{v}\cdot\nabla\boldsymbol{\sigma} + (\boldsymbol{\sigma}\mathbb{W} - \mathbb{W}\boldsymbol{\sigma})$$

$\boldsymbol{\sigma}$ ... Cauchy stress, t_M ... Maxwell relaxation time, $\mathbb{D}$ ... Strain rate

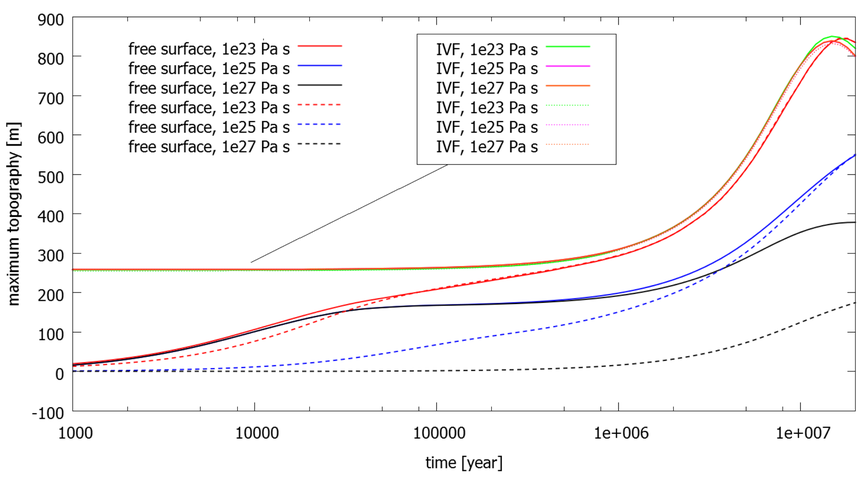
Results: case study



Topography: cylinder is fixed in space

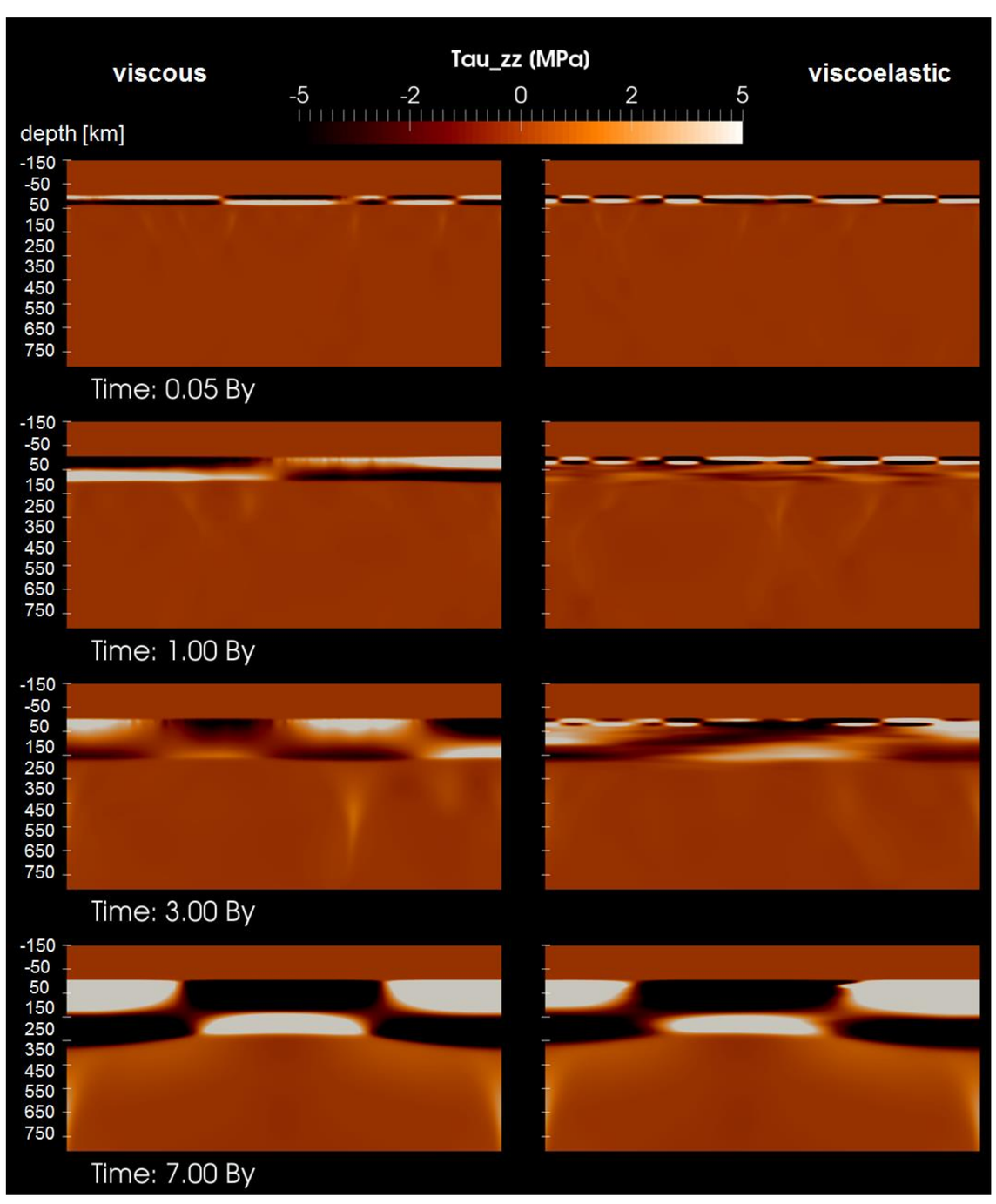


Topography: cylinder is advected with the flow



Viscoelasticity (solid lines) reduces the discrepancy between free surface and traction derived topographies, which occurs for high viscosity lids. Results from a test case by Crameri et al. 2012, extended for high viscosity, viscoelastic lids. Dashed lines show the viscous solution.

Stress memory in mantle convection



The figure shows the evolution of vertical deviatoric stress in a model with a thickening lithosphere. Viscoelastic models can remember the deformation which is taking place in the lithosphere during its formation. As a consequence, the lithosphere in the right column preserves the stress history for times comparable to the Maxwell relaxation times.