

Three-dimensional finite difference method and hybrid modeling of earthquake ground motion

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Received 28 November 2000; revised 28 September 2001; accepted 2 October 2001; published 16 August 2002.

[1] A new two-step hybrid technique (hybrid) has been developed to estimate the earthquake ground motion in three-dimensional (3-D) local models excited by external sources. The main use of the technique is fast evaluation of site effects and seismic hazard because our technique needs less computer memory and time than all-in-one source-path-site computational methods. The first step of the hybrid requires an arbitrary 3-D method (e.g., finite difference (FD), discrete wave number, ray, or analytical solution) to compute three-component excitation on a grid box surrounding a site of interest. A particular excitation contains information on the source and path effects and is saved on disk as a time history. Depending on the method used, the first step may also contain nonplanar topography. The second step (3-D FD) employs local structure and topography inside the excitation box and a procedure that exactly couples the first step results with the second step. The speedup of the hybrid method is made possible through the computation of the excitation (saved on disk) in the first step and the use of FD on irregular grids in the second step of the hybrid. The second step computational model can be just a small spatial fraction of the original source-path-site model. A practical parameter study example of a point double-couple source and regional topography model (applied in the first step) with a local low-velocity basin structure and ridge (both applied in the second step only) is presented. **INDEX TERMS:** 0902 Exploration Geophysics: Computational methods, seismic; 3210 Mathematical Geophysics: Modeling; 7212 Seismology: Earthquake ground motions and engineering; 7223 Seismology: Seismic hazard assessment and prediction; **KEYWORDS:** earthquake ground motion, seismic waves, hybrid modeling 3-D finite differences, site effects, parameter studies

1. Introduction

[2] Three-dimensional (3-D) computation of earthquake ground motion is an important task in mitigating seismic hazard. However, including source, path, and site effects into all-in-one methods leads to extreme needs of computer memory and time. An effective bypass is combining appropriate techniques for particular problems (e.g., source modeling, crustal propagation modeling, and local geology and topography modeling) in hybrid methods. Hereinafter the following abbreviations are used: FD, finite difference; DW, discrete wave number (or FK); FE, finite element; BEM, boundary element method.

[3] There are various approaches called “hybrid methods” in seismic modeling. *Kamae et al.* [1998] compute strong ground motions using Green’s functions obtained by FD for frequencies up to 1 Hz and by stochastic modeling (of the same source) for frequencies above 1 Hz. The hybrid Green’s function is retrieved by adding these two Green’s

functions for particular frequency bands. *Faeh and Suhadolc* [1994] use a 2-D hybrid approach combining modal summation and the finite difference technique to treat the combined effects of the source, path, and site response. *Zhang et al.* [1998] presented a 2.5-D elastodynamic scattering formulation that evaluates the 3-D response of 2-D scatterers, including topography. *Wen and Helmberger* [1998] developed a 2-D *P-SV* hybrid method to model localized structures near the core-mantle boundary, involving 2-D structures. They combine FD at the heterogeneous region and generalized ray theory solutions from a seismic source. After interacting with the heterogeneous structures the ground motions are propagated back to the Earth’s surface analytically by Kirchhoff’s method. Coupling BEM and FE to model 2-D *SH* scattering of surface waves in laterally heterogeneous multilayered media was presented by *Fujiwara* [1996]. Similarly, *Moczo et al.* [1997] presented 2-D computation of *P-SV* seismic motion at inhomogeneous viscoelastic topographic structures by a combination of methods: 1-D DW (source and path), 2-D FD (regional computation), and 2-D FE (topographic surface of the model). Their method was an extension of the hybrid 2-D DW-FD method of *Zahradnik and Moczo* [1996] for localized structures with a flat free surface embedded in a 1-D background medium. It combined DW

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for source modeling and path effects (1-D medium) and FD for 2-D regional model computation. An inconsistency between the 3-D source radiation and wave propagation on one side and the 2-D FD calculation on the other side [Zahradnik and Moczo, 1996] was inevitable but small enough in some special cases, such as a point source placed far from the site but close to the 2-D model plane [see also Oprsal et al., 1998]. The 2-D FD hybrid of Robertsson and Chapman [2000] computes various models differing in the details of local structure (e.g., parameter studies). The original model is computed once, and the locally altered models require calculations in a subvolume and its neighborhood only. Goh and Schmidt [1996] use a hybridization of FE, boundary integrals, and DW to solve the Helmholtz equation for seismoacoustic modeling. Lecomte [1996] introduced a three-step 2-D acoustic hybrid modeling method consisting of ray tracing, local FD solution for a deep complex structure, and extrapolation of the scattered wave field toward the receivers. Artificial scattered-field reflections from the top “extrapolating” boundary of the FD domain were avoided by a special treatment. Maggio and Quateroni [1997] developed a method combining the ordinary FE for regions of high material parameters variability with spectral element methods for homogeneous or gradually changing blocks. Faeh et al. [1994] demonstrated the advantages of their 2-D hybrid approach, combining modal summation and the FD technique, to simple 1-D techniques used for seismic zonation.

[4] Caserta et al. [1999] added a deterministic hybrid 2-D DW-FD method to stochastic noise (perturbation) to take into account the effects of small-scale heterogeneities of the crust. A hybrid approach coupling ray tracing and 2-D FD method for modeling T wave propagation from an underwater source to an on-land seismic station is proposed by Piserchia et al. [1998]. Hatayama and Fujiwara [1998] use BEM with normal modes to evaluate the excitation of secondary surface waves in 3-D basin models without (e.g., body) waves that would be present in the complete 3-D excitation of the secondary surface waves. Yokoi and Sánchez-Sesma [1998] use an indirect BEM for 3-D topographic problems requiring knowledge of analytical solution for the half-space with a flat free surface. Olsen et al. [2000] evaluate viscoelastic ground motion in a two-stage technique: In the first step (coarse gridded model) they save the stress tensor values for planes around and under the site. The second step performs 3-D computations, and the stress tensor from the first step is added to present a stress tensor at the corresponding location and time. Another powerful method to be mentioned here is a more realistic way to model high-frequency source radiation and propagation by the description of stochastic components of the wave field [Rovelli et al., 1994]. Similarly, Boore [1983] employs the FD modeling to the source, path, and site effects at low frequencies, while the higher frequencies are stochastically simulated. For an analogous approach, see also Graves [1999].

[5] Our new 3-D hybrid FD technique attempts to solve the problem of full source-path-site effects in a broad frequency band to fill the gap between other approaches. This method (described in detail in section 3) is a straightforward 3-D extension of two previously developed 2-D methods: 2-D FD method on irregular rectangular grids

[Oprsal and Zahradnik, 1999a] and a hybrid DW-FD procedure [Zahradnik and Moczo, 1996].

[6] The method consists of two steps: The first step provides 3-D source and path effects. These effects enter the second step through a special coupling. Three-dimensional ground motion computation of the second step is performed on a model site with detailed topography and low-velocity domains. The inconsistency typical of the previously mentioned 1-D DW–2-D FD hybrid method is avoided thanks to full 3-D formulations. A similar two-step approach applied in the 2-D FE-FE hybrid modeling was introduced by Bielak and Christiano [1984] and Cremonini et al. [1988]. Recently, their method was extended to the 3-D FE-FE hybrid modeling (J. Bielak et al., Domain reduction method for three-dimensional earthquake modeling in localized regions, part I, Theory, submitted to *Bulletin of the Seismological Society of America*, 2001; C. Yoshimura et al., Domain reduction method for three-dimensional earthquake modeling in localized regions, part II, Verification and applications, submitted to *Bulletin of the Seismological Society of America*, 2001).

2. Three-Dimensional FD Displacement Formulation on Irregular Grids With Topography

[7] The proposed new computational scheme is a 3-D displacement formulation scheme of second-order accuracy in space and time. The formulation is heterogeneous: The discontinuities enter the scheme via material parameters only, and only one formula is used for the whole model. The same applies for the free surface: Parameters above it are zeroed (so-called vacuum formalism).

[8] This 3-D scheme is an extension of a 2-D formulation for the P - SV case [Oprsal and Zahradnik, 1999a] on irregular rectangular grids allowing topography models. An analogous 3-D scheme on regular grids was developed by Moczo et al. [1999].

2.1. Irregular Grid

[9] One of the first to employ the irregular grid in FD was Boore [1970]. He used the vertically irregular grid in a 1-D medium. Irregular grid approaches for 2-D media are, for example, from Jastram and Tessmer [1994] (various non-uniform rectangular grids in one model) and Falk et al. [1996] (P - SV , staggered, vertically and horizontally irregular), Hestholm and Ruud [1994] (mapping topography to a flat surface, rectangular grid). Three-dimensional FD simulations on irregular grids were used, for example, by McLaughlin and Day [1994] or recently by Pitarka [1999] and Eisner and Clayton [2001].

[10] Hestholm [2001] uses 3-D FD velocity-stress formulation on a half-space with topography analytically mapped to a flat surface with eighth order of accuracy inside the homogeneous block, gradually decreasing to sixth and fourth order, and ending with second order on the free surface. The reported instabilities were avoided by using nonequidistant grids in horizontal and vertical directions ($dz \geq \frac{3}{2}dx$) or by setting the density to one tenth of its interior value at grid layer 1 at the free surface. A procedure generating FD formulas for arbitrarily spaced grids was presented by Fornberg [1988] (homogeneous medium).

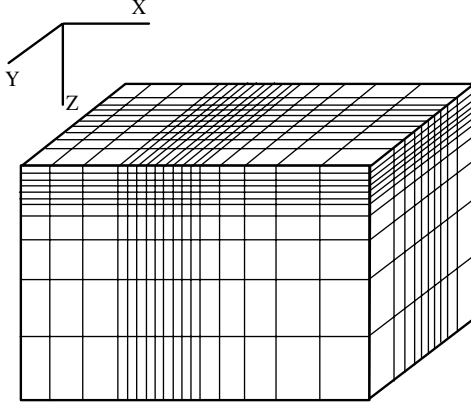


Figure 1. A schematic representation of an irregular rectangular grid. Irregularity can be defined by three 1-D vectors of DX , DY , DZ differences. Each of the vectors will contain differences and thus will define the position of relevant grid plane (e.g., $\mathbf{DX} = (DX(1), DX(2), \dots, DX(k-1))$; for the yz grid plane, k is the number of grid points in the x direction).

[11] Our irregular grid is rectangular (Figure 1) as introduced in two dimensions by Moczo [1989] (SH case, vertically irregular) and Oprsal and Zahradnik [1999a] (P - SV case, vertically and horizontally irregular). For a 3-D staggered irregular rectangular grid, see also Pitarka [1999] or Eisner and Clayton [2001]. The irregularity is defined independently in all three dimensions by 1-D vectors. Advantages of an irregular grid are twofold: Stair-case free-surface artifacts can be reduced by grid refinement, and oversampling of high-velocity regions is avoided.

2.2. Equations of Motion

[12] Seismic waves in a perfectly elastic, isotropic, heterogeneous medium may be described by second-order partial differential equations of motion. Let u , v , w denote displacement components and t time, parameters λ and μ are Lamé's elastic coefficients, ρ is the density, $\bar{f}_x$ is the x component of applied body force, etc., and $(\lambda u_x)_z = (\partial/\partial z)[\lambda(\partial u/\partial x)]$, etc. Then the displacement wave field equations read

$$\begin{aligned} &(\lambda u_x)_x + 2(\mu u_x)_x + (\mu u_y)_y + (\mu u_z)_z + (\lambda v_y)_x \\ &+ (\lambda w_z)_x + (\mu v_x)_y + (\mu w_x)_z + \bar{f}_x = \rho u_{tt} \\ &(\mu v_x)_x + (\lambda v_y)_y + 2(\mu v_y)_y + (\mu v_z)_z + (\mu u_y)_x \\ &+ (\lambda u_x)_y + (\lambda w_z)_y + (\mu w_y)_z + \bar{f}_y = \rho v_{tt} \\ &(\mu w_x)_x + (\mu w_y)_y + (\lambda w_z)_z + 2(\mu w_z)_z + (\mu u_z)_x \\ &+ (\mu v_z)_y + (\lambda u_x)_z + (\lambda v_y)_z + \bar{f}_z = \rho w_{tt}. \end{aligned} \quad (1)$$

2.3. Mixed and Nonmixed Derivative Approximations

[13] The second derivative approximations in three dimensions are analogous to those described for 2-D by Oprsal and Zahradnik [1999a]. For the mixed derivative we use only the full-form approximation from Oprsal and Zahradnik [1999a].

[14] To derive an approximation to the mixed derivative $(\partial/\partial z)[a(\partial u/\partial x)]$, approximations of the first displacement

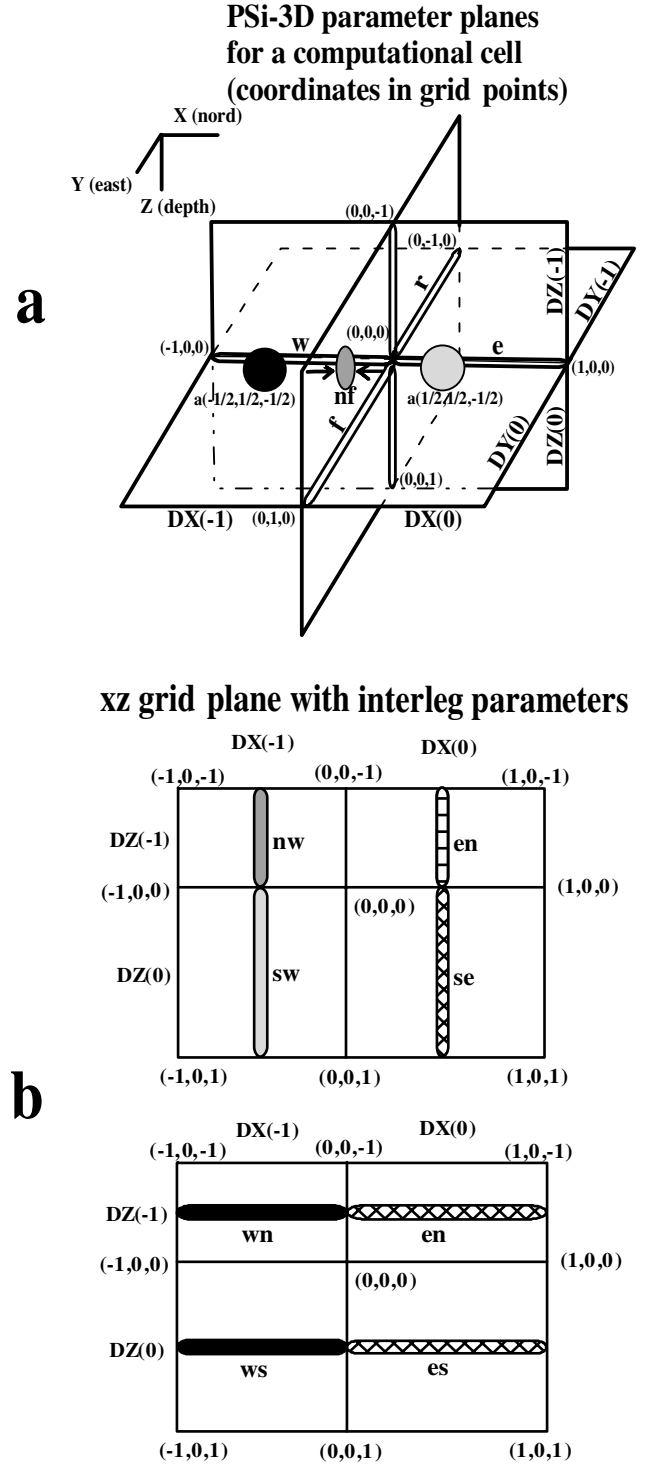


Figure 2. (a) Computational cell of $2 \times 2 \times 2$ grid points. Denomination of particular grid legs is f, r, n, s, e, w . Grid cell-centered parameter is denoted by, e.g., $a(-1/2, 1/2, -1/2)$. Coordinates are in a local rectangular coordinate system. (b) The xz grid plane part of the computational grid cell with interleg parameters $nw, sw, ne, se, wn, ws, en, es$. Irregular grid steps are $DX(i), DY(i), DZ(i); i \in \{-1, 1\}$.

derivatives and effective parameters a_{ne} , a_{se} , a_{nw} , and a_{sw} are needed at the locations between the grid lines of the xz plane (Figure 2). For material parameters used in our computation see section 2.5. This makes the scheme close to staggered schemes [Virieux, 1986] because the resulting FD formulas are equivalent to having stress values $a(\partial f/\partial x)$ between the grid lines (see, for example, equations (2) or (5)). It is, however, just a conceptual similarity since our final formulas contain only displacements, not stress. Values of either u , v , or w are represented by $f_{i,j,k}$. Indices i, j, k specify position in the computational grid cell. If

$$\frac{\partial}{\partial z} \left(a \frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial z} g,$$

then

$$\frac{\partial}{\partial z} g \doteq \frac{1}{2} \frac{\left(g_{-\frac{1}{2},0,\frac{1}{2}} + g_{\frac{1}{2},0,\frac{1}{2}} \right) - \left(g_{-\frac{1}{2},0,-\frac{1}{2}} + g_{\frac{1}{2},0,-\frac{1}{2}} \right)}{[DZ(-1) + DZ(0)]}. \quad (2)$$

After applying the mean-value theorem, the g value in point $(\frac{1}{2}, 0, \frac{1}{2})$ is

$$\begin{aligned} g_{\frac{1}{2},0,\frac{1}{2}} &\doteq \frac{a_{se}}{DX(0)} \frac{\partial f}{\partial x} \Big|_{(\frac{1}{2},0,\frac{1}{2})} [DX(0)] \\ &\doteq \frac{a_{se}}{DX(0)} (f_{1,0,\frac{1}{2}} - f_{0,0,\frac{1}{2}}). \end{aligned}$$

We again use

$$f_{1,0,\frac{1}{2}} = \frac{f_{1,0,0} + f_{1,0,1}}{2}, \quad f_{0,0,\frac{1}{2}} = \frac{f_{0,0,0} + f_{0,0,1}}{2};$$

thus

$$g_{\frac{1}{2},0,\frac{1}{2}} = \frac{a_{se}}{2DX(0)} (f_{1,0,0} + f_{1,0,1} - f_{0,0,0} - f_{0,0,1}). \quad (3)$$

The complete approximation, made up of four parts constructed similarly, is

$$\begin{aligned} \frac{\partial}{\partial z} \left(a \frac{\partial f}{\partial x} \right) &= \frac{\partial}{\partial z} g \doteq \frac{\frac{1}{2}}{DZ(-1) + DZ(0)} \\ &\cdot \left[a_{se} \frac{1}{DX(0)} (f_{1,0,0} + f_{1,0,1} - f_{0,0,0} - f_{0,0,1}) \right. \\ &+ a_{sw} \frac{1}{DX(-1)} (f_{0,0,0} + f_{0,0,1} - f_{-1,0,0} - f_{-1,0,1}) \\ &- a_{ne} \frac{1}{DX(0)} (f_{1,0,-1} + f_{1,0,0} - f_{0,0,-1} - f_{0,0,0}) \\ &\left. - a_{nw} \frac{1}{DX(-1)} (f_{0,0,-1} + f_{0,0,0} - f_{-1,0,-1} - f_{-1,0,0}) \right]. \quad (4) \end{aligned}$$

Parameters a_{se} , a_{sw} , a_{ne} , and a_{nw} are effective parameters belonging to the particular arms of the computational cell (see Figure 2). For $DZ(-1) = DZ(0)$ and $DX(-1) = DX(0)$ the equation again simplifies to the case of a regular grid

[Zahradnik, 1995]. The remaining mixed derivative approximations can be obtained by a transformation of indices in (4) (see Appendix A).

[15] For the nonmixed derivative $(\partial/\partial z) [a(\partial f/\partial z)]$ the approximation reads

$$\begin{aligned} \frac{\partial}{\partial z} \left(a \frac{\partial f}{\partial z} \right) &\doteq \frac{g_{0,0,\frac{1}{2}} - g_{0,0,-\frac{1}{2}}}{\frac{1}{2}[DZ(-1) + DZ(0)]} \\ &= 2 \frac{1}{DZ(-1) + DZ(0)} \\ &\cdot \left(a_s \frac{f_{0,0,1} - f_{0,0,0}}{DZ(0)} - a_n \frac{f_{0,0,0} - f_{0,0,-1}}{DZ(-1)} \right) \quad (5) \end{aligned}$$

with parameters a_n and a_s , shown in Figure 2. For $DZ(-1) = DZ(0)$ and $DX(-1) = DX(0)$ the equation again simplifies to the case of a regular grid [Zahradnik, 1995] (for various irregular grid FD approximations, see, for example, Láška and Hruška [1934]). The remaining nonmixed derivative approximations can be obtained by a transformation of indices in (5) (see Appendix B).

2.4. Time Integration

[16] Let the space derivatives be inserted into (1), and denote their left sides L_u , L_v , and L_w , respectively. A standard time differentiation with time step DT results in the final second-order scheme:

$$\begin{aligned} u_{0,0,0}^{M+1} &= \frac{(DT)^2}{\rho_{0,0,0}} L_u^M(u^M, v^M, w^M) + 2u_{0,0,0}^M - u_{0,0,0}^{M-1}, \\ v_{0,0,0}^{M+1} &= \frac{(DT)^2}{\rho_{0,0,0}} L_v^M(u^M, v^M, w^M) + 2v_{0,0,0}^M - v_{0,0,0}^{M-1}, \\ w_{0,0,0}^{M+1} &= \frac{(DT)^2}{\rho_{0,0,0}} L_w^M(u^M, v^M, w) + 2w_{0,0,0}^M - w_{0,0,0}^{M-1}, \end{aligned} \quad (6)$$

where M denotes the time index. Coordinates $(0, 0, 0)$ in (6) indicate positioning of the scheme in the center of the computational cell as indicated in Figure 2.

2.5. Material and Effective Parameters

[17] Let us define three steps in the creation of a computational model. First, we have a “geological model,” e.g., a series of 2-D cross sections. Second, we digitize and interpolate the geological model to obtain a “digital geological model,” a 3-D function that returns a parameter vector (e.g., $\mathbf{a} = (\lambda, \mu, \rho, Q_p, Q_s, \dots)$) for each position vector $\mathbf{X} = (x, y, z)$, x, y, z being spatial coordinates of an arbitrary point in the region of interest. Thus the digital geological model is defined as $\mathbf{a} = \mathbf{a}(\mathbf{X})$. Finally, we use $\mathbf{a}(\mathbf{X})$ to construct a “computational model,” i.e., the model used to calculate the parameters a_{se} , etc. appearing in the FD scheme. Several ways to change from the “digital model” to a computational model are described below in section 2.5.4. In the following, we denote the computational model as $a_{i,j,k}$; $i, j, k \in \{-\frac{1}{2}, \frac{1}{2}\}$, where a refers to a particular grid cell.

2.5.1. Geometric averaging

[18] Parameters a_s , a_n , a_{se} , a_{es} , a_{sw} , a_{ws} , a_{ne} , a_{en} , a_{nw} , and a_{wn} are effective parameters λ and μ of the scheme. To fulfill

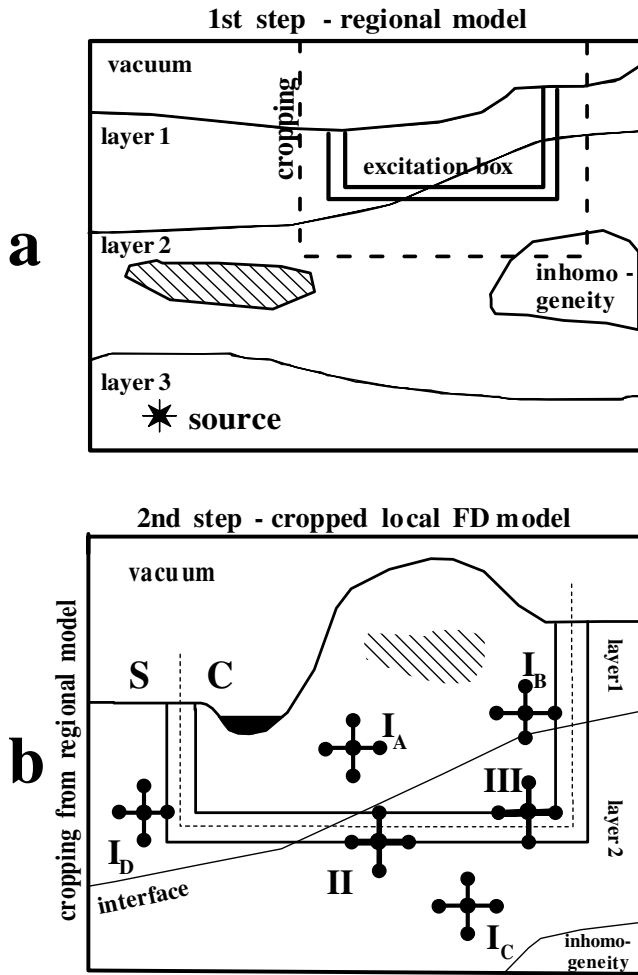


Figure 3. Hybrid method concept. (a) The first step hybrid regional model. The dashed line is the cropping edge of the computational model for the second hybrid step. (b) The second step hybrid local model. The computational model is created by cropping from a digital local geological model, new structure and topographic details inside the excitation box are added. Parts S and C correspond to areas where scattered and complete wave fields are computed, respectively. Crosses I_A, I_B, I_C, I_D, II, III are symbols of one hypothetical FD template (see section 3.1.2.1).

the boundary condition on vertical and horizontal interfaces (2-D case), those parameters should be obtained by geometric averaging. This was theoretically proven in equation (5) of Oprsal and Zahradnik [1999a] or by Zahradnik [1995]. The same applies for planar interfaces parallel to either xy or xz or yz grid planes in the 3-D case. (For the parameter legs of the finite difference computational cell, see Figure 2.)

2.5.2. Modified geometric averaging

[19] In practice, however, the parameters need further modifications. As we can see from Figure 2b, there are two directions in which effective parameters between grid legs are evaluated: a_{se} and a_{es} for the z and x directions, respectively. Theoretical justification (in terms of correct boundary conditions) of geometric averaging was found for internal interfaces that are parallel to one of the grid lines in

the 2-D case [Zahradnik, 1995] or to one of the grid planes in the 3-D case (not shown here) and for a free surface aligned with the grid lines (see section 2.6). Generally, a_{se} may differ from a_{es} for either parallel and inclined interfaces or a free surface intersecting a particular parameter leg. There are several cases when this happens.

1. If a free surface intersects one of a_{se} , a_{es} parameter legs, then such parameter is zero valued by definition (standard geometric averaging). In that case, the other one must be artificially zero valued too: $a_{se}, a_{es} \rightarrow a_{se} = a_{es} = 0$. A complete description of this necessary stabilization (for topography) is given by Oprsal and Zahradnik [1999a]. This is fully compliant with the free-surface boundary condition.

2. Models with high v_p/v_s ratio contrast (e.g., $v_p/v_s = 5$ and $v_p/v_s = 2$) may suffer a principal instability, reported and stabilized in Figure 4 of Oprsal and Zahradnik [1999b]. The stabilization is simple: Put $a_{se} = a_{es}$. In our practical computation for the 2-D case [Oprsal and Zahradnik, 1999b] we used $a_{se}, a_{es} \rightarrow \min(a_{se}, a_{es})$. This automatically also includes case 1. The modification changes the computational model slightly but only within one grid cell. Our 3-D code includes modifications 1 and 2 implicitly. Thus $a_{se} = a_{es}$ by definition. Therefore we need only one of these two parameters in the FD equations. Treatment of density is detailed in section 2.5.3.

2.5.3. Implicit modifications of geometric averaging

[20] Both aspects described above are solved by defining the original computational model parameters in the center of a grid cell, not on any grid plane. There are eight grid cells belonging to a computational scheme (Figure 2). We assume that the values of the λ , μ , ρ parameters are defined in the center of each grid cell, and we denote them as

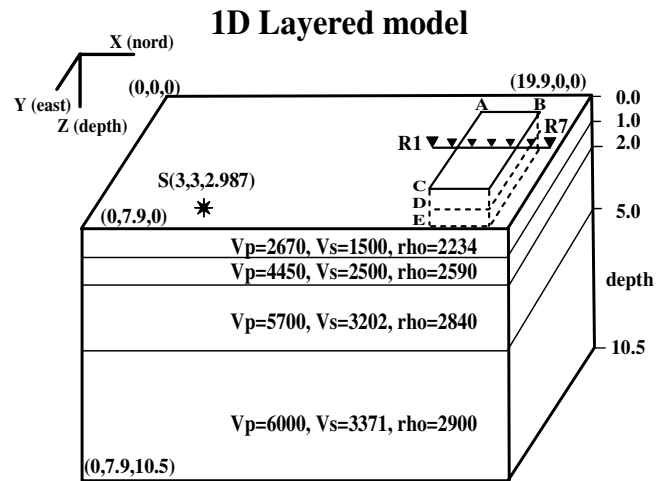


Figure 4. The 1-D layered model. Two shallow excitation boxes far from the source. The depth of the boxes is a fraction of the penetration depth of the surface waves. S is the source, R1–R7 are receivers placed on the free surface; R3–R5 are inside the excitation box. A(17.300,1.900,0), B(19.100,1.900,0), C(17.300,5.900,0), D(17.300,5.900,0.700), E(17.300,5.900,1.600), R1(16.600,3.900,0), R7(19.600,3.900,0) m. The coordinates are in km, v_p , v_s velocities are in m/s, density ρ is in kg/m³.

$a_{i,j,k}; i, j, k \in \{-\frac{1}{2}, \frac{1}{2}\}$. Then for λ and μ interleg parameters we define

$$a_{se} = a_{es} = a_{\frac{1}{2},0,\frac{1}{2}} = \frac{1}{2} \sum_{j \in \{-\frac{1}{2}, \frac{1}{2}\}} a_{\frac{1}{2},j,\frac{1}{2}}; \quad (7)$$

similarly, we evaluate the parameter

$$\begin{aligned} a_s &= a_{0,0,\frac{1}{2}} = \frac{1}{2} (a_{se} + a_{sw}) = \frac{1}{2} (a_{fs} + a_{rs}) \\ &= \frac{1}{4} \sum_{i,j \in \{-\frac{1}{2}, \frac{1}{2}\}} a_{i,j,\frac{1}{2}}. \end{aligned}$$

Analogous evaluation is performed for the remaining λ and μ parameters. For example, $a_{nf} = a_{fn} = \frac{1}{2} (a_{-\frac{1}{2},\frac{1}{2},-\frac{1}{2}} + a_{\frac{1}{2},\frac{1}{2},-\frac{1}{2}})$ (Figure 2a). Since the scheme uses ρ in point $(0, 0, 0)$, we compute its value:

$$\rho_{0,0,0} = \frac{1}{8} \sum_{i,j,k \in \{-\frac{1}{2}, \frac{1}{2}\}} \rho_{i,j,k}. \quad (9)$$

2.5.4. Creation of the computational model

[21] The computational grid cell-centered parameters (e.g., $a_{\frac{1}{2},\frac{1}{2},\frac{1}{2}}$) that are stored in RAM are taken from a prescribed digital geological model. If just point values of the digital geological model parameters are used to create the computational model (all numerical examples in this paper), then the curved interfaces deform into grid lines and become staircase shaped, including the free surface. Note that the free surface is always staircase shaped in our method (see sections 2.5.1 and 2.5.2, case 1). For a point above the free surface, $a_{\frac{1}{2},\frac{1}{2},-\frac{1}{2}} = 0$, and for a point under it, $a_{\frac{1}{2},\frac{1}{2},\frac{1}{2}} = A$. The digital geology free surface may be at any depth $z \in (-\frac{1}{2}, \frac{1}{2})$, but the computational model is shifted into $z = 0$. Parameters on this (shifted) free surface are $a_{\frac{1}{2},0,\frac{1}{2}} = A/2$ (equation (7)).

[22] Following the vacuum formalism, the values of λ and μ are zeroed above the free surface. The densities are not exactly zeroed but are formally set to very small values to permit computation of equations (6) in the whole model, vacuum included.

[23] The above treatment satisfies the traction-free condition on a generally nonplanar free surface. It is equivalent to the conditions described by *Oprsal and Zahradnik* [1999a] (generalized for three dimensions). Let $\bar{\rho}$ be the density below the surface. Then, for the grid points at the planar (horizontal or vertical) parts of the surface $\rho_{0,0,0} = \bar{\rho}/2$, while for 2-D outer and inner corners, $\rho_{0,0,0} = \frac{1}{4}\bar{\rho}$, $\rho_{0,0,0} = \frac{3}{4}\bar{\rho}$, respectively; for 3-D outer and inner corners, $\rho_{0,0,0} = \frac{1}{8}\bar{\rho}$, $\rho_{0,0,0} = \frac{7}{8}\bar{\rho}$, respectively. The legs being a part of a planar free surface must have their effective parameters halved with respect to values inside, legs being a part of a 2-D corner at free surface must be quartered. The vacuum formalism for centered grid cell point values used with (7), (8), and (9) satisfies the treatment described above. These rules follow from the requirement that the FD scheme automatically approximates the free-surface condition.

[24] There are various sophisticated ways to smooth the internal staircase-shaped interfaces by evaluating computational model parameters from a digital geology model.

These methods are beyond the scope of this article, but as an example, we mention the procedure introduced by *Muir et al.* [1992] for the case of an inhomogeneous isotropic medium with similar elastic parameters. *Muir et al.* [1992] evaluate average slownesses p_p, p_s for each grid cell from a certain number of points evenly spread across a particular grid cell (e.g., $3 \times 3 \times 3$ points). The average density may be retrieved as an arithmetic average of densities at the same points, or we can just keep its point values. Consequently, parameters λ and μ are retrieved. The free surface stays staircase shaped. Averaging slownesses is a way to keep the propagation time equal to that in the original model. This is effective also in the case of a few low-velocity layers appearing within one grid step. For a more general case with very divergent elastic parameters where averaging P and S properties independently is not possible, see *Muir et al.* [1992].

2.5.5. Memory requirements

[25] If the intercell parameters are stored in the memory, then three of the material parameter values (λ, μ, ρ) are needed for every spatial grid cell. In this case a model of $187 \times 177 \times 78$ grid points occupies just 177 MB of RAM.

2.6. Consistency With Free-Surface Boundary Condition

[26] Analogous to the consistency analysis in two dimensions [*Zahradnik*, 1995; *Oprsal and Zahradnik*, 1999a], the behavior of our scheme for a typical model surface can be investigated by inserting a 3-D Taylor expansion in spatial coordinates into the FD approximations in section 2.3 and substituting into equations (6). A temporal expansion is also carried out. For simplicity, let us assume a regular grid with $DX(i) = DY(i) = DZ(i) = h$ and a homogeneous medium with elastic parameters λ_0, μ_0 and density ρ_0 . Point $(0, 0, 0)$ of the computational grid cell (Figure 2) is placed on the horizontal planar free surface. Then (after using (7), (8), and (9)) we have values of the parameters entering the FD approximations in section 2.3: $\mu_n = \mu_{en} = \mu_{wn} = \mu_{fn} = \mu_{rn} = 0$, $\mu_s = \mu_{es} = \mu_{ws} = \mu_{fs} = \mu_{rs} = \mu_0$, $\mu_{ew} = \mu_{es} = \mu_{ef} = \mu_{wf} = \mu_0/2$. The same applies for λ , density is $\rho = \rho_0/2$. After using the Taylor expansion of $\mathbf{u} = (u, v, w)$ about point $(0, 0, 0)$ we get (corresponding to equations (6)) a correct free-surface boundary condition:

$$\begin{aligned} 0 + o(t^2) &= \mu(u_z + w_x) + o(h), \\ 0 + o(t^2) &= \mu(v_z + w_y) + o(h), \\ 0 + o(t^2) &= (\lambda + 2\mu)w_z + \lambda(u_x + v_y) + o(h), \end{aligned} \quad (10)$$

where $u_z = \partial u / \partial z$, for example. Equations (10) show that our second-order scheme (in the vacuum formalism) automatically turns into the first-order approximations to the free-surface condition. Similar results are found for other surfaces parallel to the grid planes or corners (not shown here).

3. FD Hybrid in Three Dimensions

[27] Our hybrid modeling procedure uses the method of *Alterman and Karal* [1968] originally developed for the representation of seismic sources. In 2-D hybrid FD modeling (P - SV case), this procedure was introduced by *Zahrad-*

nik and Moczo [1996] for a planar free surface and by Caserta et al. [1999] for topography models. A 3-D version of the hybrid is developed in the present paper.

3.1. Principle of Hybrid FD Method

[28] The method consists of two steps. The site of interest is entirely enclosed within a box called the excitation box. It consists of a set of parallel inner and outer planes. Figure 3 shows a 2-D section of such a situation in detail.

3.1.1. First step

[29] In the first step, displacements on the excitation box planes are computed by any method (FD, ray, FE, DW, or analytically) by propagation from the source through a general first step model. Let this displacement wave field be called the background field ($U_b = (u, v, w)_b$). The box may contain any topography or local structures, but local low-velocity structures and detailed topography inside the box are usually not present in this step. Time history of displacements on the excitation box is stored on disk for the second step.

3.1.2. Second step

[30] The second step is performed by 3-D FD in a model with topography and local (low velocity) structures. The computational model may be just a very small detail cut out of the first step model (Figure 3a). The necessary rule of thumb is to keep the new cropped model unchanged with respect to the first step one, except for the medium inside the excitation box, including the excitation box planes (Figure 3). The only change allowed outside the excitation box is cropping the second step model to reduce its size, and that is the goal of the method. Thus this cropping is an approximation that neglects interaction of the scattered field with cropped parts of the model such as deeper layers or distant inhomogeneities (Figure 3). The sources of the first hybrid step are not present at this stage and are represented by the excitation (obtained at the first step) incorporated into the second step FD template by so-called hybrid coupling.

3.1.2.1. Hybrid coupling

[31] The whole second step model is evaluated with one FD approximation. The dotted line of Figure 3b shows a symbolic division of the computational FD area into two parts, C and S. Wave fields computed in areas C and S are complete ($U_c = (u, v, w)_c$) and scattered ($U_s = (u, v, w)_s$) wave fields, respectively. Evaluation of the second-order FD scheme branches into three possibilities.

1. All values used in the computational cell belong to either the S or C area so that no grid point of the computational cell lies on the excitation grid plane of other areas as depicted by I_A , I_B , I_C or I_D in Figure 3b. Evaluation of the FD approximation proceeds as in a standard FD computation. $U_c = (u, v, w)_c$, $U_s = (u, v, w)_s$ wave fields are computed.

2. The center of the computational cell lies in the S area (outer excitation grid plane) and one or more stencil grid points lie in the C area (inner excitation grid plane), as depicted by II in Figure 3b. Evaluation of the approximation proceeds as in a standard FD computation, but corresponding U_b values (saved on disk in the first step) of grid points lying on the inner excitation grid plane are subtracted from U_c values (current computational memory):

$$U_s = U_c - U_b. \quad (11)$$

Thus all of the input values entering the FD approximation are U_s ; the scattered wave field (U_s) is evaluated by the FD template.

3. The center of the computational cell lies in the C area (inner excitation grid plane) and one or more stencil grid points lie in the S area (outer excitation grid plane), as depicted by III in Figure 3b. Evaluation of the approximation proceeds as in a standard FD computation, but corresponding U_b values (saved on disk in the first step) of grid points lying on the outer excitation grid plane are added to the U_s values (current computational memory):

$$U_c = U_s + U_b. \quad (12)$$

Thus all of the input values entering the FD approximation are U_c ; the complete wave field (U_c) is evaluated by the FD template.

3.1.2.2. General hybrid coupling

[32] A general principle of the hybrid coupling is to keep the discontinuity between the scattered and complete wave fields on the excitation boundary. This difference (needed just on the excitation boundary) is equal to the background wave field computed in the first hybrid step.

3.2. Replication Test

[33] The excitation box with hybrid coupling represents a boundary condition in space and time for all signals coming to the box from outside. Coupling assures transparency for the scattered field and at, the same time, represents all outer sources from the first step of the hybrid computation.

[34] Let us suppose the presence of no sources except for the excitation in the second step, and let displacement at the first two time levels be

$$U_{(c,s)}^0 = U_{(c,s)}^1 = 0 \quad (13)$$

in the whole model and let there be no change between the first and second step models (except for cropping as described in section 3.1.2). M denotes present time level. When computing along the excitation planes as described in section 3.1.2.1, possibilities 2 and 3, we get from (11) and (12) the neighboring values

$$U_s^M = U_c^M - U_b^M = 0 \quad (14)$$

$$U_c^M = U_s^M + U_b^M = U_b^M, \quad (15)$$

respectively. Considering (6), (13), (14), and (15) and the case described in section 3.1.2.1, possibility 1, we get

$$U_c^M = U_b^M, \quad U_s^M = 0 \quad (16)$$

for complete and scattered field areas, respectively, at time level $M \geq 3$ ($M \geq 0$).

[35] Because of the known U_b^M (from the first step of the hybrid method), the procedure given by (14) (inner excitation grid plane) and (15) (outer excitation grid plane) is repeated with the same results (U_b^M and 0, respectively) for the whole time history. This means that the second step of the hybrid method, computed for the (cropped) model from

the first step of the hybrid (with corresponding excitation U_b^M), results in a zero-value scattered wave field outside the box ($U_s = 0$) and that the time history of the complete wave field inside the box replicates the first step solution ($U_c = U_b$). This is called replication test. Numerically, $U_s \doteq 0$, $U_c \doteq U_b$, with relative difference of amplitudes typically $< 10^{-7}$ for the same grid points (on the excitation box), time step, and method in both hybrid computations (here FD-FD hybrid).

[36] Results of the replication test check the consistency between first and second steps. Coordinate systems, time and space interpolation (section 3.6), coupling, representation of models, and excitation are tested. Because $U_s = 0$ in a properly computed replication test, there is no need for nonreflecting boundaries (section 3.5) in this test. The same applies for cropping or changing of the second step model in the S area (Figure 3).

[37] Note that a replication test for a topography model performed on a finer grid than the first step is, in fact, only partially justified since the staircase free-surface approximation slightly modifies the topography, and therefore this is not an exact replication test. To avoid this, it is possible to prescribe the coarse grid topography approximation for the fine grid model in the second hybrid step making the replication test fully valid.

3.3. Scattered Field

[38] After a valid replication test a change of the model inside the excitation box can be made. Wave field U_c corresponds to the source, path, and new site structure effects, and U_s corresponds to scattering of U_b by the changed structure.

[39] It is not usual to present U_s wave field together with U_c as we do in this paper; however, it is necessary for checking the validity of the replication test and for showing how the structure changes (from first to second step) affect the incoming wave field. If we also compute U_b outside the excitation box and construct $U_c = U_s + U_b$, we get a standard representation (U_c everywhere). Another way to obtain U_c is to change the excitation box to cover the area of interest. At the same time we would lose the possibility of detailed insight into the wave phenomena.

3.4. Source Modeling

[40] Our new 3-D code (see section 2) can perform both steps of the hybrid technique (i.e., FD-FD hybrid). In the case of FD used for the first step we use the source descriptions by means of equivalent single, dipole, and double-couple body forces [Aboudi, 1971; Aki and Richards, 1980]. The FD “point” sources may be extended to finite extent sources by description of the body forces distributed in space and time.

3.5. Nonreflecting Boundaries

[41] Nonreflecting boundaries of *Emeryman and Stephen* [1983] are always employed at model boundaries. Tapers by *Cerjan et al.* [1985] complement the nonreflecting boundaries when needed.

3.6. Interpolation

[42] The points of the excitation box in the first step are usually much coarser in space and time than those of the

second step. Saving coarse excitation data requires less disk memory. Therefore a quadruple-linear (3-D space and time) interpolation is used to obtain the appropriate U_b time marching history.

[43] A good measure of the quality of the interpolation may be obtained by performing the replication test on different grids in the first and second steps, respectively. The following is an example for a homogeneous model. The size of the model was approximately $2\Lambda_p$ (Λ_p being the prevailing S wavelength), and the receivers were placed on a vertical seismic profile in the vicinity of the center of the model inside the excitation box. FD-FD hybrid with double-couple point source was used.

[44] Relative error (RE) is defined as a relative change in the corresponding local extremes of displacement obtained in the first and second steps, respectively. RE is typically < 0.002 for pure time interpolation, with first and second step excitation points identical and time step halved (keeping the time step unchanged gives $RE \approx 10^{-7}$, as described in section 3.2). Typical RE were 0.01 if the excitation box grid planes of the first and second steps were identical (while the grid points did not necessarily have to be identical).

[45] For two grids, the coarse one with dx_c, dy_c, dz_c grid steps, dt_c time step and the fine one $dx_f = dx_c/2, dy_f = dy_c/2, dz_f = dz_c/2, dt_f = dt_c/2$, the RE was typically 0.02 (0.01–0.03). A systematic time shift $\delta t \in < 0, dt_f >$ for entire seismograms appears to be due to the simplicity of the linear interpolation. The shift was zero ($\delta t = 0$) in the case of the inside planes of the first step excitation box, coinciding with the inner grid planes of the second step excitation box.

4. Hybrid of Higher Order

[46] Higher-order FD schemes may be used similarly to the way described in section 3. The same holds for hybrid coupling. Let us suppose we have a fourth-order FD scheme. Similarly, as in Figure 3, the excitation box consists of two inner and two outer excitation (sub)boxes consisting of planes belonging to the C and S areas, respectively. Time histories $U(u, v, w)$ are saved on disk in the first step. The second step is the same as in section 3.1.2, and it uses coupling described in section 3.1.2.1.

5. Validation Experiment

[47] This FD method on regular grids compared to DW method is presented by *Moczo et al.* [1999, Figures 1–4] for $v_p/v_s \in 1, 2, 4$. The results were very good for $v_p/v_s = 2$ and $v_p/v_s = 3$, while a phase shift and amplitude disagreement in late phases of surface waves were found for $v_p/v_s = 4$ [*Moczo et al.*, [1999, Figure 4]]. Tests of our 3-D method on regular versus irregular grids, including abrupt changes in meshing, showed very good agreement similar to the 2-D case [*Oprsal and Zahradnik*, 1999a].

6. Numerical Examples

[48] Rather simple models presented in this section may be computed in a single shot by appropriate methods, as FD, DW, or the ray method. However, the hybrid method (FD-FD) is used to show its particular features as cou-

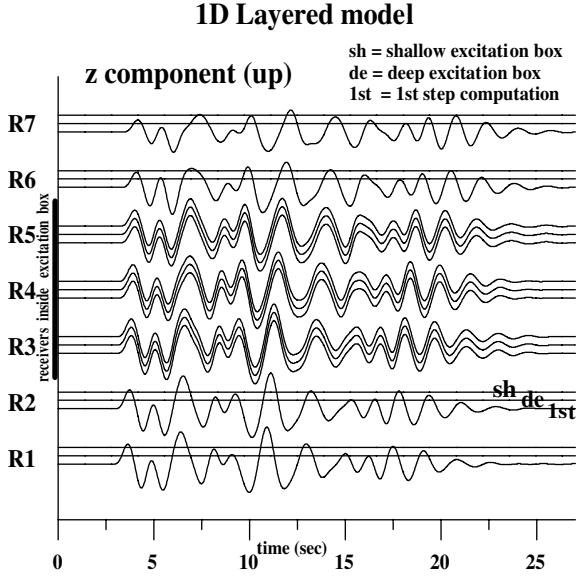


Figure 5. Replication test, comparison of the solution for the 1-D layered model at receivers R1–R7; sh, de, and 1st stand for shallow box, deep box replication test, and first step solution, respectively. The agreement is excellent for all three traces and receivers R3–R5 inside the excitation box. Traces sh and de have zero values for receivers outside the excitation box.

pling, replication test, scattered field propagation, and independence of the second step results on the first step model. All of the presented examples use point values of λ , μ , ρ to define the computational model parameters (see section 2.5.4).

6.1. The 1-D Layered Model With Flat Surface

[49] The model is depicted in Figure 4. The model consists of three layers over a homogeneous half-space. The grid is irregular in the z direction with grid steps $dx = 100$ m, $dy = 100$ m, $dz|_{z=2200\text{ m}} = 100$ m, $dz|_{z=2346.1\text{ m}} = 146.1$ m, $dz|_{z=10458\text{ m}} = 213.5$ m. The source is a horizontal point force f_x in the north direction with time history:

$$\bar{f}_x|_{t \in (0, T)} = \frac{1}{q} \left[\sin\left(2\pi \frac{t}{T}\right) - \frac{1}{2} \sin\left(4\pi \frac{t}{T}\right) \right], \quad (17)$$

$$\bar{f}_x|_{t \notin (0, T)} = 0,$$

where the normalization constant is $q = 1.287231$ and duration $T = 2.07$ s, $T[\text{s}] = 3.1/\text{highest frequency} [\text{Hz}]$, where highest frequency is the frequency above which the amplitude spectrum is $<1\%$ of the maximum spectral value and t is time in seconds.

[50] Two excitation boxes were tested in this experiment. Those are rectangular parallelepipeds, whose horizontal extent and position are identical, defined by points A, B, C in Figure 4. Their depth (700 and 1600 m) is depicted by points D and E for the shallow and deep boxes, respectively. The shallow box lies entirely in the uppermost layer, while the deep box extends approximately to the midlevel of the second layer. Thus most of the layers for the deep

box case, and all layers for the shallow box case, do not exist in the second step model. Receivers R1–R7 are placed on the planar free surface with constant spacing of 500 m.

[51] The hybrid method is FD-FD. Figure 5 shows comparisons between the straight first step solution and two replication tests with the shallow and deep excitation boxes. The maximum value of displacement is $U(u, v, w)_{\max} = w_{\max} = +3.623 \times 10^{-6}$ m. All three solutions are almost identical (the relative difference in amplitudes between each two solutions is $<10^{-6}$) for the whole time history at receivers inside the box (R3–R5). The replication test results are identical for deep and shallow boxes, with displacements equal to zero outside the excitation box in both tests, as desired.

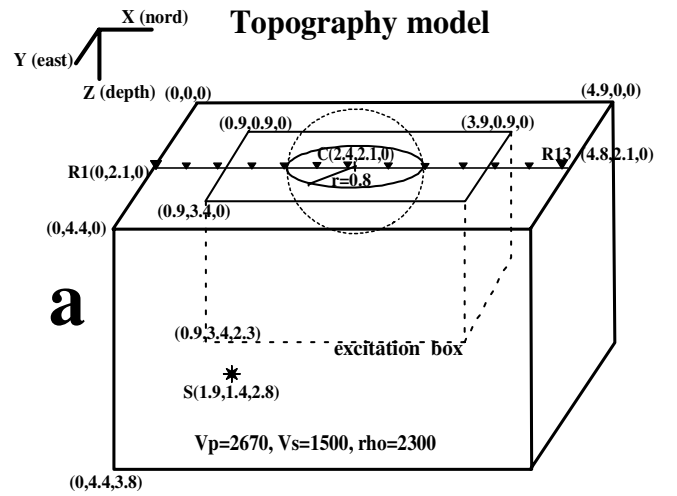
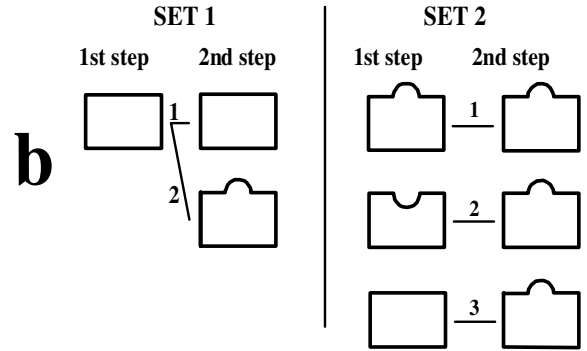


Figure 6. Homogeneous topography models over a half-space. (a) Three models represented by flat surface topography, hemispheric hill topography, and valley topography. S is the source, receivers R1–R7 are placed on the flat surface, the hemisphere has center C, and diameter r . The coordinates are in km, v_p , v_s velocities are in m/s, and density ρ is in kg/m^3 . (b) Set 1 represents a replication test for the flat surface model 1 and a hybrid computation with hill topo in the second step of hybrid 2. Set 2 represents three different excitations computed for three corresponding first step models: 1, hill topo; 2, valley topo; 3, flat topo. These excitations are used in the second step with hill topography (identical for 1, 2, 3 in set 2).

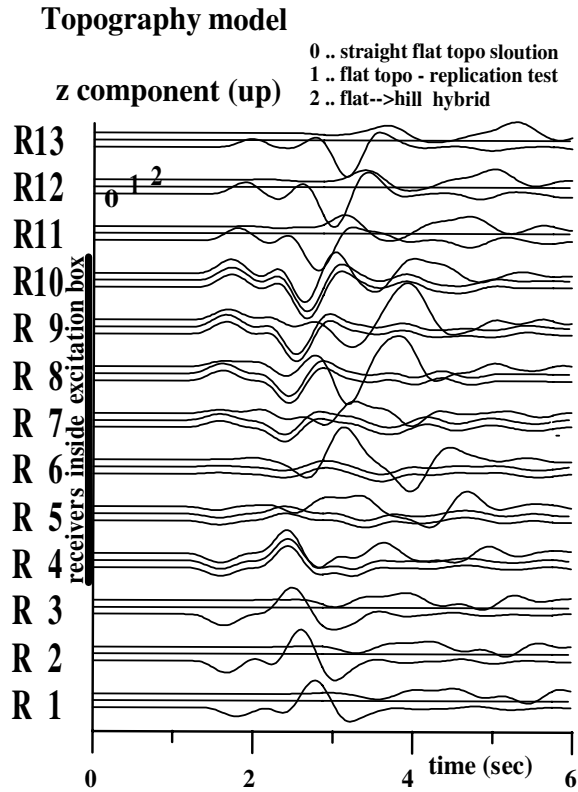


Figure 7. Set 1 of Figure 6b. Straight solution for flat surface, replication test, and hybrid computation with flat surface (first step), hill topography (second step); compared in traces 0, 1, and 2, respectively (see section 6.2.1).

[52] This agreement holds not only for P and S waves but also for surface waves arriving at 11 s. The wave field complexity at these epicentral distances (13–17 km), seen from comparison of the source duration $T = 2.07$ s and the 23-s duration of the synthetics, is mainly due to multiple reflections and interference in the surface layers. The surface waves with prevailing period of 1.9 s have wavelengths of 2850–6400 m. This means that the depths of both deep and shallow boxes are just fractions of the penetration depth and that most of the layers for the deep box case, and all of the layers for the shallow box case, do not exist in the second step model. Nevertheless, the coupling algorithm is able to fully represent such source and path effects in every detail. Independence of the results on the depth of the box numerically validates that the coupling described in section 3.1.2.1 is a good representation of boundary conditions in space and time.

6.2. Topography Model

[53] There are three topography models over a homogeneous half-space ($v_p = 2670$ m/s, $v_s = 1500$ m/s, $\rho = 2300$ kg/m³) used in the following computations: flat surface, hemispheric hill, and hemispheric valley. The hill and valley shapes are defined by a sphere with center C and radius $r = 800$ m (Figure 6a). The “hill” has the same material parameters as the underlying half-space, the “valley” is empty. All changes of the model topography are inside the excitation box area. The grid is regular with constant grid steps $dx = dy = dz = 100$ m. The source is a horizontal point

force $\bar{f}_x$ in the north direction, its time history is described in (17), with $T = 2.07$ s. This computation incorporates two sets of FD-FD hybrids as shown in Figure 6b.

6.2.1. Set 1 hybrid results

[54] We computed a flat surface model and performed the replication test, and finally, we evaluated hybrid results for the hill model. The match between the straight solution and the replication test is evident for traces 0 and 1 in each receiver triplet of Figure 7. The agreement is perfect inside the excitation box, and the replication test traces have zero values outside the box, as it should be for good coupling (see section 3.2). Then we attempt a change from planar surface to a hemispheric hill in the second step. The results (trace 2) are different from those for a flat free surface (traces 0 and 1) everywhere (Figure 7). A strong site response is evident for receivers R4–R10 in the complete wave field, ~ 2 –3 s after the beginning of excitation. A nonzero scattered wave field is apparent for all receivers outside the excitation box, especially for R11–R13. Notice a pronounced time shift and difference between

Topography model: 2nd step hill model

0 .. straight hill solution
1 .. hill - replication test
2 .. valley->hill hybrid
3 .. flat->hill hybrid

z component (up)

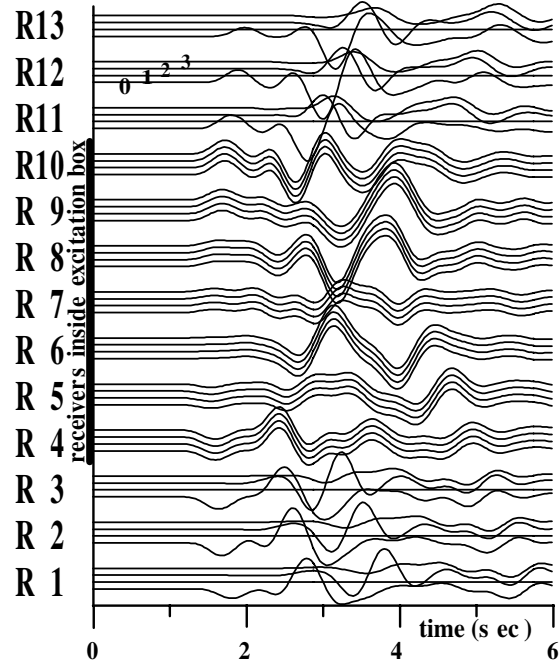


Figure 8. Set 2 of Figure 6b. Independence of first step structure inside the excitation box on the second step results. Solution for the first step of the hybrid is performed for three models which differ inside the excitation box: hill, valley, and flat topographies. Hybrid (second step) is computed three times for the hemispheric hill topography in all three different excitation cases, shown in traces 1, 2, and 3, respectively. Trace 1 is a replication test (unchanged structure for both hybrid steps). Trace 0 is first step solution for the hill model for comparison (see section 6.2.2). Trace 2 represents the same case as trace 3 of Figure 7.

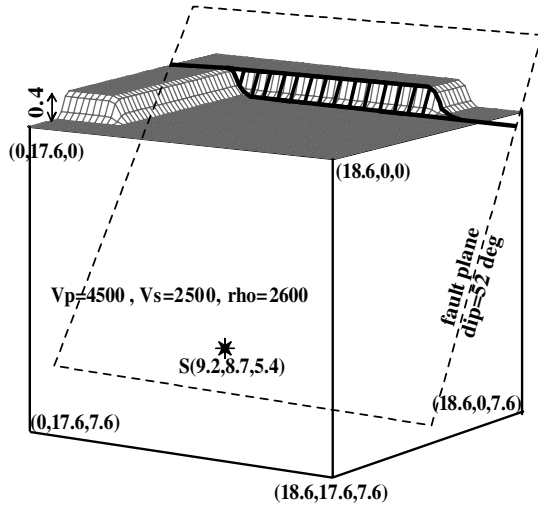


Figure 9. Basin and topography model. This model is used in the first step of the hybrid computation. Source S is on the fault plane that intersects with the flat surface at the thick line and crosshatched area. The topographic plateau is 400 m high, coordinates are in km, v_p , v_s velocities are in m/s, and density ρ is in kg/m^3 .

corresponding incoming wave and scattered wave at traces 0 and 2 of R13 caused by diffraction and reflection at the hill topography.

6.2.2. Set 2 hybrid results

[55] This set (Figure 6b) shows an important property of the hybrid method called “independence”: We computed three first step excitations for hill, flat, and valley topographies inside the excitation box. The model outside the box was unchanged in all three cases and for both hybrid steps. After that we performed the second step of the hybrid with hill topography inside the excitation box. The results

are shown in Figure 8. Traces 0 and 1 compare the direct computation (here for the hill model) and its replication test, respectively. Traces 2 and 3 correspond to valley $\rightarrow$ hill and flat $\rightarrow$ hill hybrid solutions, respectively (for example, valley $\rightarrow$ hill means that the first step of hybrid was performed for valley topography inside the box and the second step was performed for hill topography inside the box). The agreement is perfect for receivers inside the excitation box. The results differ, as expected, only in the scattered field part. The scattered field for the hill $\rightarrow$ hill hybrid has zero value because (in this case) it is a replication test. Notice the delay of the scattered wave field onsets (traces 2 and 3) compared to the straight solution (trace 0). Solutions in trace 2 of Figure 7 and trace 3 of Figure 8 are equal.

[56] Although there were various topography details inside the excitation box during the first step, the second step results are identical in all three cases (inside the box). This demonstrates the independence of the second step results on the first step structure inside the excitation box. This feature is not limited to just topography modifications but also applies to structural changes inside the excitation box. The results demonstrate that the second step complete wave field is not affected by our choice of the structure or topography considered inside the excitation box (excitation planes excluded) during the first hybrid step. The only influence of the inside-box structure on the resulting wave field is in the second hybrid step. For example, we could calculate “flat topo” second step using the “hill topo” excitation with the same results (not shown here) as in the direct flat topo model.

[57] Because of independence we need not strictly specify any rules on how to construct the first step model inside the excitation box. This useful feature of the hybrid method finds practical utilization in complex models, such as the ones in the forthcoming section. A recommendation for the first step model inside the box could be that the topography

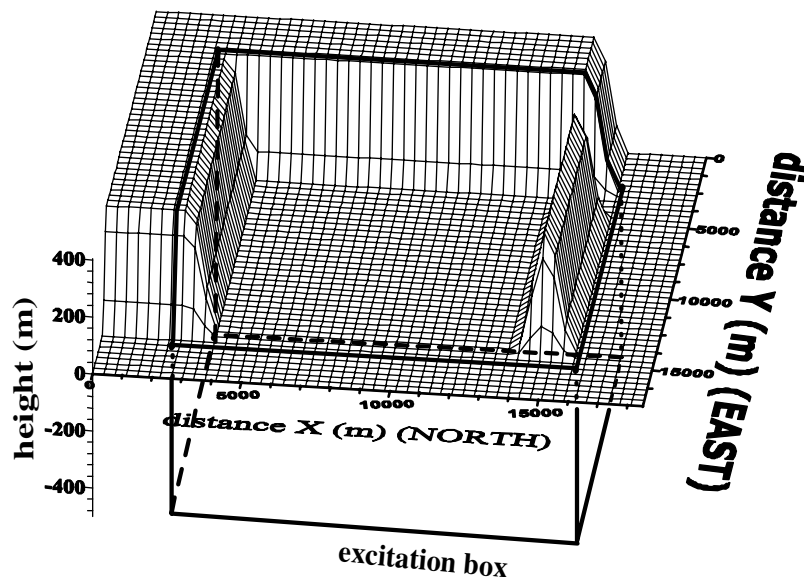


Figure 10. Topography model for the second step of the hybrid. The new topography feature, a ridge, is 400 m high (as the surrounding plateau), and it is situated inside the excitation box. Coordinates are in m.

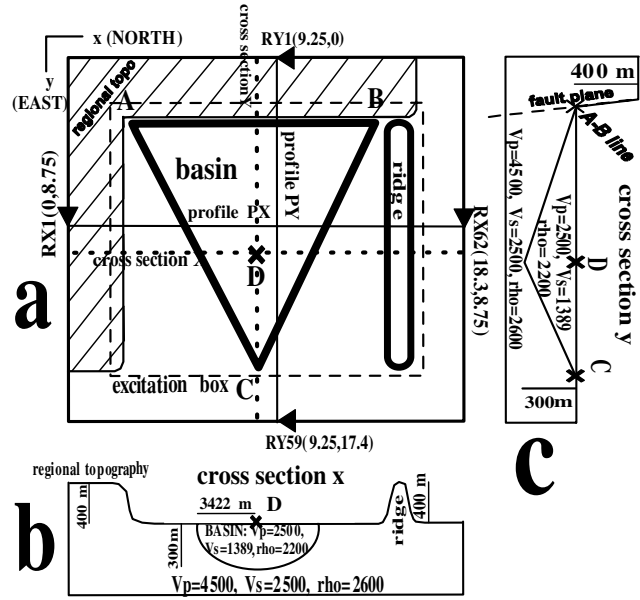


Figure 11. Layout of the second hybrid step. The crosshatched area is the topographic plateau (Figure 9), present in both steps. A ridge and an ellipsoidal valley are depicted by thick lines. The cross sections in Figures 11b and 11c correspond to SX and SY as they are depicted in Figure 11a. These sections intersect each other at point D. Note that the receiver profiles do not exactly coincide with the cross sections.

is similar to the second step model and it is a simple structure without low velocities (see section 6.3).

6.3. Basin and Topography Model

[58] This experiment exploits an FD-FD hybrid combined with irregular grids. A rather complex model (Figure 9) is designed for the first hybrid step to demonstrate the broad applicability of the method. The first step model is a homogeneous half-space with a flat surface region, surrounded on two sides by a 400-m-high plateau extending to infinity ($v_p = 4500$ m/s, $v_s = 2500$ m/s, and $\rho = 2600$ kg/m³).

[59] The second step model adds two more features: A 400-m-high ridge (Figures 10 and 11), and triangular low-velocity basin with $v_p = 2500$ m/s, $v_s = 1389$ m/s, $\rho = 2200$ kg/m³ (Figure 11). The basin cross section is elliptical (Figures 11a and 11b).

[60] The grid in the first step is regular ($dx = dy = dz = 100$ m). Wave propagation velocities and coordinates are shown in Figure 11. The point double-couple source with strike $\phi = 0^\circ$, dip $\delta = 52^\circ$, rake $\lambda = -90^\circ$, and source function as in (17) with $T = 1.24$ s, is situated under the flat part of the surface on a 52° dipping fault plane, marked in the topography by the 400-m scarp (Figure 9). Receivers are placed at a depth of $z = 0$ m (i.e., on the lower flat topography surface) in two perpendicular horizontal profiles, PX (receivers RX1–RX62) and PY (receivers RY1–RY59), with constant spacing of 300 m. Results for both profiles for the x (north) component of displacement are depicted together for each of the four models (Figures 12a–12d). In addition, Figures 12e and 12f show the y (east) and z (up) components of displacement computed for the fourth model, respectively. Two sets of snapshots (x component)

for the first step solution, the replication test, and three models with topography and structural variations in the second step are presented in Figures 13 and 14 for times $t = 1.8$ s and $t = 4.2$ s, respectively.

[61] The results of the replication test performed on the original regular coarse grid ($dx = dy = dz = 100$ m, $dt = 0.0075$ s) are presented in Figure 12a. Notice that profile PY is located near to (and parallel to) the nodal plane for the x component of displacement. This is why the amplitudes on PY are very low, having the advantage that any change in the solution can be clearly detected. The scattered field has zero value (see also Figures 13 and 14) as desired, and the complete wave field has typical relative errors 10^{-7} .

[62] The second step of the hybrid computation was performed for three models specified in sections 6.3.1–6.3.3, all with a common first step. Meshing was refined ($dx = dy = dz|_{z=0}^{z=1000} = 50$ m, $dz|_{z=0}^{z=2500} = 100$ m) causing a slight change of the step-like free surface topography parts (see section 2.5).

6.3.1. Ridge, no basin

[63] The second step of the hybrid method has the 400-m-high ridge inside the excitation box added to the first step model (Figure 10). Results are shown in Figure 12b (see also Figures 13b and 14b). The PX profile shows a reflected wave propagating approximately from receivers RX47 down to RX1 generated at the ridge (thus going out of the excitation box as a scattered wave). The same waves propagating in the opposite direction can be seen at receivers RX47–RX62 for times 3–6 s. Receivers RY1–RY9 and RY51–RY59 show scattered field radiating out of the excitation box (as expected), especially in time interval $t \in < 5, 7.5 >$ s. The strong effect of the ridge on either U_s and U_c is apparent in snapshot at Figure 14b ($t = 4.2$ s).

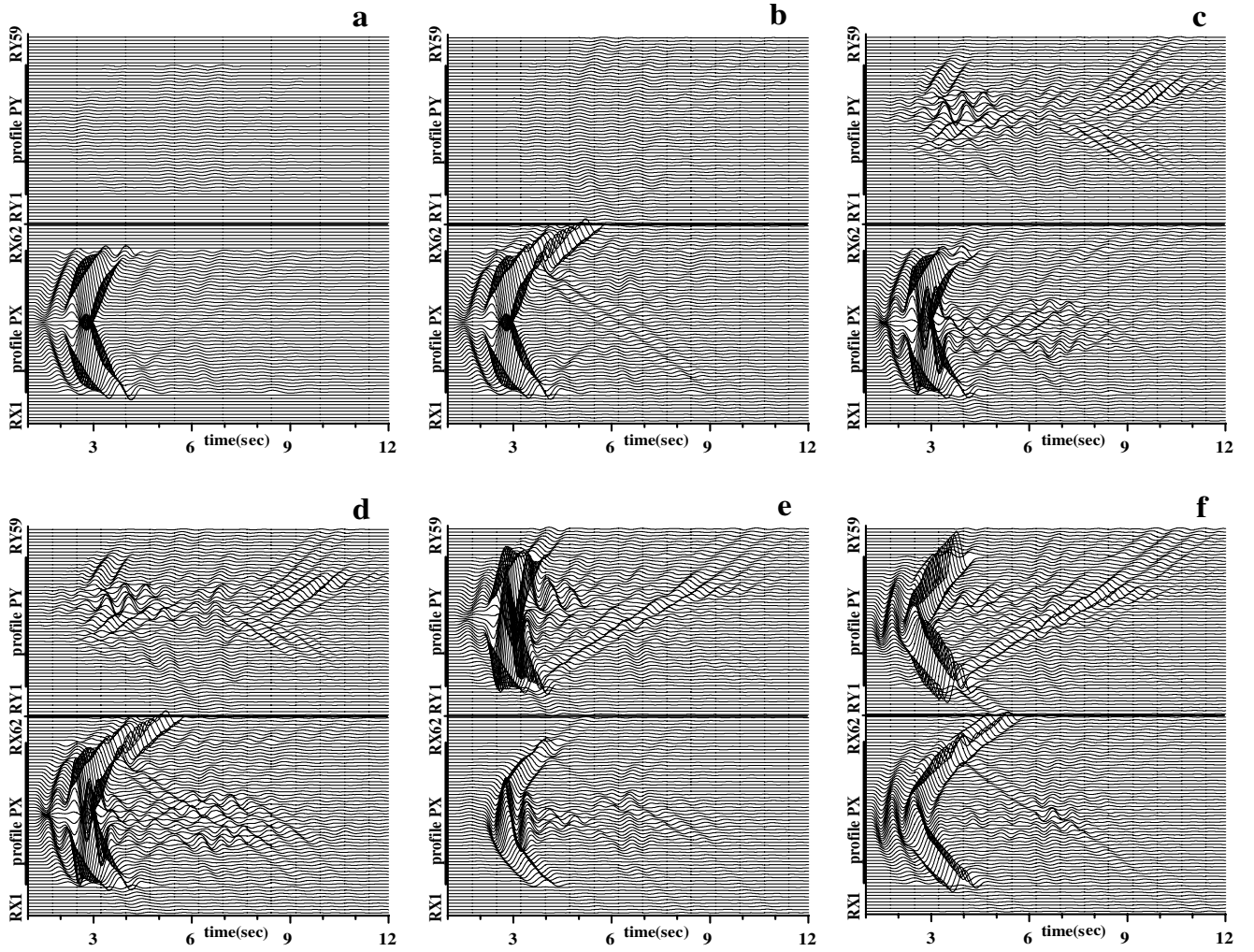


Figure 12. Basin and topographic model results (displacements). Receivers RX1–RX62 and RY1–RY59 on profiles PX and PY are indicated on the vertical axis. The thick indicators of the vertical axis mark locations inside the excitation box. The amplitude scale in Figures 12a–12d is the same; Figures 12e and 12f have the amplitudes halved. (a) Replication test on a coarse grid for topography plateau only (x component). (b) The second step of the hybrid with a 400-m-high ridge inside the excitation box (x component). (c) The second step of the hybrid with a low-velocity basin inside the excitation box (x component). (d, e, f) Second step model with both ridge and basin present (x , y , z components, respectively). For snapshots of U_s and U_c corresponding to Figures 12a–12d, see Figures 13 and 14.

6.3.2. Basin, no ridge

[64] The second step of the hybrid method has the low-velocity basin inside the excitation box added to the first step model (Figure 10). Details of the basin are described in section 6.3 and Figure 11. Results are shown in Figure 12c. Profiles PX and PY show the scattered wave field at $t > 3.5$ s. An interesting point is that in contrast to the first onset ($t \in < 1, 2 >$ s), the scattered wave field is not very strong until $t \approx 3.5$ s (see also Figures 13c and 14c). This is due to small refraction of the waves into the horizontal directions of propagation. Trapping of the waves (see Figure 13c) is apparent in synthetics on both profiles, and it is mostly responsible for the late scattered wave field (U_s) phases. Compared to the case with ridge only, U_s is weaker than the wave field refracted by the ridge (section 6.3.1 and Figure 12b) but is stronger for $t > 3.5$ s,

especially for $t \in < 8, 12 >$ s because of the radiation from the basin. Notice the clear difference between synthetics for U_s and U_c in Figure 12c.

6.3.3. Basin and ridge

[65] This second step model combines the two features studied separately in sections 6.3, 6.3.1 and 6.3.2. Therefore it shows both the ridge refraction (Figure 14d) and the trapping of the energy in the basin (Figure 13d). The snapshots recorded on the free surface show that the refraction effect of the topography is considerably stronger than the shallow basin effect. The synthetics are shown in Figures 12d, 12e, and 12f for x , y , and z components, respectively. Changes compared to Figure 12c are due to the strong ridge effect. For example, amplitudes in the “basin and ridge” model are weaker than in the “ridge, no basin” at receivers RY51–RY59 for $t \in < 9.5, 11 >$ s (see section 6.3.1). This is because

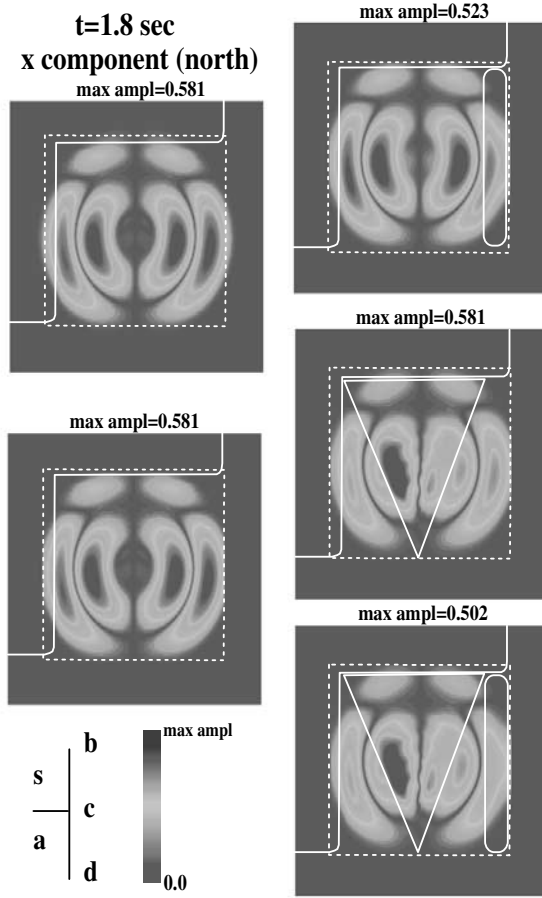


Figure 13. Snapshots of absolute value of the horizontal (x , north) component of displacement at $t = 1.8$ s. The snapshots are taken in the plane of zero altitude ($z = 0$). The thin white lines are outlines of the 400-m-high plateau, ridge, and basin; the thin dashed line is outline of the excitation box. The wave field shown is U_c everywhere (Figure 13s), U_c inside and U_s outside the excitation box (Figures 13a–13d, corresponding to Figures 12a–12d, respectively). The results are normalized to the maximum amplitude during the simulation of the fourth model. (s) The first step solution. (a) The replication test. (b) The second step of the hybrid with a 400-m-high ridge inside the excitation box. (c) The second step of the hybrid with basin inside the excitation box. (d, e, f) The second step of hybrid with ridge and basin inside the excitation box (x , y , z components, respectively).

part of the energy is being trapped in the basin (and refracted by it) from the beginning, and that energy cannot be diffracted by the ridge later on.

7. Conclusions

[66] A 3-D FD displacement hybrid method for an irregular rectangular grid is presented. The main motivation for developing the method was an effective 3-D FD simulation of wave propagation including general seismic source and path and evaluation of site effects and seismic hazard.

[67] The method uses a single FD approximation to the elastodynamic equation in the whole computational model. The free surface topography is approximated by a staircase-like boundary, and it is treated by the vacuum formalism. The internal boundaries are approximated by averaged material parameters. A hybrid method takes into account source, path, and site effects in two steps. The first step may be treated by any 3-D method for a model containing the source, regional structure, and regional topography. A special coupling bounds the first step results with the second step computation. The second step is performed by the 3-D FD method on a local site model on an irregular grid with detailed structure and topography. The hybrid method saves computer memory and time, especially in cases with high-frequency content, high material parameters contrasts, and large-scale difference between the local and regional models. Thus the hybrid is also suitable for parametric studies as various local models may be computed with relatively low demands of computer memory and time.

Appendix A: Complete Mixed Derivative Approximations

[68] To create the remaining mixed derivative approximations (here we write $(au_x)_x = (\partial/\partial x)[a(\partial f/\partial x)]$, etc.),

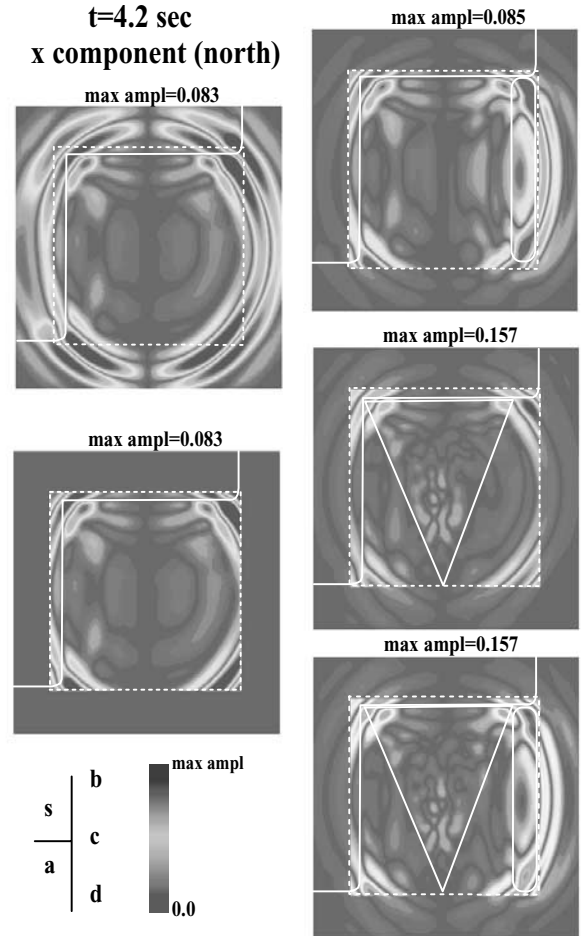


Figure 14. Snapshots of absolute value of the horizontal (x , north) component of displacement at $t = 4.2$ s. See Figure 13.

we can use the following transformations of indices in equation (4) in section 2.3:

$$(\mathbf{a} \mathbf{f}_x)_z \rightarrow (\mathbf{a} \mathbf{f}_y)_z: \quad (\text{A1})$$

$$f_{i,0,k} \rightarrow f_{0,i,k}, a_{es} \rightarrow a_{fs}, a_{ws} \rightarrow a_{rs}, a_{en} \rightarrow a_{fn}, a_{wn} \rightarrow a_{rn}, \\ DX(m) \rightarrow DY(m), DZ(m) \rightarrow DZ(m);$$

$$(\mathbf{a} \mathbf{f}_x)_z \rightarrow (\mathbf{a} \mathbf{f}_y)_x: \quad (\text{A2})$$

$$f_{i,0,k} \rightarrow f_{k,i,0}, a_{es} \rightarrow a_{ef}, a_{ws} \rightarrow a_{er}, a_{en} \rightarrow a_{wf}, a_{wn} \rightarrow a_{wr}, \\ DZ(m) \rightarrow DX(m), DX(m) \rightarrow DY(m);$$

$$(\mathbf{a} \mathbf{f}_x)_z \rightarrow (\mathbf{a} \mathbf{f}_z)_x: \quad (\text{A3})$$

$$f_{i,0,k} \rightarrow f_{k,0,i}, a_{es} \rightarrow a_{es}, a_{ws} \rightarrow a_{en}, a_{en} \rightarrow a_{ws}, a_{wn} \rightarrow a_{wn}, \\ DZ(m) \rightarrow DX(m), DX(m) \rightarrow DZ(m);$$

$$(\mathbf{a} \mathbf{f}_x)_z \rightarrow (\mathbf{a} \mathbf{f}_z)_y: \quad (\text{A4})$$

$$f_{i,0,k} \rightarrow f_{0,k,i}, a_{es} \rightarrow a_{fs}, a_{ws} \rightarrow a_{fn}, a_{en} \rightarrow a_{rs}, a_{wn} \rightarrow a_{rn}, \\ DZ(m) \rightarrow DY(m), DX(m) \rightarrow DZ(m);$$

$$(\mathbf{a} \mathbf{f}_x)_z \rightarrow (\mathbf{a} \mathbf{f}_x)_y: \quad (\text{A5})$$

$$f_{i,0,k} \rightarrow f_{i,k,0}, a_{es} \rightarrow a_{ef}, a_{ws} \rightarrow a_{wf}, a_{en} \rightarrow a_{er}, a_{wn} \rightarrow a_{wr}, \\ DX(m) \rightarrow DX(m), DZ(m) \rightarrow DY(m). \text{ Let us, moreover, suppose that } a_{kl} = a_{lk}; \text{ the reasons are given in section 2.5.}$$

Appendix B: Complete Nonmixed Derivative Approximations

[69] To create the remaining nonmixed derivative approximations, we can use the following transformations of indices in equation (5) in section 2.3:

$$(\mathbf{a} \mathbf{f}_z)_z \rightarrow (\mathbf{a} \mathbf{f}_y)_y: \quad (\text{B1})$$

$$f_{0,0,k} \rightarrow f_{0,k,0}, a_s \rightarrow a_f, a_n \rightarrow a_r, DZ(m) \rightarrow DY(m);$$

$$(\mathbf{a} \mathbf{f}_z)_z \rightarrow (\mathbf{a} \mathbf{f}_x)_x: \quad (\text{B2})$$

$$f_{0,0,k} \rightarrow f_{k,0,0}, a_s \rightarrow a_e, a_n \rightarrow a_w, DZ(m) \rightarrow DX(m).$$

[70] **Acknowledgments.** The manuscript benefited from helpful comments of editors F. Albarède and A. Dainty and reviews of R. Graves and S. Hestholm. This research was financially supported by research project of Czech Republic MSMT J13/98-113200004 and ME354, Grant Agency of Czech Republic 205/00/0902, GAUK grant 176/2000/B GEO/MFF, NATO Collaborative Linkage grant EST.CLG.976035, EU projects EVG1-CT-1999-00001 PRESAP and EVG1-CT-2000-00023 SAFE (BBW 00.0336). This paper is contribution 1201 of the Geophysical Institute of ETH Zurich.

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