

Prediction of magnitude of the largest potentially induced seismic event

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Abstract We propose a method for determining the possible magnitude of a potentially largest induced seismic event derived from the Gutenberg–Richter law and an estimate of total released seismic moment. We emphasize that the presented relationship is valid for induced (not triggered) seismicity, as the total seismic moment of triggered seismicity is not bound by the injection. The ratio of the moment released by the largest event and weaker events is determined by the constants a and b of the Gutenberg–Richter law. We show that for a total released seismic moment, it is possible to estimate number of events greater than a given magnitude. We determine the formula for the moment magnitude of a probable largest seismic event with one occurrence within the recurrence interval (given by one volumetric change caused by mining or injecting). Finally, we compare theoretical and measured values of the moment magnitudes of the largest induced seismic events for selected geothermal and hydraulic fracturing projects.

Keywords Magnitude · Seismic moment · b value · Induced events · Microseismicity · Hydraulic fracturing · Enhanced geothermal systems

1 Introduction

The volumetric changes in underground (e.g., fluid injection, gas or rock extraction) can trigger or induce micro-earthquakes of variable magnitudes (e.g., Rutledge and Phillips 2003; Mahrer et al. 2005; Eisner et al. 2011; Lasocki and Orlecka-Sikora 2008). Such seismicity can be very complex because of the combination of accumulated regional stress on fault systems and physical communication (e.g., fluid flow, pressure-front propagation, stress transfer, etc.) between parts of such a system (McGarr 1976; Barton et al. 1995; Hickman et al. 1997; Blewitt et al. 2002). At the same time, limitations on induced seismic events may be well-estimated as the injected energy is known and the affected volume is limited (McGarr 1976, and references therein; Shapiro et al. 2010).

The goal of this study is to provide a realistic estimate of the maximum induced, but not triggered, event magnitude from an anticipated total seismic moment (scalar value of released seismic moment) and the ratio between the number of large and small events (Gutenberg–Richter law). This approach is valid only for induced seismic events, as the triggered events can be significantly larger (Dahm et al. 2013; Shapiro et al. 2013). In other words, we focus on cases where the seismic energy released by the induced events is relatively higher than that for

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triggered events as have been observed in numerous case studies discussed in public domain (e.g., McGarr 1976; Baisch et al. 2009a; Hallo et al. 2012). For such cases of induced seismicity, we presume that the regional tectonic stress field does not play the main role in determining the size of the maximum seismic event, or that the fault system is activated only in a very limited volume. The regional stress or the shape of the volume deformation may still influence the mechanism of the induced events (McGarr 1976; Shapiro et al. 2013). The only known case study where the total seismic moment exceeded the estimate, based on the injected volume, and derived from McGarr (1976) are the aftershocks recorded at the potentially triggered sequence at the Rocky Mountain Arsenal (e.g., Healy et al. 1968). All other mentioned case studies are consistent with McGarr's empirical prediction (Hallo et al. 2012).

2 The method

We apply three simple empirical relationships, which show the characterization of the event magnitude–frequency distribution of the selected region, the cumulative (or total) seismic moment estimate, and the moment–magnitude relationship:

- a. The Gutenberg–Richter law (Gutenberg and Richter 1954) relates number of events N greater than a given magnitude M appearing within a time interval:

$$\log_{10} N = a - bM, \quad (1)$$

where a and b are positive constants (called a and b values). The b value (slope) characterizes the ratio of a number of small versus large events. A larger b value means a higher proportion of smaller to larger events. The a value is related to seismic activity of the area, where 10^a is the total number of events of $M \geq 0$ in a time interval. The variation in the parameters of the Gutenberg–Richter for different sites can be interpreted as a result of the variation in the relevant elastic parameters as discussed by Christensen and Olami (1992) or the stress regime (Schorlemmer et al. 2005). Obviously, the constants a and b depend on a chosen magnitude scale (e.g., local or moment magnitude). As most of the applications in microseismic monitoring use moment

magnitude, we therefore derive further relationships for moment magnitudes M_w (the a and b values are relevant to the moment magnitude in the whole study).

- b. Total seismic moment is a sum of the scalar seismic moments $\sum M_0$ of events (McGarr 1976; Aki and Richards 1980, chapter 4.5). The $\sum M_0$ is the upper limit of the modulus of summed tensor moments having variable mechanisms, where equality only applies if all moment tensors are self-similar. The induced events by hydraulic fracturing have typically very similar mechanisms (e.g., Rutledge and Phillips 2003) due to ambient stress field or geometry of the volume deformation (McGarr 1976). The formulae derived in this study depend only on the total seismic moment. If another, more suitable, estimate of the total seismic moment is found, it can be applied by analogy.
- c. The moment–magnitude–energy relationship used in the study is given by Kanamori (1977):

$$M_w = \frac{2}{3} (\log_{10}(M_0) - 9.1), \quad (2)$$

where M_0 is the scalar seismic moment in Newton meters. However, alternative relationships between moment and magnitude can be used. In this study, we use the relationship for moment magnitude given by Eq. (2) as it is the most common in microseismic monitoring (e.g., Urbancic et al. 1996; Maxwell et al. 2010).

In the following derivations, we utilized the combined relations from above Sections a, b, and c using an approach analogous to Anderson and Luco (1983). The final estimated moment is scaled to the moment magnitude (Kanamori 1977); however, the proposed methodology is not dependent on this particular choice and any moment–magnitude relationship can be used. Hence, in our case, the known current b value (related to the ratio of the number of small to large events), the total scalar moment potentially released in the area, and the magnitude–moment relationship determine the magnitude of the largest event that can occur within the vicinity of the injections (or more generally volumetric changes).

2.1 Maximum magnitude

The magnitude distribution of events, combined in the total seismic moment $\sum M_0$ is estimated from the Gutenberg–Richter law (Gutenberg and Richter 1954) and moment magnitude formulae (Kanamori 1977). Let $M_0(M_w)$ be the released scalar seismic moment of a particular event of the magnitude M_w . Then, the upper limit of the total released seismic moment is the sum of the seismic moments of all events (i.e., integral of the seismic moment over number of events, depending on the magnitude):

$$\sum M_0 = \int_{M_w^{\min}}^{M_w^{\max}} M_0(M_w) dN(M_w), \quad (3)$$

where the dependence of seismic moment on the magnitude $M_0(M_w)$ can be obtained from Eq. (2).

To define the number of events $dN(M_w)$ having the seismic moment M_w (i.e., events from the magnitude interval $[M_w, M_w + dM_w]$), we use Gutenberg–Richter law (1). The number of events can be obtained as absolute value of the differential of the Gutenberg–Richter law (1):

$$dN(M_w) = |-b \cdot \ln(10) \cdot 10^{a-bM_w}| dM_w = b \cdot \ln(10) \cdot 10^{a-bM_w} dM_w, \quad (4)$$

where the b value is positive by definition. By applying Eqs. (3) to (4), we obtain:

$$\sum M_0 = \frac{b \cdot 10^a \cdot 10^{0.1}}{\left(\frac{3}{2} - b\right)} \left(10^{(M_w^{\max}(\frac{3}{2}-b))} - 10^{(M_w^{\min}(\frac{3}{2}-b))} \right), \quad (5)$$

where $M_w^{\min}$ and $M_w^{\max}$ are the minimum and maximum considered magnitudes, respectively. Let us assume that $M_w^{\max}$ is the magnitude of the largest event that occurs once within a recurrence interval (e.g., in this study, the duration of injection). Equation (4) is a continuous function, which is analogous to an un-normalized probability density function; however, the number of events has to have an integer value. Then, the finite integral of Eq. (4) between the interval $M_w^{\max} - \delta$ and $M_w^{\max} + \delta$ has to be equal to one to have just one realization of the largest event. The finite integral of Eq. (4) leads to expression:

$$a = bM_w^{\max} - \log_{10}(10^{b\delta} - 10^{-b\delta}). \quad (6)$$

Equation (6), describes the generally unknown a value as a function of a probabilistic half-bin size δ and b value, where the half-bin size is defined around the maximum magnitude in the Gutenberg–Richter distribution. This interval is parameter of the model and it has to be calculated in order to fulfill the condition that the finite integral over this bin is equal to one (one event of such magnitude in a recurrence interval). The half-bin size δ has been calculated by numerical evaluation of Eq. (5) to comply with the condition: $(\lim_{b \rightarrow 0} (\max(M_0)/\sum M_0) = 1)$, that is, as $b \rightarrow 0$, all moment is released in a single event. The resulting δ is then a constant ($\delta = 0.1448$) for $M_w^{\min} = -\infty$; this value changes for cases having a low number of events above $M_w^{\min}$ as shown in Table 1. Moreover, the δ is infinitesimally small in the case where the whole seismic moment is incorporated in one event with magnitude $M_w^{\min} = M_w^{\max}$.

Equations (5) and (6) together show the relationship among $M_w^{\max}$, $M_w^{\min}$, $\sum M_0$, and the b value. To better illustrate this relationship, we show numerical solutions for the dependence of the ratio between seismic moment of the largest event $\max(M_0)$ (Eq. 2 where $M_w = M_w^{\max}$) and the total released seismic moment $\sum M_0$ on b value (Fig. 1). For the case of $M_w^{\min} = -\infty$ (Fig. 1, solid line),

Table 1 Numerically calculated constants δ for five models with different $M_w^{\min}$

$\sum M_0$	$M_w^{\min}$				
	$-\infty$	-7	-3	0	2
10^5	0.1448	0.1448	0.0872	–	–
10^6	0.1448	0.1448	0.1390	–	–
10^7	0.1448	0.1448	0.1442	–	–
10^8	0.1448	0.1448	0.1447	–	–
10^9	0.1448	0.1448	0.1448	–	–
10^{10}	0.1448	0.1448	0.1448	0.1265	–
10^{11}	0.1448	0.1448	0.1448	0.1430	–
10^{12}	0.1448	0.1448	0.1448	0.1445	–
10^{13}	0.1448	0.1448	0.1448	0.1447	0.1265
10^{14}	0.1448	0.1448	0.1448	0.1448	0.1430
10^{15}	0.1448	0.1448	0.1448	0.1448	0.1445

the ratio has the same dependence on b value for various $\sum M_0$ and it is greater than 0 for b values up to 1.5. For b values equal or greater than 1.5, the total seismic moment and energy released by small events increases to infinitely large value. This means that the proportional part of the seismic moment of the largest event is equal to 0, which is not empirically observed in geophysics. In the case of a catalogue with limited $M_w^{\min}$ (Fig. 1, dashed lines), the ratio is greater than 0 for b values in the open interval from 0 to infinity, as the energy released by small events cannot increase to the infinite value. Moreover, the dependence of the ratio on b value differs for various $\sum M_0$. The ratio is similar to the case of $M_w^{\min} = -\infty$ for $\sum M_0$ much greater than $\min(M_0)$ (Eq. 2 where $M_w = M_w^{\min}$), and it is constant (equal to 1) for the limiting case where $\sum M_0 = \min(M_0) = \max(M_0)$ (the case where the whole seismic moment is incorporated in one event with magnitude $M_w^{\min} = M_w^{\max}$).

Now, let the Gutenberg–Richter law be valid for whole magnitude range from finite $M_w^{\max}$ to $M_w^{\min} \rightarrow -\infty$ (i.e., $10^{(M_w^{\min}(\frac{3}{2}-b))} = 0$) and let the positive b value be lower than 1.5. Then, using Eqs. (5) and (6), we get relationship determining the moment magnitude of the

largest possible seismic event ($M_w^{\max}$):

$$M_w^{\max} = \frac{2}{3} \left(\log_{10} \left(\frac{\sum M_0 \left(\frac{3}{2} - b \right)}{b \cdot 10^{9.1}} \right) + \log_{10} (10^{b\delta} - 10^{-b\delta}) \right), \quad (7)$$

where $\delta = 0.1448$. The values of the possible largest seismic events calculated by Eq. (7) are shown in Fig. 2 as function of the total seismic moment and b value. Because Eq. (7) is a special form of Eq. (5), it is possible to calculate values of the possible largest seismic events for models with limited $M_w^{\min}$ by the original Eq. (5) using the appropriate δ from Table 1. We show one realization for $M_w^{\min} = -3$ in Fig. 3. All dashed lines for different b values in Fig. 3 are defined for $\sum M_0$ larger than $10^{4.6}$ (which is the seismic moment of an event of magnitude $M_w = -3$) corresponding to the case where the whole seismic moment is incorporated in one event with magnitude $M_w^{\min} = M_w^{\max} = -3$.

The most critical factor for the correct magnitude prediction by Eq. (7) is the value of the total seismic moment (Figs. 2 and 3). The possible error in Eq. (7) caused by inaccurate seismic moment (neglecting b

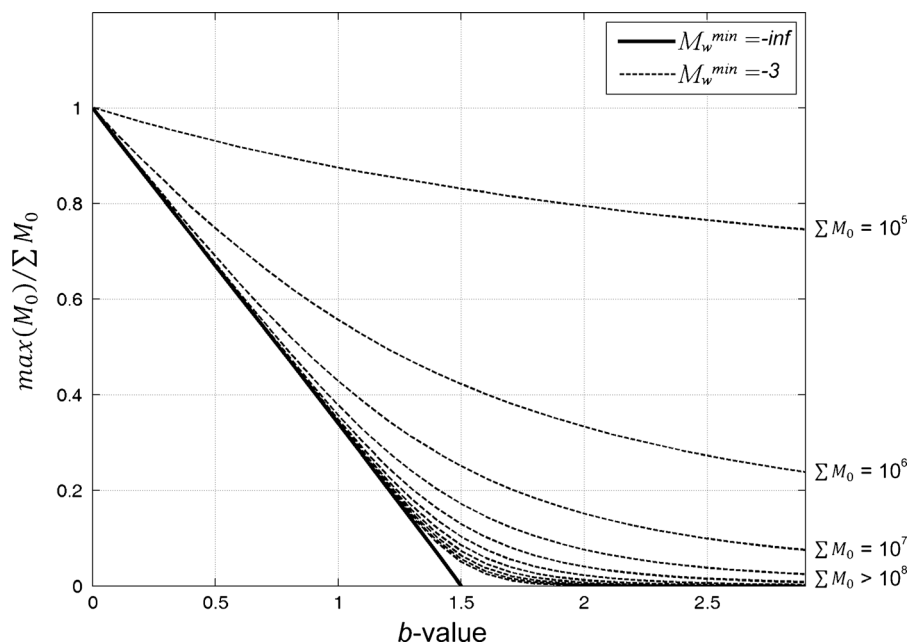


Fig. 1 Ratio between the theoretical value of moment of the largest event and the total released seismic moment, as function of the b value. The total seismic moment is incorporated in events with magnitudes from the interval $[-\infty, M_w^{\max}]$ (solid line) and from interval $[-3, M_w^{\max}]$ (dashed lines)

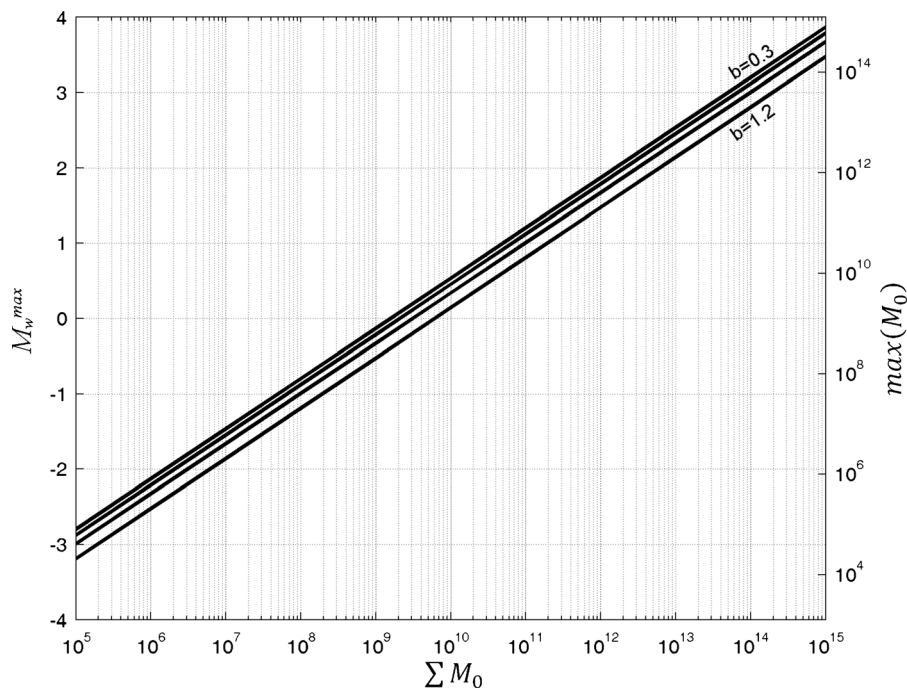


Fig. 2 Theoretical moment magnitude of the largest seismic event dependent on the total seismic moment. The total seismic moment is incorporated in events with magnitudes from the interval $[-\infty, M_w^{\max}]$. The individual lines correspond to different b values $\{0.3, 0.6, 0.9, \text{ and } 1.2\}$

value error) is then:

$$\text{err}(M_w^{\max}) = \frac{2}{3} \log_{10} \left(\frac{\text{assumed } \sum M_0}{\text{real } \sum M_0} \right). \quad (8)$$

Then, for example, a incorrectly estimated total seismic moment $\sum M_0$ by a factor of 2, results in a -0.2 lower moment magnitude $M_w^{\max}$.

2.2 Total seismic moment $\sum M_0$ —practical issues

Knowledge of the total induced seismic moment is particularly important in mining, geothermal, and hydrocarbon production (McGarr 1976; Maxwell et al. 2009). It is also the main input parameter for Eq. (7), for determining the magnitude of the possible largest seismic event.

McGarr (1976) proposed total seismic moment (scalar value of released seismic moment) by a model in which the stress is completely released seismically by small to moderate events of the prevailing moment tensor. The total seismic moment $\sum M_0$ is then:

$$\sum M_0 = K\mu|\Delta V|, \quad (9)$$

where ΔV is the volume change, μ is the shear modulus and factor K ranges from $1/2$ to $4/3$, depending on the induced sources mechanism. The shear strain model of McGarr (1976) is valid for solid rock excavation, and is usually upper bound for seismicity induced by hydraulic fracturing and other hydraulic stimulations (Hallo et al. 2012).

Equation (9) does not incorporate aseismic deformation within the medium, inelastic effects, and nonlinearity. Hallo et al. (2012) show that Eq. (9) can be modified to account for aseismic slip to explain observed seismic moments. It is possible to define the “seismic efficiency ratio” (SER) as an analogy to the “seismic injection efficiency” of Maxwell et al. (2009). The SER is a ratio of the released to the theoretical scalar cumulative moment. The range of observed SER has been estimated for different types of stimulations in Hallo et al. (2012). In this paper, we present revised values: $\text{SER} \in [10^{-2}, 1]$ for enhanced geothermal systems (EGS) and saltwater disposals; and $\text{SER} \in [5 \times 10^{-7}, 5 \times 10^{-4}]$ for hydraulic fracturing of rocks. The theoretical $\sum M_0$ with

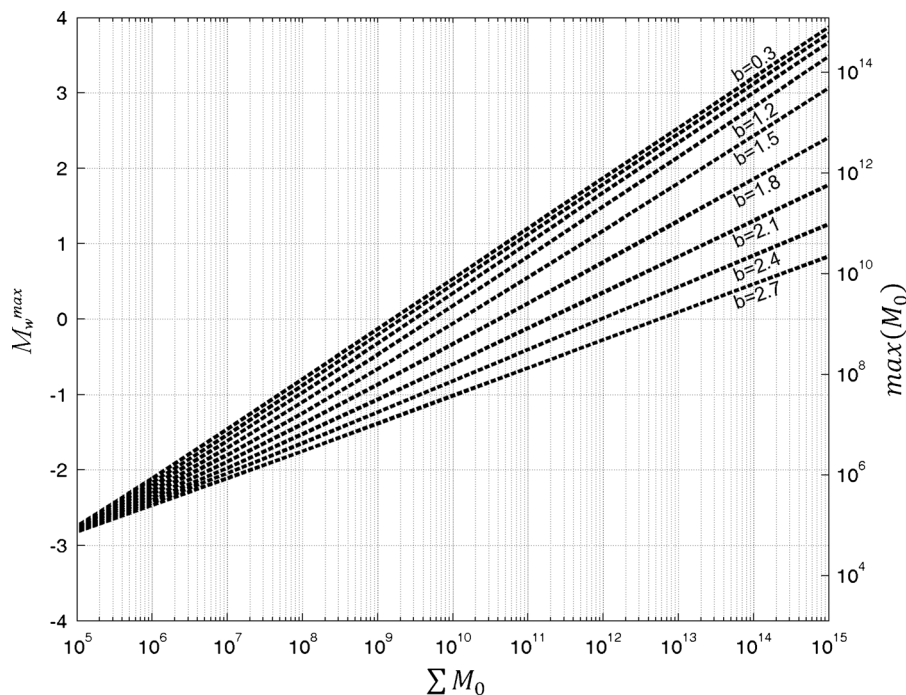


Fig. 3 Theoretical moment magnitude of the largest seismic event dependent on the total seismic moment. The total seismic moment is incorporated in events with magnitudes

from the interval $[-3, M_w^{\max}]$. The individual lines correspond to different b values $\{0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, \text{ and } 2.7\}$

incorporated aseismic deformation within the medium is given by:

$$\sum M_0 = \text{SER} \cdot K\mu |\Delta V|. \quad (10)$$

Equation (10) results in lower values of the total seismic moment released especially for fluid injection into soft rock (hydraulic fracturing). The values of the ratio $\sum M_0/|\Delta V|$ and SER for numerous case studies (hydraulic fracturing, enhanced geothermal system stimulation, and saltwater disposal injections) are given in Table 2. Revised values of SER mentioned above are sufficient for estimation of the most probable range of released total seismic moment; however, the variability of SER and the physical model of SER is still poorly understood.

2.3 Case studies

The accuracy of the model has been tested by comparison between measured maximum magnitudes from real case studies and values of $M_w^{\max}$ given by Eq. (7). The case studies are given in Table 2, and resulting measured

and theoretical values of maximum magnitudes are compared in Figs. 4 and 5. The theoretical values of $M_w^{\max}$ have been calculated by Eq. (7) for three different b values from known (measured) $\sum M_0$ of each case (Fig. 4). Then, the theoretical values of $M_w^{\max}$ have been estimated from known injected volumes, where $\sum M_0$ is given by Eq. (10) as the most probable interval of total seismic moment within typical limits of SER (Fig. 5). Finally, the maximum possible $M_w^{\max}$ estimated from known injected volume has been calculated for the case where the injected energy is completely released seismically (Eq. (9), Fig. 5).

From the listed examples, we note the following case studies:

The EGS stimulation in Basel in 2006 (N. 2 in Table 2): The measured $M_w^{\max}=3.2$, computed $M_w^{\max}$ from measured $\sum M_0$ is 3.6 (for b value=0.67), range of the most probable $M_w^{\max}$ from estimated $\sum M_0$ by Eq. (10) is 3.4 to 2.0 (for b value=0.67), and maximum possible $M_w^{\max}$ from estimated $\sum M_0$ by Eq. (9) is 3.4.

The hydraulic fracturing example is, e.g., stage 1 of a Woodford Shale (USA) project (N. 14 in Table 2): The measured $M_w^{\max}=-1.9$, computed $M_w^{\max}$ from measured

Table 2 List of case studies (EGS in hard rocks and hydraulic fracturing in soft rocks) used for demonstration of the method

N.	Site	Rock	Phase	ΣM_0	$\frac{\Sigma M_0}{V}$	SER	b	Measured $M_w^{\max}$	References
1	Bad Urach (Germany)	Metamorphic rock	2003	9.5E+11	3.0E+08	1.3E-02	–	–	Baisch et al. (2009a)
2	Basel (Switzerland)	Granite	2006	5.8E+14	5.0E+10	2.3E+00	0.71	3.2	Bachmann et al. (2011); ECOS (2002); Mukuhira et al. (2008)
3	Cooper Basin (AUS)	Fractured granite	2003	6.0E+14	3.0E+10	1.4E+00	0.52	–	Baisch et al. (2009a); USGS (2012)
4	Cooper Basin (AUS)	Fractured granite	2005	2.3E+14	1.0E+10	4.6E-01	–	2.6	Baisch et al. (2009a); Baisch et al. (2009b)
5	Soultz (France)	Granite	2000	1.4E+14	6.0E+09	2.7E-01	–	2.2	Baisch et al. (2009a); Charlety et al. (2007)
6	Soultz (France)	Granite	2003	1.0E+14	3.0E+09	1.4E-01	–	2.5	Baisch et al. (2009a); Charlety et al. (2007)
7	Soultz (France)	Granite	2004	9.3E+13	1.0E+10	4.6E-01	–	2.1	Baisch et al. (2009a); Charlety et al. (2007)
8	Soultz (France)	Granite	2005	8.6E+13	7.0E+09	3.2E-01	–	2.4	Baisch et al. (2009a); Charlety et al. (2007)
9	Dallas-Fort (USA)	Barnett shale	2008–9	5.1E+14	1.2E+09	1.3E+00	0.89	3.3	Frohlich et al. (2011);
10	Blackpool (UK)	Bowland shale	2011	2.7E+12	3.1E+08	3.4E-01	0.49	2.2	Eisner et al. (2011)
11	Cotton Valley (USA)	Sands and shales	A	1.8E+09	1.3E+06	3.4E-04	–	–1.4	Rutledge et al. (2004)
12	Cotton Valley (USA)	Sands and shales	B	2.7E+09	2.2E+06	5.7E-04	–	–1.4	Rutledge et al. (2004)
13	Cotton Valley (USA)	Sands and shales	E	7.8E+09	2.0E+07	5.1E-03	–	–1.0	Rutledge et al. (2004)
14	Oklahoma (USA)	Woodford shale	Stage 1	1.6E+07	(0.7 to 1.5)E+04	(0.7 to 1.5)E-05	1.31	–1.9	Vulgamore et al. (2007)
15	Oklahoma (USA)	Woodford shale	Stage 2	4.8E+07	(2.0 to 4.1)E+04	(2.2 to 4.4)E-05	1.53	–1.8	Vulgamore et al. (2007)
16	Oklahoma (USA)	Woodford shale	Stage 3	4.9E+06	(2.0 to 4.1)E+03	(2.2 to 4.4)E-06	1.22	–2.3	Vulgamore et al. (2007)
17	Oklahoma (USA)	Woodford shale	Stage 4	2.5E+06	(1.1 to 2.1)E+03	(1.1 to 2.3)E-06	1.18	–2.1	Vulgamore et al. (2007)
18	Oklahoma (USA)	Woodford shale	Stage 5	1.2E+06	(0.5 to 1.0)E+03	(0.6 to 1.1)E-06	1.11	–2.3	Vulgamore et al. (2007)

ΣM_0 , SER, and b values are measured values for a particular stimulation. b -values are relevant to the moment magnitude scale

$\sum M_0$ is -1.9 (for b value = 1.33), range of the most probable $M_w^{\max}$ from estimated $\sum M_0$ by Eq. (10) is -0.7 to -2.9 (for b value = 1.33), and maximum possible $M_w^{\max}$ from estimated $\sum M_0$ by Eq. (9) is 1.5.

Differences of measured $M_w^{\max}$ and computed $M_w^{\max}$ (from measured $\sum M_0$, Fig. 4) are less than 0.5 for EGS with known b values, and less than 0.25 for hydraulic fracturing projects with known b values. All measured $M_w^{\max}$ are smaller than maximum possible value estimated from $\sum M_0$ by Eq. (9) (Fig. 5). Almost all measured $M_w^{\max}$ have values within the most probable interval estimated from $\sum M_0$ by Eq. (10) (Fig. 5). The only exception is hydraulic fracturing in the Bowland shale in 2011; however, the measured $M_w^{\max}$ is still smaller than maximum possible value estimate. The most probable magnitude intervals from estimated $\sum M_0$ by Eq. (10) have half-widths 0.7 for EGS and 1.1 for hydraulic fracturing, and they are governed by the uncertainty of SER values.

The overall agreement of measured and computed $M_w^{\max}$ through various projects is relatively good,

considering that the measured values of $M_w^{\max}$, $\sum M_0$, and b values in Table 2 and Figs. 4 and 5 are determined by various methods and are also subject to measurement errors.

3 Discussion of b values

The relationship in Eq. (7) is valid only for b values < 1.5 . If the b value is greater or equal to 1.5, then the most energy is radiated at lower magnitudes and the Gutenberg–Richter relationship cannot hold for infinitely small magnitudes. Another view of $b < 1.5$ is via topological fractal dimension. Aki (1981) suggests that the Gutenberg–Richter relation of a given b value is equivalent to the fractal distribution with the fractal topological dimension $D = 2b$, provided $M_0 \sim R^3$, where R is the characteristic size of the source. The fractal dimension is related to the fault geometry in 2D or 3D space (Turcotte 1989; King 1983; Sammis et al. 1993). The dimensional analysis of fractal sets (e.g., Manin

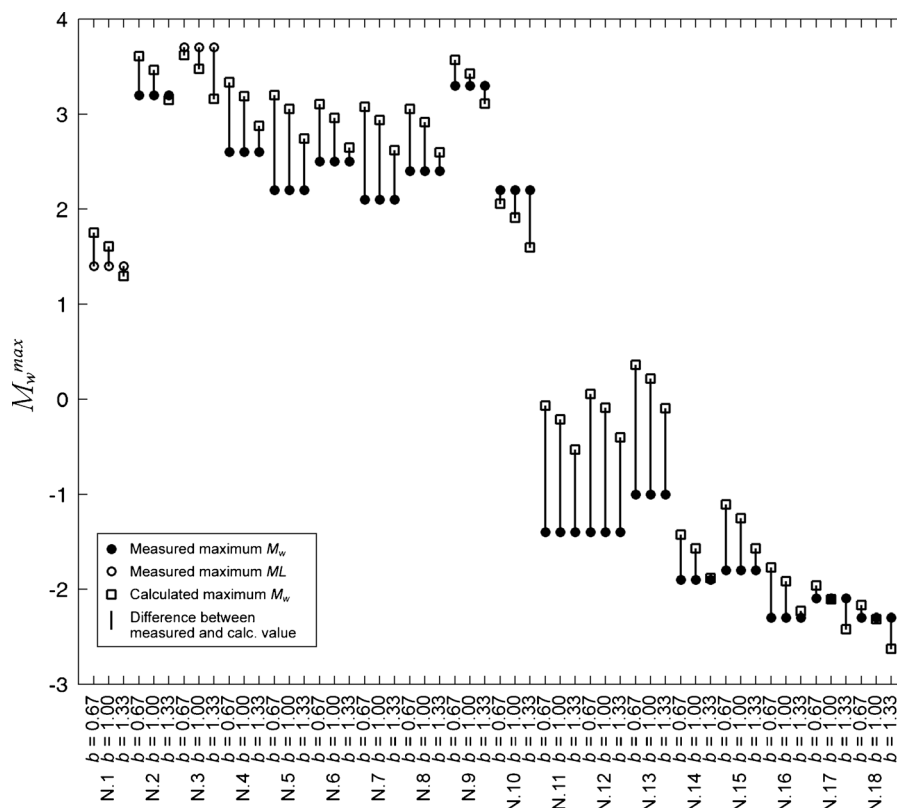


Fig. 4 Comparison of measured and calculated moment magnitudes of largest induced seismic events for the case studies from Table 2. Theoretical values are computed from known total seismic moment by Eq. (7). b values are relevant to the moment magnitude scale

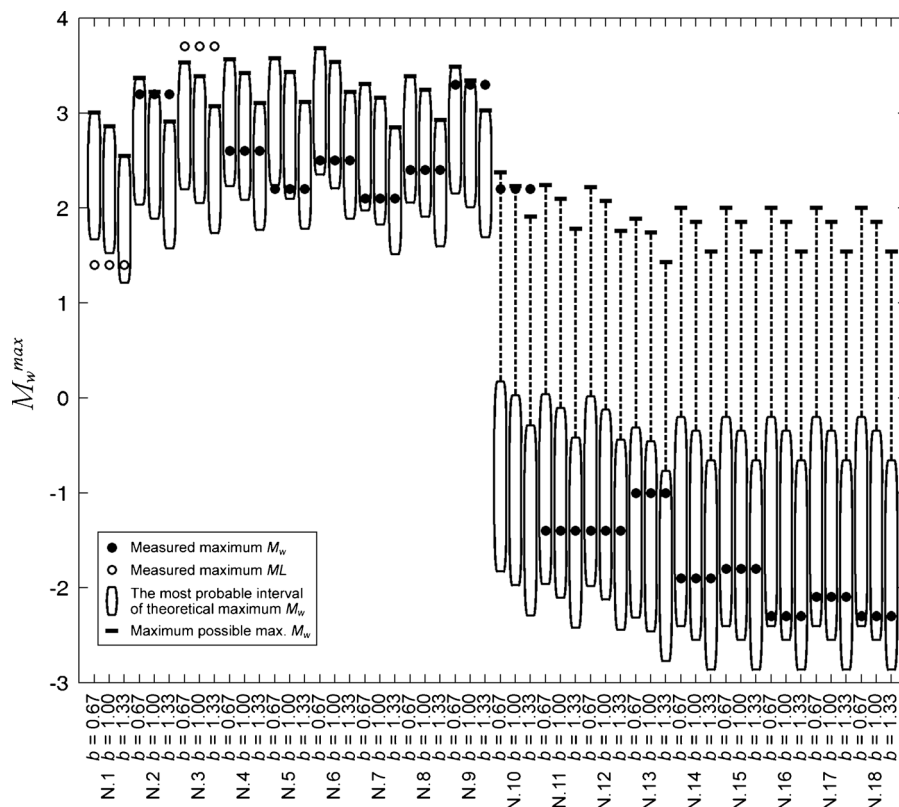


Fig. 5 Comparison of measured and calculated moment magnitudes of largest induced seismic events for case studies from Table 2. Theoretical values are estimated from known injected volume (with $SER \in [10^{-2}, 1]$ for hard rock, and

$SER \in [5 \times 10^{-7}, 5 \times 10^{-4}]$ for soft rock). Values of μ for different rock types are averaged from limit values given in Mavko et al. (2009). b values are relevant to the moment magnitude scale

2006) shows that 2D faulting systems (objects of $D=2$ such as fault planes) can create fractals with dimension $D \leq 3$. The fractal dimension in this model faulting system should be smaller than 3 because the (3D) volume occupied by the faulting systems is finite, hence measurable by dimension $D=3$ (in cubic meter). Therefore, under the conditions provided above, the real (not apparent or phenomenological) b value should be smaller than 1.5. The difference between the apparent and theoretically maximum possible b value leads to the hypothesis that a single b value, of typically low accuracy, may be a poor estimate of local seismicity, especially if applied to very limited datasets of induced seismicity. As mentioned before, planar faults should not create anything larger than 3D fractal.

The above-mentioned relation between dimension D and b values (Kanamori and Anderson 1975), is a limited model, and some studies suggest different relationships between dimensionality D and b values (e.g.,

Hirata 1989; Henderson et al. 1994; Barton et al. 1999; Scholz 2002). In this study, we use a model with constant b values in the interval $[M_w^{\min}, M_w^{\max}]$ while other case studies show a more complex magnitude frequency distribution (e.g., Lasocki and Orlecka-Sikora 2008; Anderson and Luco 1983).

4 Conclusion

We propose a method and geomechanical model to determine the magnitude of a possible largest induced seismic event derived from the Gutenberg–Richter law and an estimate of total released seismic moment. The uncertainty of the resultant magnitude is then mainly governed by the accuracy of total released seismic moment estimation. Application of this model on various case studies from EGS and hydraulic fracturing stimulations showed the uncertainty smaller than 0.5

magnitude for EGS and smaller than 0.25 magnitude for hydraulic fracturing cases. Additionally, we show the prediction of possible magnitude from a known volume of injected fluid in the form of an interval of the most probable maximum magnitude and its maximum possible value.

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References

- Aki K (1981) A probabilistic synthesis of precursors phenomena. Earthquake prediction. Maurice Ewing series, 4 ed. Simson and Richards, AGU: Washington D.C., pp 566–575.
- Aki K, Richards PG (1980) Quantitative seismology theory and methods. W.H. Freeman, San Francisco
- Anderson JG, Luco JE (1983) Consequences of slip rate constraints on earthquake occurrence relation. Bull Seism Soc Am 73:471–496
- Bachmann CE, Wiemer S, Woessner J, Hainzl S (2011) Statistical analysis of the induced Basel 2006 earthquake sequence: introducing a probability-based monitoring approach for enhanced geothermal systems. Geophys J Int 186:793–807
- Baisch S, Carbon D, Dannwolf U, Delacou B, Devaux M, Dunand F, Jung R, Koller M, Martin C, Sartori M, Secanell R, Vörös R (2009a) Deep heat mining Basel—seismic risk analysis. SERIANEX study prepared for the Departement für Wirtschaft, Soziales und Umwelt des Kantons Basel-Stadt, Amt für Umwelt und Energie. <http://www.wsu.bs.ch/geothermie>.
- Baisch S, Vörös R, Weidler R, Wyborn D (2009b) Investigation of fault mechanisms during geothermal reservoir stimulation experiments in the Cooper Basin, Australia. Bull Seismol Soc Am 99(1):148–158
- Barton CA, Zoback MD, Moos D (1995) Fluid flow along potentially active faults in crystalline rock. Geology 23:683–686
- Barton D, Foulger G, Henderson J, Julian B (1999) Frequency magnitude statistics and spatial correlation dimensions of earthquakes at Long Valley caldera, California. Geophys J Int 138:563–570
- Blewitt G, Coolbaugh M, Holt W, Kreemer C, Davis JL, Bennet RA (2002) Targeting of potential geothermal resources in the Great Basin from regional relationships between geodetic strain and geological structures. Transactions Geothermal Resources Council 26:523–526
- Charlery J, Cuenot N, Dorbath L, Dorbath C, Haessler H, Frogneux M (2007) Large earthquakes during hydraulic stimulations at the geothermal site of Soultz-sous-Forets. International Journal of Rock Mechanics & Mining Sciences 44:1091–1105
- Christensen K, Olami Z (1992) Variation of the Gutenberg-Richter b values and nontrivial temporal correlations in a spring-block model for earthquakes. J Geophys Res 97(B6):8729–8735
- Dahm T, Becker D, Bischoff M, Cesca S, Dost B, Fritschen R, Hainzl S, Klose CD, Kühn D, Lasocki S, Meier T, Ohmberger M, Rivalta E, Wegler U (2013) Recommendation for the discrimination of human-related and natural seismicity. J Seismol 17:197–202
- ECOS (2002) Swiss Seismological Service, Earthquake Hazard and Risk, Earthquake Catalog of Switzerland. <http://histserver.ethz.ch>. Accessed 7 Aug 2013.
- Eisner L, Janská E, Oprsal I, Matoušek P (2011) Seismic analysis of the events in the vicinity of the Preese Hall well. In: Pater CJ, Baisch S, Geomechanical study of Bowland Shale seismicity—appendix 3. <http://www.cuadrillaresources.com/wp-content/uploads/2012/02/Geomechanical-Study-Appendix-3-2.11.2011.pdf>.
- Frohlich C, Hayward C, Stump B, Potter E (2011) The Dallas–Fort Worth earthquake sequence: October 2008 through May 2009. Bull Seismol Soc Am 101(1):327–340
- Gutenberg B, Richter CF (1954) Seismicity of the earth and associated phenomena, 2nd edn. Princeton University Press, Princeton
- Hallo M, Eisner L, Mohammed YA (2012) Expected level of seismic activity caused by volumetric changes. First Break 30:97–100
- Healy JH, Rubey WW, Griggs DT, Raleigh CB (1968) The Denver earthquakes. Science 161:1,301–1,310
- Henderson J, Main I, Pearce R, Takeya M (1994) Seismicity in north-eastern Brazil: fractal clustering and the evolution of the b value. Geophys J Int 116:217–226
- Hickman S, Barton CA, Zoback MD, Morin R, Sass J, Benoit R (1997) In situ stress and fracture permeability along the Stillwater fault zone, Dixie Valley, Nevada. Int J Rock Mech Min Sci 34(3):414
- Hirata T (1989) A correlation between the b value and the fractal dimension of earthquakes. J Geophys Res 94:7507–7514
- Kanamori H (1977) The energy release in great earthquakes. J Geophys Res 82(20):2981–2987
- Kanamori H, Anderson L (1975) Theoretical basis of some empirical relations in seismology. Bull Seism Soc Am 65:1073–1095
- King G (1983) The accommodation of large strains in the upper lithosphere of the earth and other solids by self-similar fault systems: the geometrical origin of b -value. Pageoph 121: 761–815
- Lasocki S, Orlecka-Sikora B (2008) Seismic hazard assessment under complex source size distribution of mining-induced seismicity. Tectonophysics 456(1–2):28–37
- Mahrer K, Ake J, Block L, O’Connell D, Bundy J (2005) Injecting brine and inducing seismicity at the world’s deepest injection well, Paradox Valley, Southwest Colorado. Dev Water Sci 52:361–375
- Manin YI (2006) The notion of dimension in geometry and algebra. Bull Am Math Soc New Ser 43(2):139–161
- Mavko G, Mukerji T, Dvorkin J (2009) The rock physics handbook, second edition: tools for seismic analysis of porous media. Cambridge University Press, Cambridge
- Maxwell SC, Shemeta J, Campbell E, Quirk D (2009) Microseismic deformation rate monitoring. EAGE Passive Seismic Workshop, Cyprus, A18.
- Maxwell SC, Rutledge J, Jones R, Fehler M (2010) Petroleum reservoir characterization using downhole microseismic monitoring. Geophysics 75(5):129–137

- McGarr A (1976) Seismic moments and volume changes. *J Geophys Res* 81(8):1487–1494
- Mukuhira Y, Asanuma H, Niitsuma H, Schanz U, Haring M (2008) Characterization of microseismic events with larger magnitude collected at Basel, Switzerland in 2006. *Geothermal Resources Council Transactions* 32:87–94
- Rutledge JT, Phillips WS (2003) Hydraulic stimulation of natural fractures as revealed by induced microearthquakes, Carthage Cotton Valley gas field, East Texas. *Geophysics* 68:441–452. doi:10.1190/1.1567214
- Rutledge JT, Phillips WS, Mayerhofer MJ (2004) Faulting induced by forced fluid injection and fluid flow forced by faulting: an interpretation of hydraulic-fracture microseismicity, Carthage Cotton Valley Gas Field. *Bull Seismol Soc Am* 94(5):1817–1830
- Sammis CG, Saito M, King GCP (1993) Fractals and chaos in the earth sciences. *Pure App Geo (Pageoph Topical Volumes)* 138(4):532–706
- Scholz C (2002) *The mechanics of earthquakes and faulting*, 2nd edn. Columbia University, New York. ISBN 9780521655408
- Schorlemmer D, Wiemer S, Wyss M (2005) Variations in earthquake-size distribution across different stress regimes. *Nature* 437:539–542
- Shapiro SA, Dinske C, Langenbruch C, Wenzel F (2010) Seismological index and magnitude probability of earthquakes induced during reservoir fluid stimulations. *Lead Edge* 29:304–309
- Shapiro SA, Krüger OS, Dinske C (2013) Probability of inducing given-magnitude earthquakes by perturbing finite volumes of rocks. *J Geophys Res Solid Earth* 118:1–19. doi:10.1002/jgrb.50264
- Turcotte DL (1989) A fractal approach to probabilistic seismic hazard assessment. *Tectonophysics* 167:171–177
- Urbancic TI, Trifu CI, Mercer RA, Feustel AJ, Alexander JAG (1996) Automatic time-domain calculation of source parameters for the analysis of induced seismicity. *Bulletin of Seismological society of America* 86(5):1627–1633
- USGS (2012) United States Geological Survey's (USGS) Earthquake Hazards Program, Global Earthquake Search. <http://earthquake.usgs.gov/earthquakes/eqarchives/>. Accessed 7 Aug 2013.
- Vulgamore T, Clawson T, Pope C, Wolhart S, Mayerhofer M, Machovoe S, Waltman C (2007) Applying hydraulic fracture diagnostics to optimize stimulations in the Woodford Shale, SPE Annual Technical Conference and Exhibition, paper SPE 110029, Anaheim.