Lecture course on

Sea level variations and global geodynamics

3. The "Sea Level Equation" - theory and applications

(November 9, start 13:10)

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Outline

1). Surface loads (“ice” and “water loads”)

2). Response to surface loads

3). “The Sea Level Equation” (with discussion)

4). Special topics: global RSL variations, a SLE benchmark, and present-day geodetic variations.

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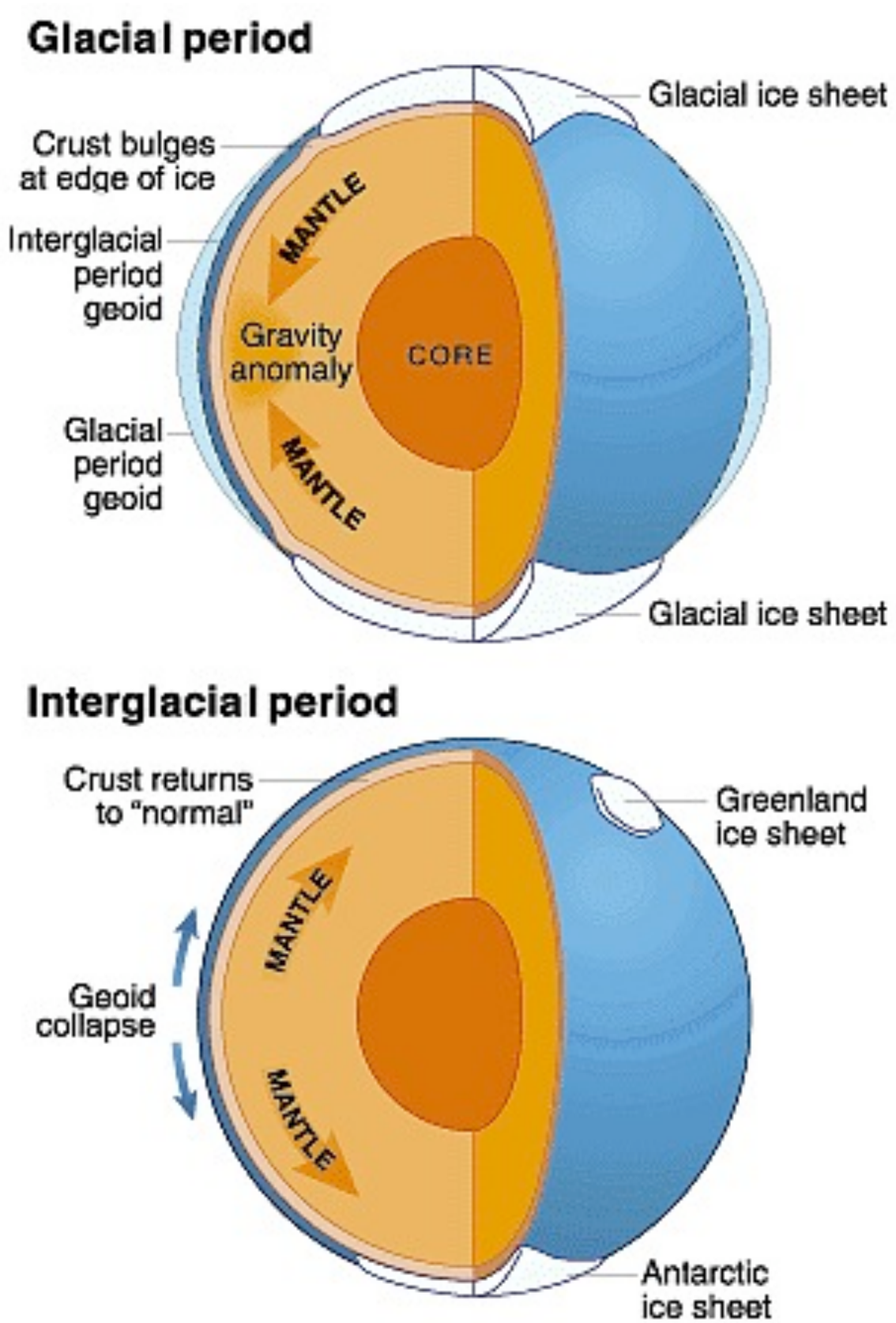


Surface loads:

ice and water
but also volcanoes...
time-dependent,
globally distributed,
often unknown a priori,
and interacting,
...

A **general load** can be decomposed as:

$$\mathcal{L}(\omega, t) = L_i + L_o$$

with:

$$\omega = (\theta, \lambda) \quad t = \text{ time,}$$

and:

$$L_i = \text{ice load}$$

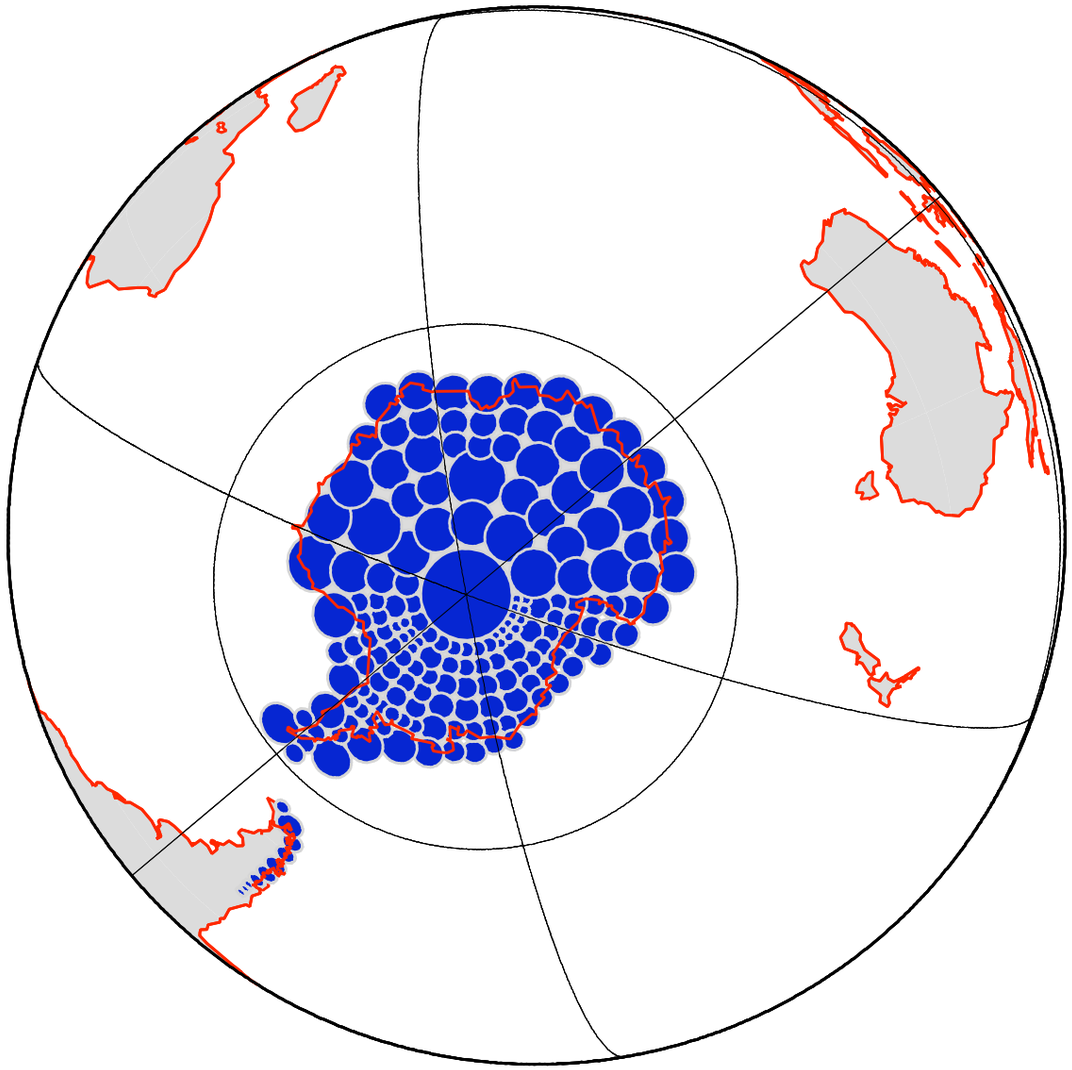
$$L_o = \text{ocean (water) load}$$

that describe changes in the weight of the ice sheets and the oceans, respectively:

L_i describes "Glacial isostasy"

L_o describes "Hydro isostasy"

Ice loads



Ice loads are related to ice thickness *variations,* NOT to absolute thickness (*):

$T(\omega, t)$ ice thickness

$I(\omega, t) = T(\omega, t) - T(\omega, t_0)$ ice thickness variation

The *"ice load"* is defined as:

$$L_i(\omega, t) = \frac{\text{Ice mass variation}}{\text{Unit surface}} = \rho_i I(\omega, t)$$

Hence the *variation* of ice sheets mass is:

$$m_i(t) = a^2 \int_i d\omega L_i(\omega, t)$$

(*) Note: the SLE does not include any "ABSOLUTE" quantity, only variations...

If the ice load has *fixed margins*, the load can be "factorized":

$$L_i(\omega, t) = \sigma(\omega) f(t)$$

where:

$\sigma(\omega)$ "load function" a closed form often exists

$f(t)$ "load time-history" e. g., in the form of steps...

Generally the load time history has a step-like form:

$$f(t) = f_0 + \sum_i^N (f_{i+1} - f_i)[H(t - t_{i+1}) - H(t - t_i)]$$

The case of a complex ice load with *time-evolving margins* can be dealt with by a **combination** of factorized loads of this type.

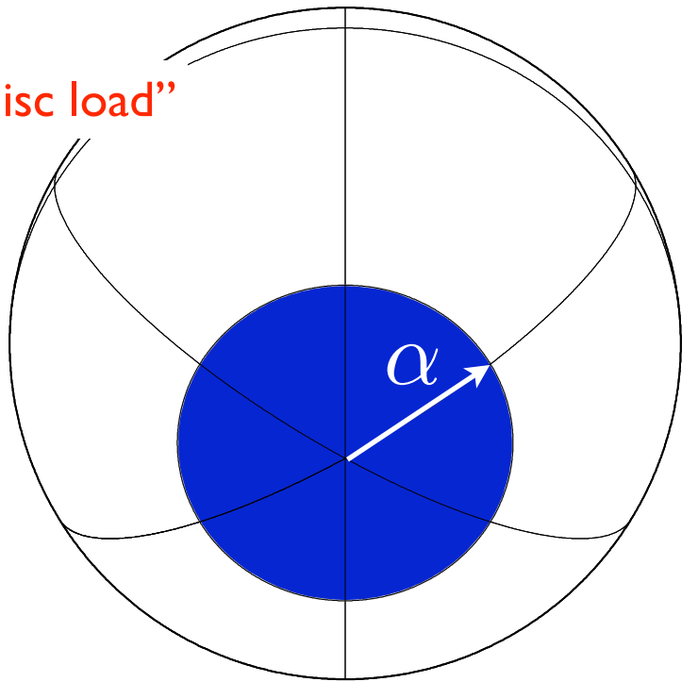
Northern hemisphere glaciation during the last ice ages. The set up of 3 to 4 km thick ice sheets caused a sea level lowering of about 120 m.

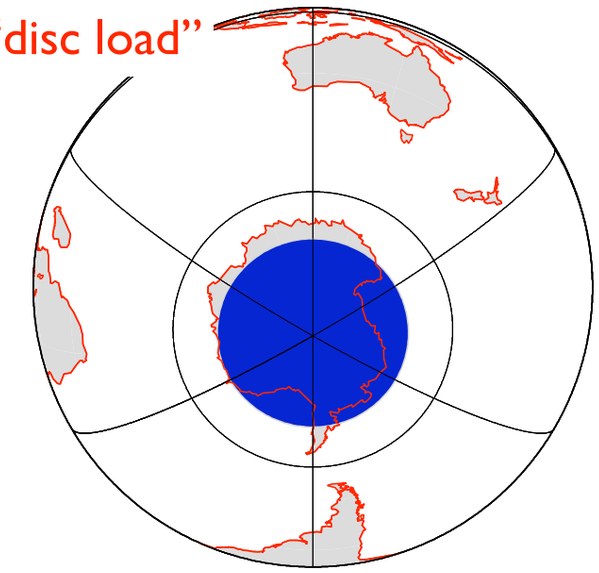
R
M
Alps
B
St
M
Himalaya
Ural
C
Greenland
North
Pole
W
Gobi
NORTH
AMERICA
P
Rocky Mountains
A
Summer
sea ice
LA
T
Winter sea ice

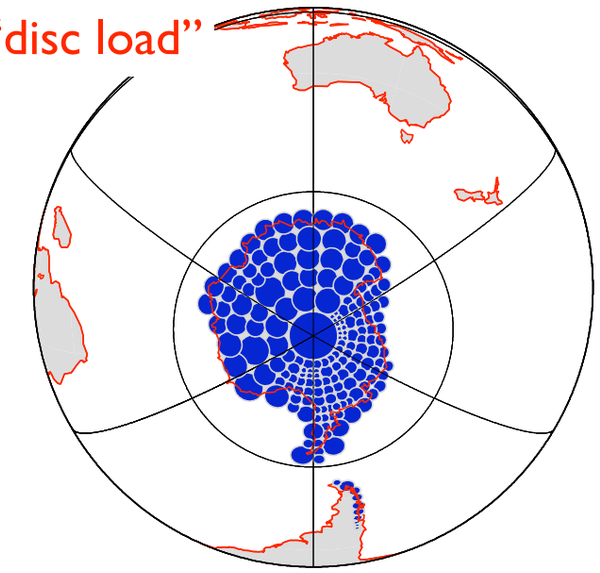
Ice loads northern hemisphere LGM

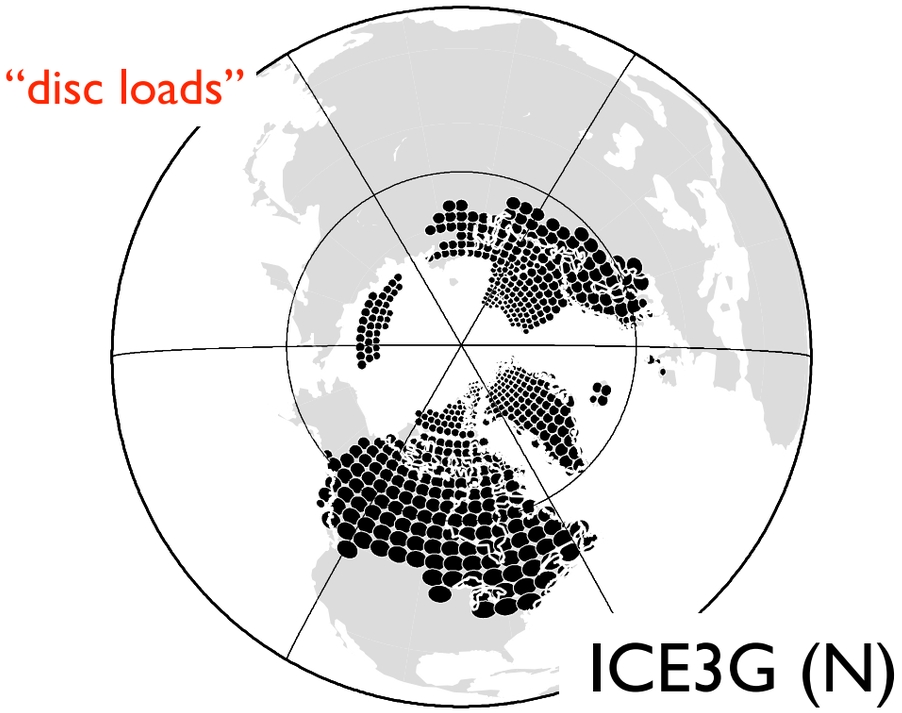
Ice loads

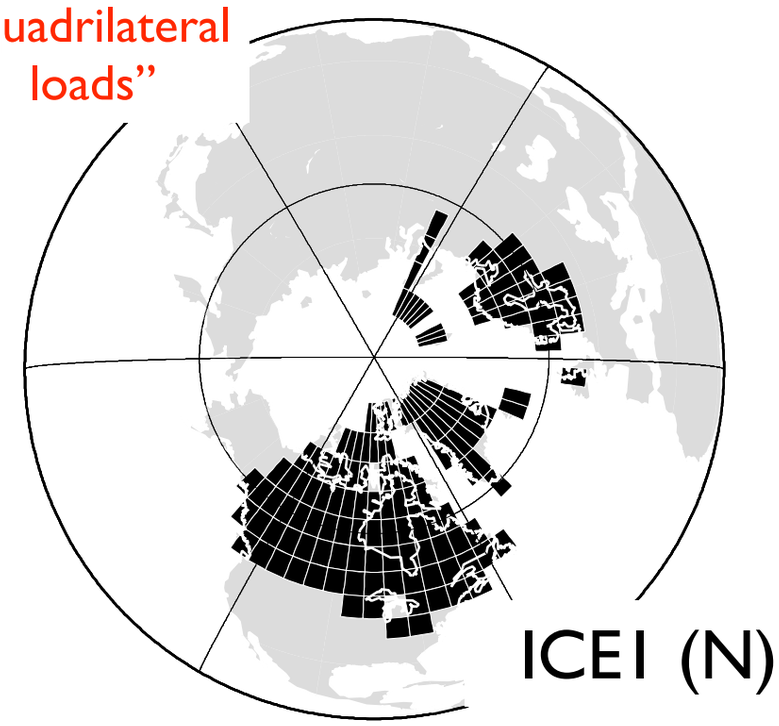
examples of ice loads





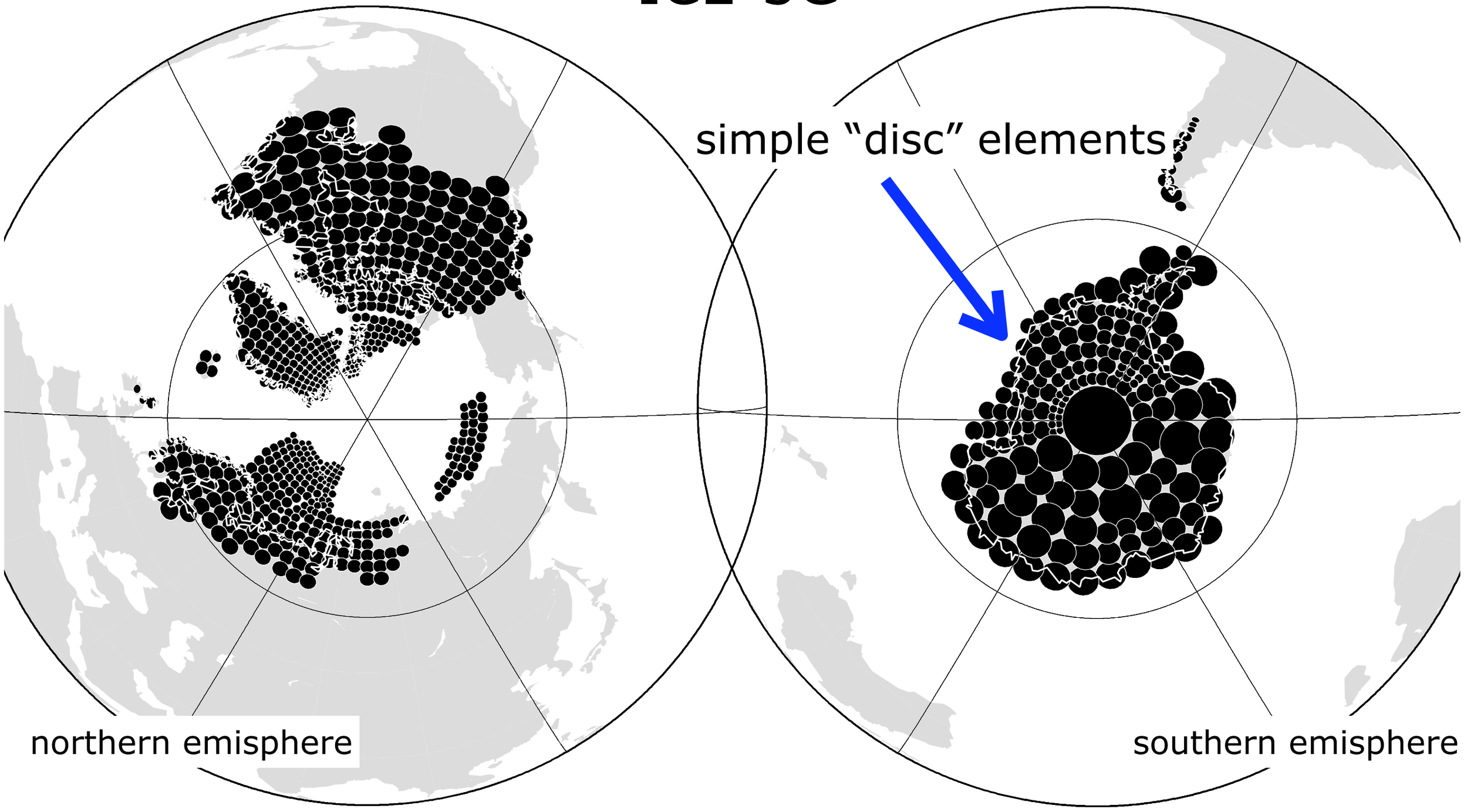






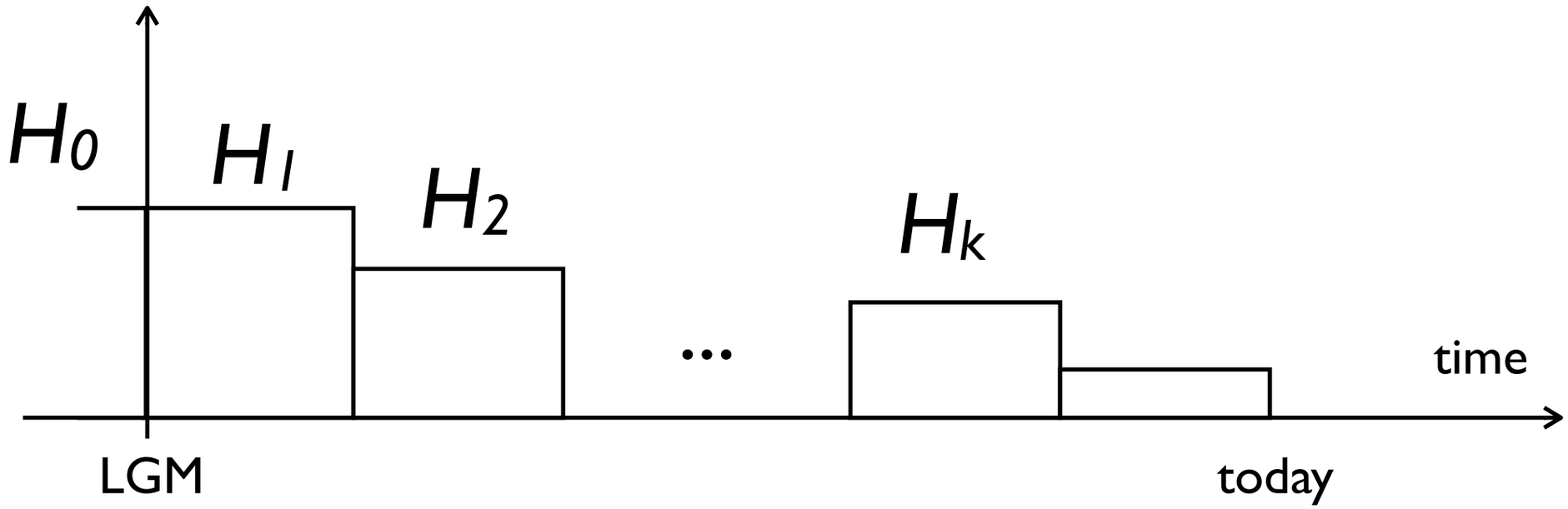
examples of ice loads

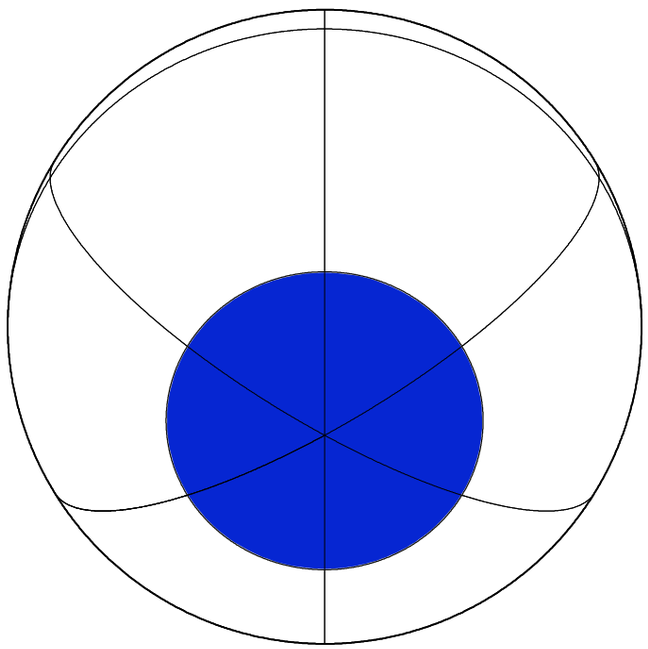
ICE-3G

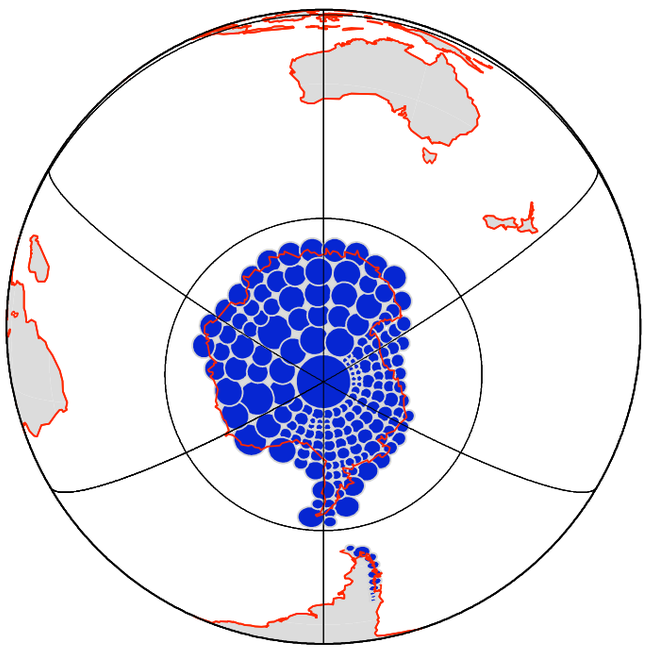


Tushingham, A. M., Peltier, W. R., 1991.
Ice-3G: a new global model of late Pleistocene deglaciation based upon geophysical prediction of post--glacial sea level change. Journal of Geophysical Research 96, 4497-4523.

The thickness of each disc varies in a “piecewise manner”
(Isostatic equilibrium *may be* assumed before the LGM)







ice1.dat
The full ICE1
Authors: Peltier and Andrews, GJRAS, 1976.
153 number of elements
20   header lines
1   lines for each element
18   time since the LGM (kyrs)
1   lenght of each time step
5   size of the 'rectangular' elements (deg)

For each ice element, the data are presented in one line, with the following meaning:
colatitude and longitude of the centroid of the ('rectangular') elements (deg), and
disc element thickness (m) at various times (as integers). The first value is
thickness before the beginning of deglaciation (i.e. from -infinity to 18 kyrs BP),
the 18 elements that follow are ice thickness at 1 kyrs intervals up to present time.
File SH1.F of SELEN reads this file and indicates with h_0 ice thickness
before melting begins, h_1 between 18 and 17 kyrs BP, ..., and with h_18 ice thickness
between 1 and 0 kyrs BP. With h_19=h_18 SELEN denotes the ice (constant) thickness in
the future. GS July 28 2007.

```
 10 265 500 300 300 300 300 200 200 200 200 0 0 0 0 0 0 0 0 0 0
 10 270 1000 850 850 850 850 800 800 800 800 900 900 500 500 500 500 500 500 500 500
 10 275 2000 1250 1250 1250 1250 1200 1200 1200 1200 500 500 0 0 0 0 0 0 0 0
 10 280 2000 1250 1250 1250 1250 1200 1200 1200 1200 1000 1000 500 500 500 500 500 500 500 500
 10 285 1000 650 650 650 650 600 600 600 600 200 200 500 500 500 500 500 500 500 500
 15 250 200 300 300 300 300 300 300 0 0 0 0 0 0 0 0 0 0 0 0
 15 255 200 1000 1000 1000 1000 1000 1000 0 0 0 0 0 0 0 0 0 0 0 0
 15 260 1000 1400 1400 1400 1400 1400 1400 0 0 0 0 0 0 0 0 0 0 0 0
 15 265 1000 1500 1500 1500 1500 1500 1500 600 600 0 0 0 0 0 0 0 0 0 0
 15 270 1000 1200 1200 1200 1200 1200 1200 900 900 700 700 0 0 0 0 0 0 0 0
 15 275 1000 900 900 900 900 800 800 800 800 900 900 500 500 500 500 500 500 500 500
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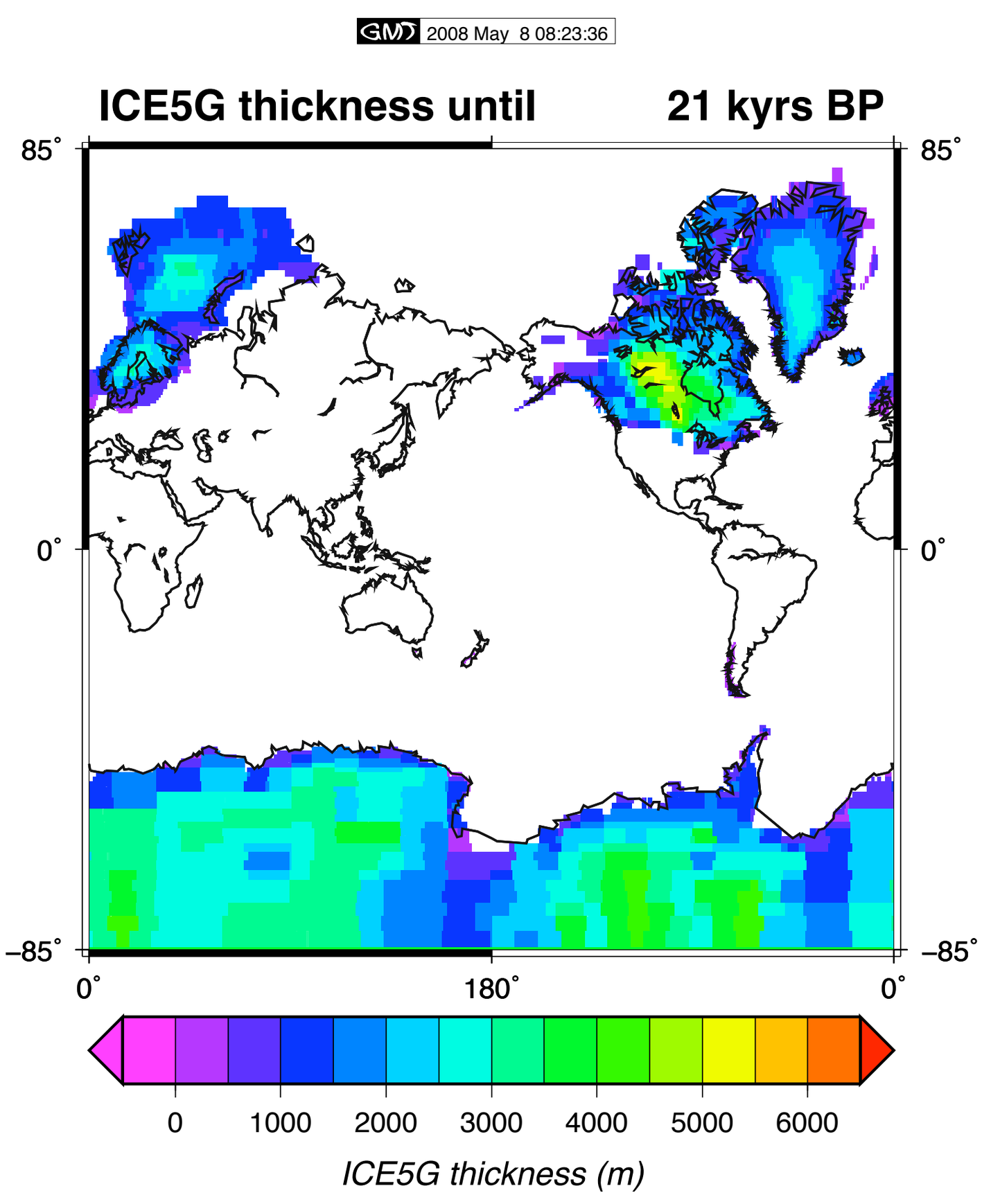
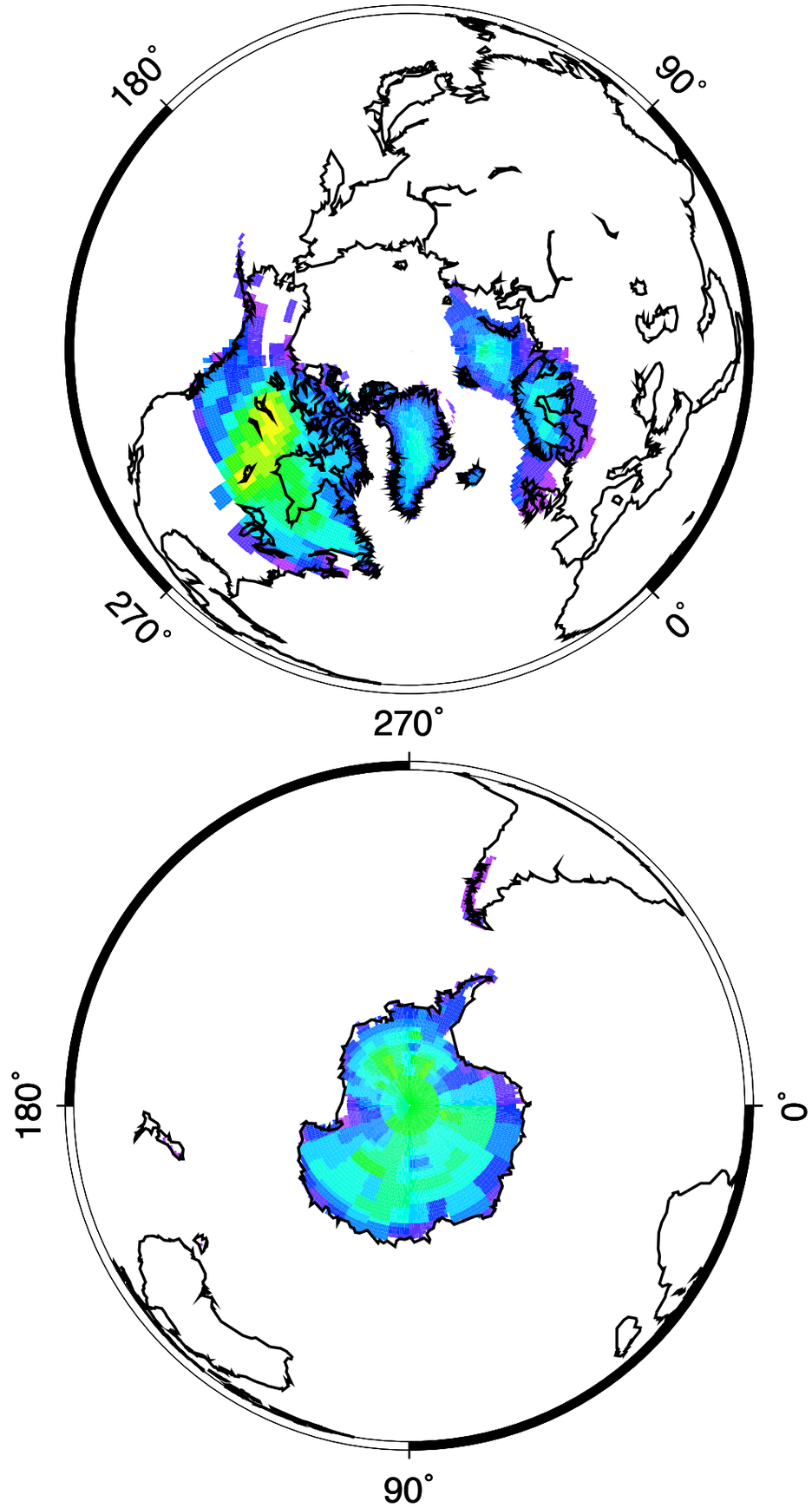
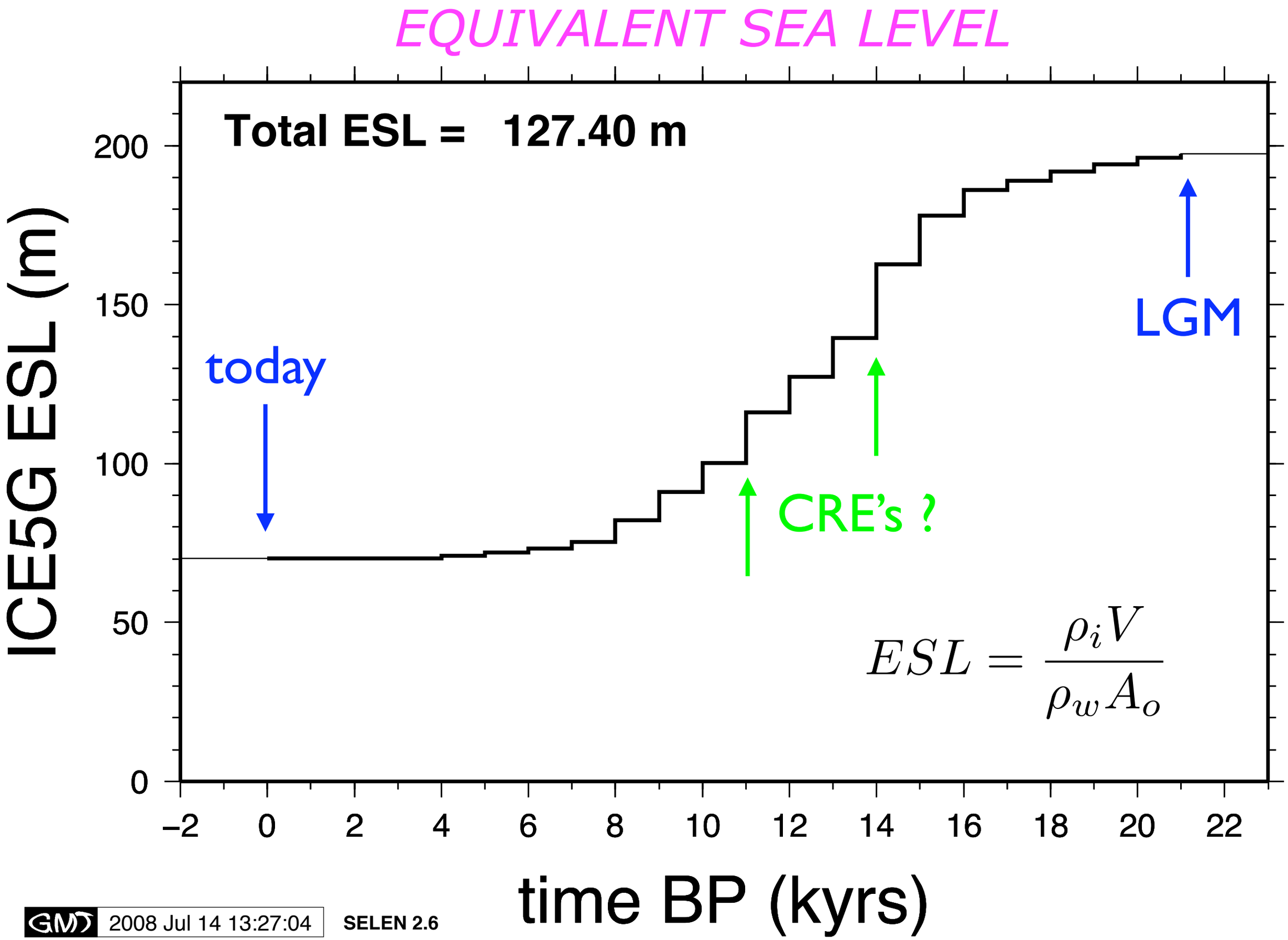
Ice loads

examples of ice loads - ICE5G

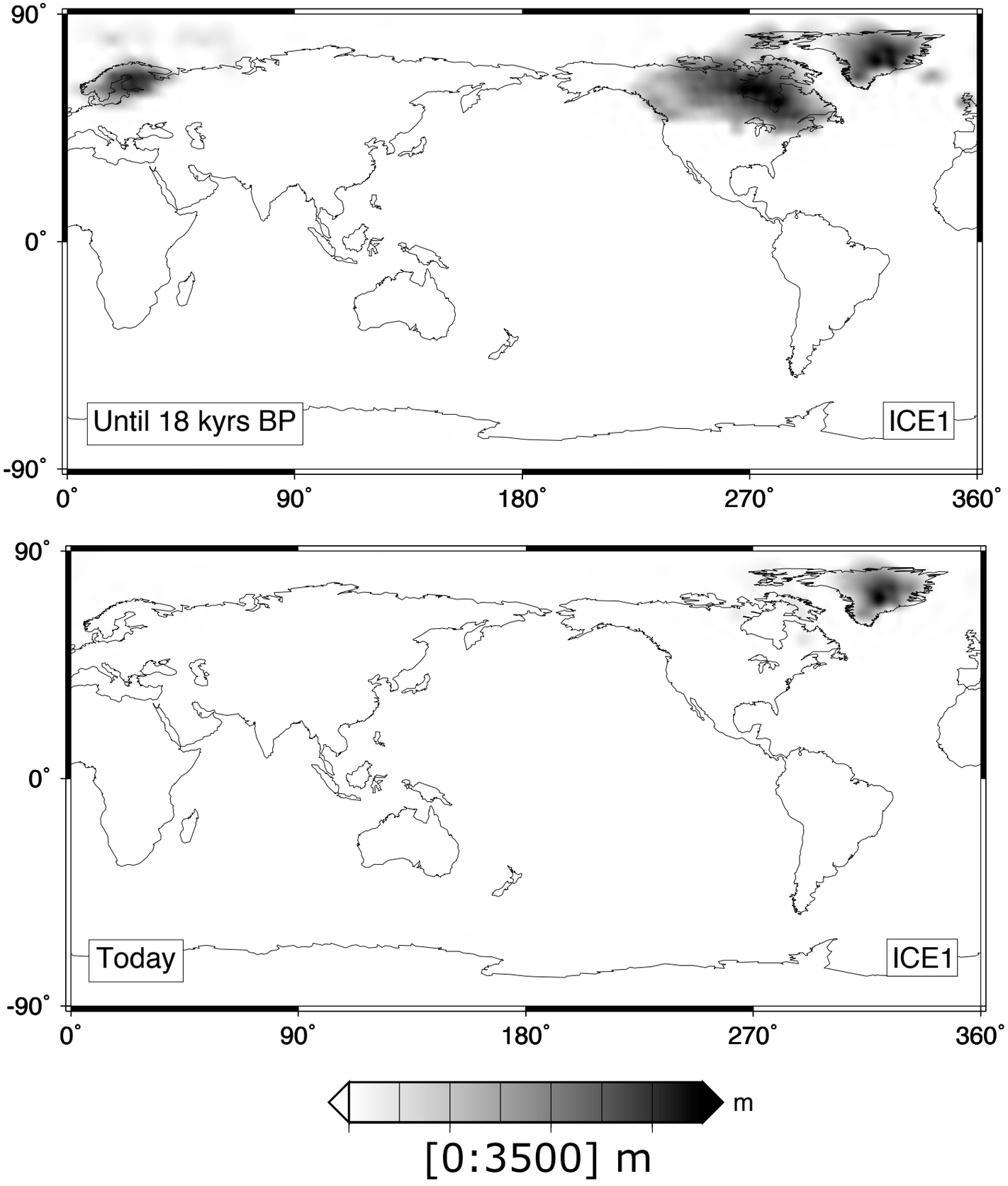





EQUIVALENT SEA LEVEL
Total ESL = 127.40 m
ICE5G ESL (m)
200
150
100
50
0
today
LGM
CRE's ?
$ESL = \frac{\rho_i V}{\rho_w A_o}$
–2
0
2
4
6
8
10
12
14
16
18
20
22
time BP (kyrs)
GMT 2008 Jul 14 13:27:04
SELEN 2.6

Ice loads

examples: ICE-1 and ICE-3G



ICE1(LGM)

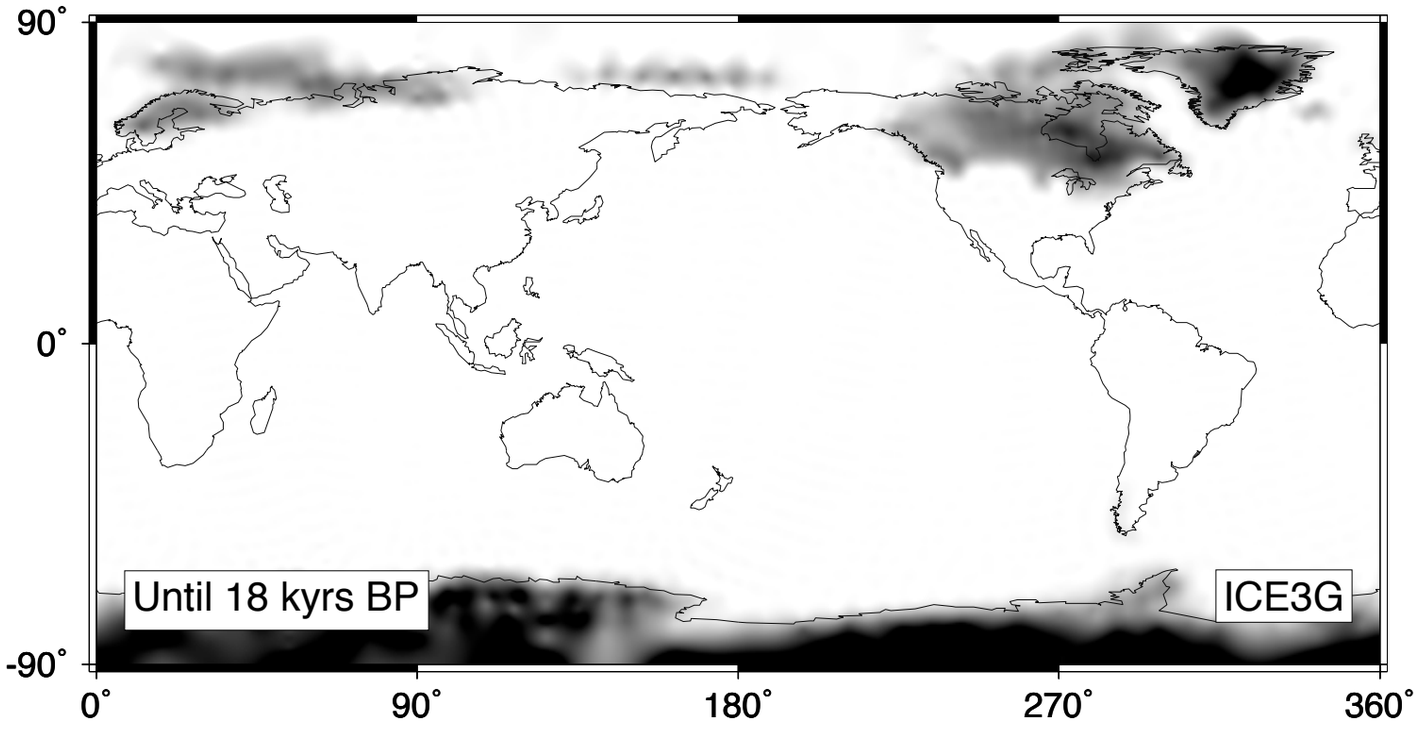
Peltier & Andrews, 1976
ESL = 80m
Based on glaciology

ICE1(0 BP)

Peltier & Andrews, 1976
ESL = 80m
Based on glaciology

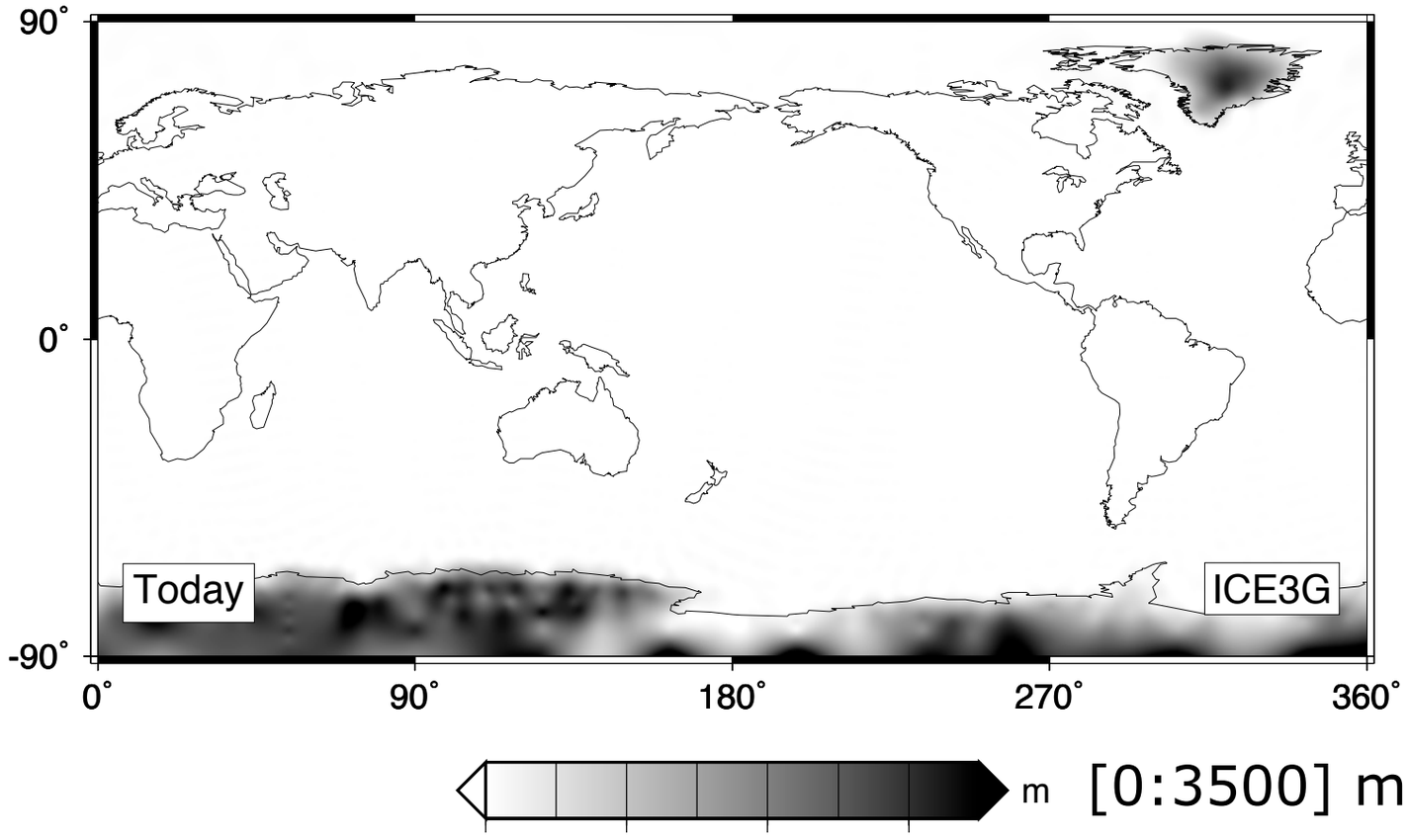
Ice loads

examples: ICE-1 and ICE-3G



ICE3G (LGM)

Tushingham & Peltier, 1992
ESL = 120 m
Recently updated to ICE5G
Based on glaciology and constrained by RSL data

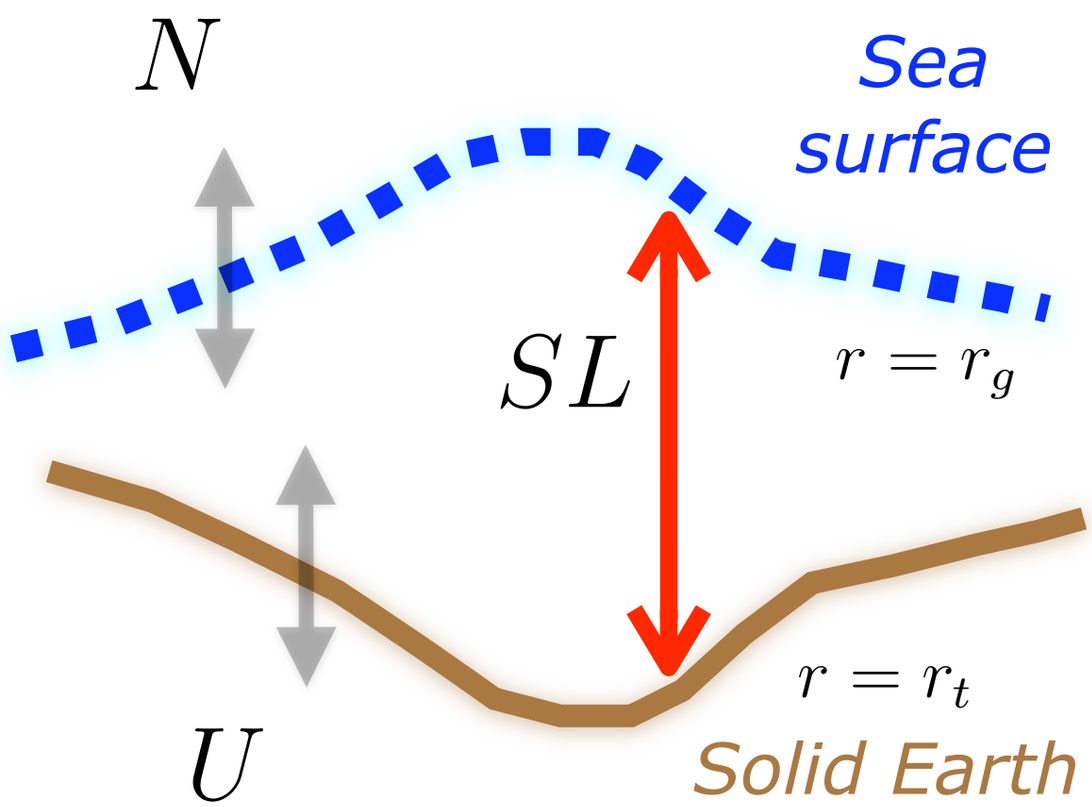


ICE3G (0 BP)

Tushingham & Peltier, 1992
ESL = 120 m
Recently updated to ICE5G
Based on glaciology and constrained by RSL data

(melt)water load

Before introducing the "water load", we define important quantities:



Illusttratrion of Sea Level

$SL = r_g - r_t$ Sea Level

N Sea surface height variation

U Vertical displacement

Sea Level change:

$$SL - SL_0 = S$$

Sea Level change = geoid height variation - vertical displacement

"SEA LEVEL EQUATION" (SLE): $S = N - U$

The "water load" is defined as:

$$L_o(\omega,t) = \frac{\text{Oceans mass variation}}{\text{Unit surface}}$$

or equivalently:

$$L_o(\omega,t) = \rho_w I(\omega,t)\mathcal{O}(\omega,t)$$

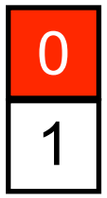
with:

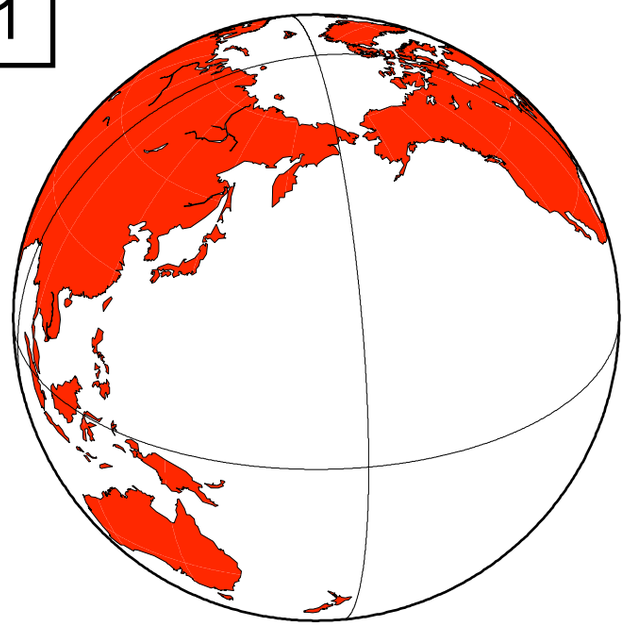
ρ_w water density

$S(\omega,t)$ sea level change

$\mathcal{O}(\omega,t)$ (time-dependent) ocean function

$$\mathcal{O}(\omega,t) = \begin{cases} 1, & \omega \in \text{oceans} \\ 0, & \omega \notin \text{oceans} \end{cases}$$





"Ocean" function

The water load can be estimated assuming:

1) *a rigid Earth:* Love numbers = 0, and

2) *no gravitational interactions*: Newton constant = 0.

In this case, the sea level variations are "*EUSTATIC*", from "*eustatism*" a word coined by the english geologist Eduard Suess (1903)

$$S = \text{constant}$$

Eduard Suess
(20. August 1831 in London - † 26. April 1914 in Wien)*

war ein österreichischer Geologe und Politiker des 19. Jahrhunderts und wurde bekannt als Experte für den tektonischen Bau der Alpen. Auf ihn sind zwei wesentliche palä-geographische Entdeckungen zurückzuführen: der ehemalige Superkontinent Gondwana und das Tethysmeer.

The law of eustasy can be stated as follows:

$$S(\omega,t) = -\frac{\text{Ice mass variation}}{\text{Water density} \times \text{Area of the oceans}}$$

In other words:

$$S = S^E(t) = -\frac{m_i(t)}{\rho_w A_o}$$ Sea level change is everywhere the *SAME*

$$L_o^E(\omega,t) = -\frac{m_i(t)}{A_o}\mathcal{O}(\omega,t)$$ Eustatic water load is *constant* over the oceans

$$m_i(t) = M_i(t) - M_i(t_r)$$ Ice mass *variation*

$$M_i(t)$$ Effective ice mass

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The Green function can be used to compute the response of the Earth to FINITE-SIZED loads *(this is the reason of its usefulness!)*.

The vertical displacement U and the variation of the gravitational potential Φ at any point ω and time t result from changes of both ice and water mass distributions at any other point ω' and times $t' \leq t$.

Mathematically, this can be stated by *convolution integrals* over the surface of the Earth:

$$\left\{ \begin{array}{c} U \\ \Phi \end{array} \right\}(\omega, t) = \int_{-\infty}^{t} dt' \int_{e} dA' \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\}(\alpha, t - t') \mathcal{L}(\omega', t')$$

where $\mathcal{L}(\omega, t)$ is TOTAL surface load and $dA = a^2 \sin\theta d\theta d\lambda$ is the element of area on the surface of the sphere.

We introduce the simplified notation:

$$(G \otimes_{\mathcal{D}} L)(\omega, t) \equiv \int_{-\infty}^{t} dt' \int_{\mathcal{D}} dA' \; G(\alpha, t - t') L(\omega', t')$$

where $\mathcal{D}$ is a subset of the surface of the Earth, $L(\omega, t)$ is a generic surface load, and *G* is a given Green function.

Thus:

$$\left\{ \begin{array}{c} U \\ \Phi \end{array} \right\} (\omega, t) = \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_e \mathcal{L} = \left\{ \begin{array}{c} U_i + U_o \\ \Phi_i + \Phi_o \end{array} \right\}$$

where:

$$\left\{ \begin{array}{c} U_i \\ \Phi_i \end{array} \right\} (\omega, t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_i \rho_i I, \quad \text{and} \quad \left\{ \begin{array}{c} U_o \\ \Phi_o \end{array} \right\} (\omega, t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_o \rho_w S$$

are the “ice” and “ocean components” of *U* and Φ , respectively.

An observation. In the case of RIGID EARTH,

$G_u = 0$ but: $G_\phi \neq 0$

because of the "1" term (the "DIRECT" effect) in the expression for the GF for the gravity potential.

Hence, from:

$$\left\{ \begin{array}{c} U_i \\ \Phi_i \end{array} \right\}(\omega,t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_i \rho_i I, \quad \text{and} \quad \left\{ \begin{array}{c} U_o \\ \Phi_o \end{array} \right\}(\omega,t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_o \rho_w S$$

we see that, for a RIGID EARTH: $U_i = U_o = 0$ but:

$\Phi_i \neq 0$ Gravitational interaction: ice sheets - oceans

$\Phi_o \neq 0$ Gravitational interaction: oceans - oceans
(we say that the oceans are "self-gravitating").

Equations:

$$\left\{ \begin{array}{c} U_i \\ \Phi_i \end{array} \right\}(\omega,t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_i \rho_i I, \quad \text{and} \quad \left\{ \begin{array}{c} U_o \\ \Phi_o \end{array} \right\}(\omega,t) \equiv \left\{ \begin{array}{c} G_u \\ G_\phi \end{array} \right\} \otimes_o \rho_w S$$

pose a serious problem:

Solving e. g. for U_i and U_o requires knowledge of I and S. While I may be "assumed" to be known a-priori (e. g., from geological evidence), S will be only known after vertical displacements (and geoid) are determined !

So, to compute S, we need U (and the geoid), but to compute U (and the geoid), we need S !

How to escape from this *circular argument* ?

There are ***two*** possible solutions!

The ***first*** is to assume a-priori eustatic sea--level variations.

This leads to what I call the "simplified GIA problem", in which an *eustatic water load* is assumed (*). The implified GIA problem has been dealt with in Lesson 2 ---> see the benchmark paper.

The major limitation of this approach is that it cannot accurately describe sea level variations in the *far-field of the ice sheets,* where water load is comparable (or in excess to) ice load.

In the near-field, the simplified GIA problem usually provides accurate solutions...

We can do better, but we must follow a ***second***, more difficult route...

(*) Spada G., and Stocchi, P., 2006. The Sea Level Equation, Theory and Numerical Examples, ISBN: 88-548-0384-7, 96 pp., Aracne, Roma.

Is sea level change uniform across the oceans,
far away from the (former) ice sheets ?

RSL observations

"RSL zones"

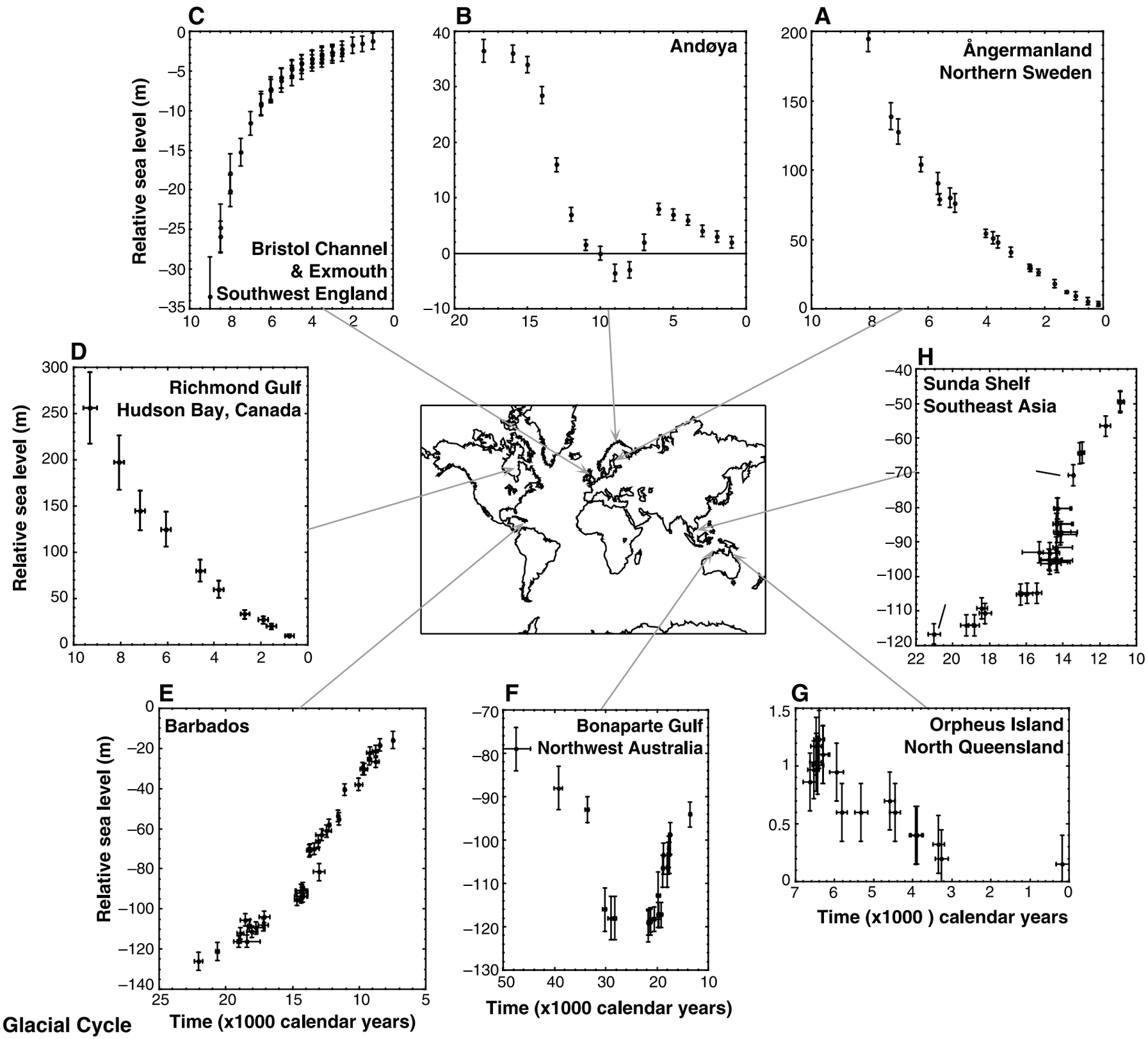

Fig. 2. Observed spatial variability of sea level change since the time of the LGM from tectonically stable areas or areas where the tectonic rate is known and has been removed from the observed signal. (**A**) Ångerman, Gulf of Bothnia, Sweden (*13*). (**B**) Andøya, Nordland, Norway (*12*). (**C**) South of England (*14*). (**D**) Hudson Bay, Canada (*4*). (**E**) Barbados (*16*–*18*). (**F**) Bonaparte Gulf, northwest Australia (*27*). (**G**) Orpheus Island, North Queensland, Australia [(*23*) and unpublished Australian National University data]. (**H**) Sunda Shelf, southeast Asia (*15*). Note the different time and amplitude scales. In the examples illustrated, all observed depths or elevations of the sea level indicators have been reduced to mean sea level. All time scales are in calendar years.

Downloaded from www.sciencemag.org

Sea Level Change Through the Last Glacial Cycle
Kurt Lambeck, *et al.*
Science **292**, 679 (2001);
DOI: 10.1126/science.1059549

no, it is not.

We definitively need a
“Gravitationally Self-Consistent”
theory for Sea Level change!

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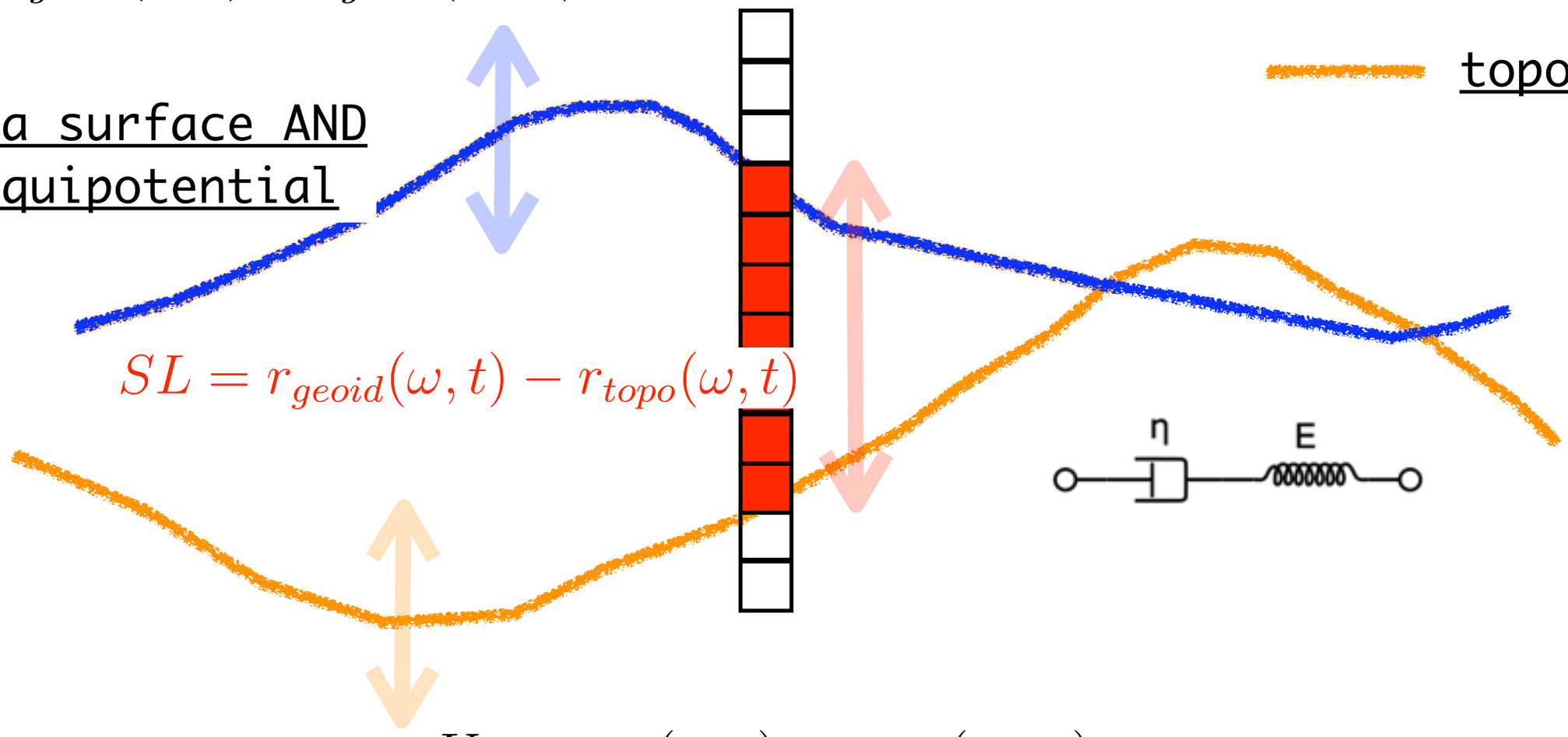
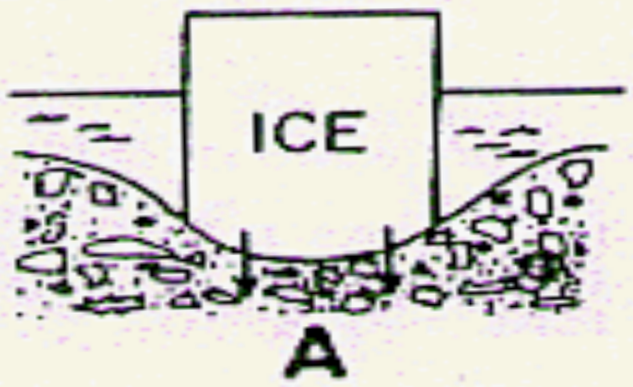
$$N = r_{geoid}(\omega, t) - r_{geoid}(\omega, t_0)$$



$$S = SL(\omega, t) - SL(\omega, t_0) = N - U$$

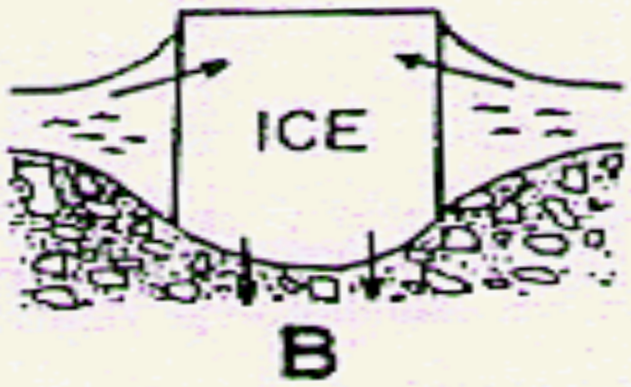
"Sea Level Equation"

ICE LOADING

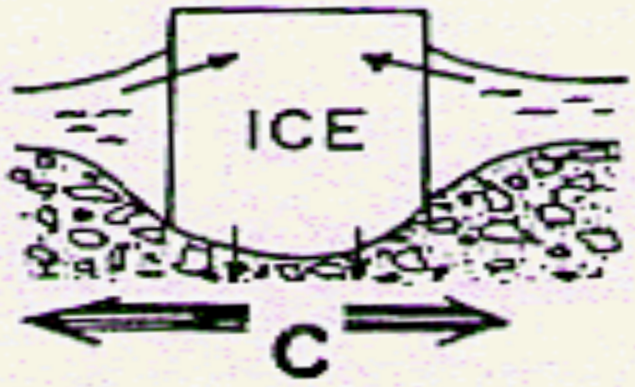


A

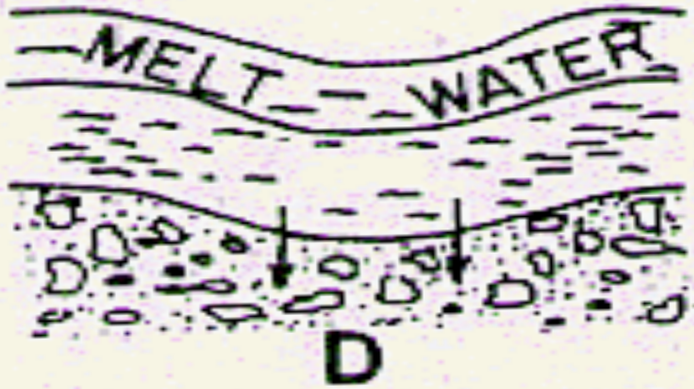
GRAVITATIONAL ATTRACTION



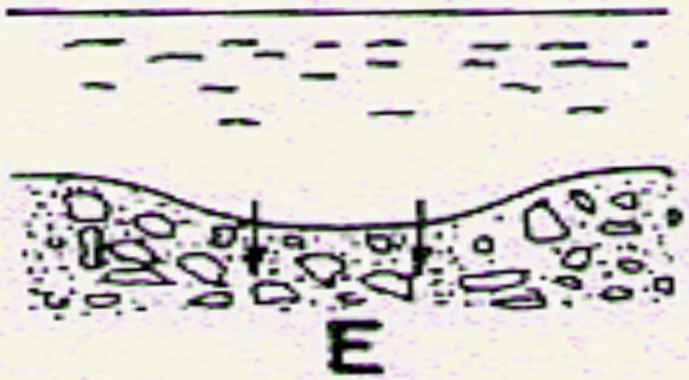
B



C



D



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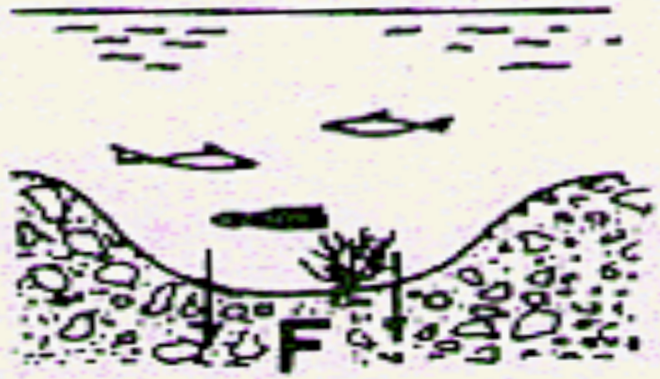
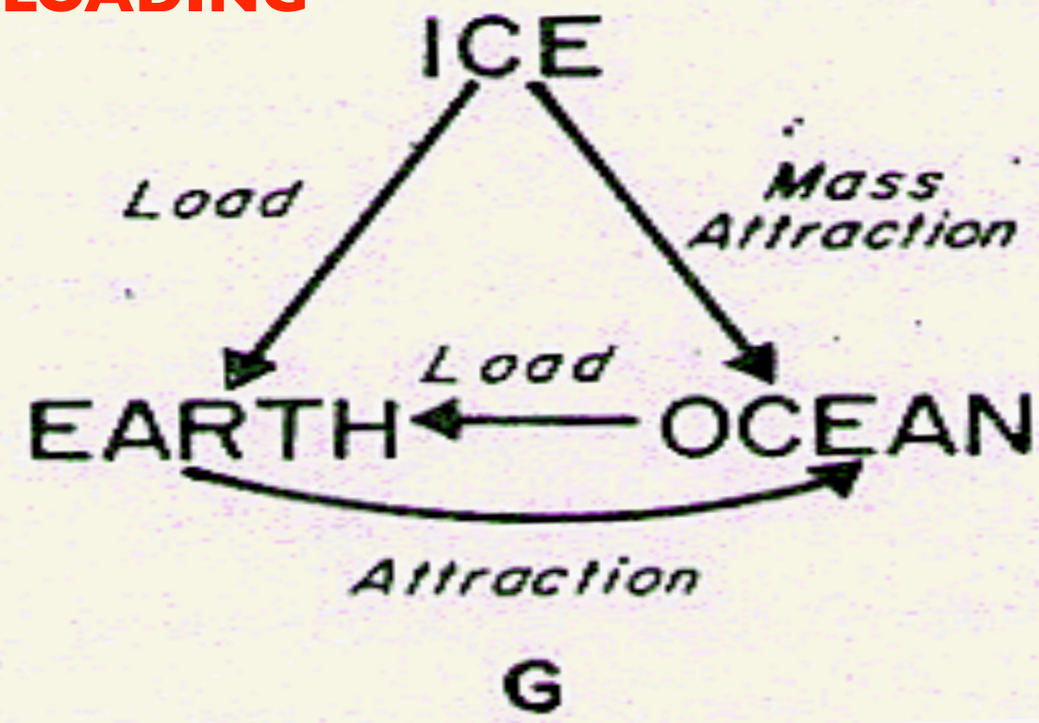


F

MELT WATER LOADING

DELAYED RESPONSE



G

Clark & Lingle, 1979

Write the sea surface variation as: $$N = \frac{\Phi}{\gamma} + c$$

By the elementary form of the SLE: $$S = \frac{\Phi}{\gamma} + c - U$$

Impose mass conservation: $$c = -\frac{m_i}{\rho_w A_o} - \overline{\left(\frac{\Phi}{\gamma} - U\right)}$$

Hence:

$$S = \left(\frac{\Phi}{\gamma} - U\right) + S^E - \overline{\left(\frac{\Phi}{\gamma} - U\right)}$$

Using the "convolution" expressions for Φ and U finally gives:

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

with:

$S =$ sea level change

$\rho_i, \rho_w =$ ice and water density

$G_s =$ sea level Green function

$I =$ ice thickness variation

$m_i =$ ice mass variation

$A_o =$ area of the oceans

$\otimes_i, \otimes_o =$ 3(2+1)D convolutions

$\overline{(\ldots)} =$ ocean average

More details can be found in these papers:

✓Farrell, W. E., Clark, J. A., 1976. On postglacial sea-level, Geophysical Journal of the Royal Astronomical Society 46, 647-667.

✓Mitrovica, J. X., Milne, G. A., 2003. On post-glacial sea--level: I. General theory. Geophysical Journal International, doi:10.1046/j.1365-246X. 2003.01942.x

✓Spada, G., Stocchi, P., 2007. SELEN: a Fortran 90 program for solving the "Sea Level Equation", Comput. and Geosci., 33 (4), 538-562. doi: 10.1016/ j.cageo.2006.08.006.

✓Spada G., & Stocchi, P., 2006. The Sea Level Equation, Theory and Numerical Examples, ISBN: 88-548-0384-7, 96 pp., Aracne, Roma.

A discussion about the SLE
(in 10 points and remarks)

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S + S^E - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

1) SL Green function.

The SLE contains the "Sea Level Green function", which combines the vertical displacement and gravity potential GFs:

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

$$\frac{G_s}{\gamma}(\alpha, t) \equiv \frac{G_\phi}{\gamma} - G_u$$

Depends upon "h" and "k", not on "l"

Horizontal movements are *caused* by sea level change S, but they *do not influence* S.

Their importance has been recognized since the 90's in de-glaciated areas. Rule of thumb: 10 times smaller that vertical motions...

2) Geoid.

"The" equipotential surface that represents the free surface of the oceans.

The SLE ensures that the time-evolving geoid is always an equipotential surface.

3) Mass conservation.

The SLE accounts automatically for *mass conservation* within the system composed by (oceans + ice sheets + solid Earth). The atmosphere is ignored in GIA modeling...

In GIA computations, density of ice and water are assumed to be *constant. Regardless of temperature variations of the oceans, dynamics of the ice sheets, etc...*

Regarding ice: ice is a "boundary condition". No physical coupling with bedrock, except for the effect of weight.

More observations on the SLE...

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

4) Nature of the SLE. The SLE is a **linear**, ***integral equation***, since the **unknown** *S* also appears within convolution integrals !

This reflects the simple idea that sea level change at one point ω and time t depends on sea level change at any point ω' and at any time $t' \leq t$!

In other words... << the problem of sea level change can not be reduced to a regional scale >>.

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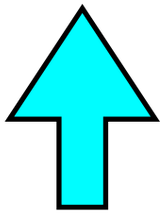

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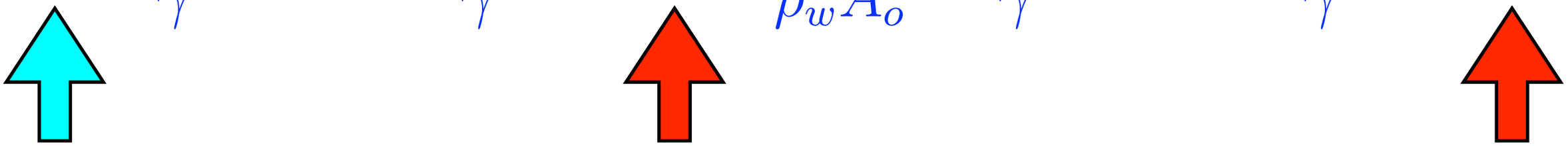

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More observations...

5) Eustasy. The SLE has a very special solution when several terms are dropped:

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

We are left with:

$$S \equiv S^E = -\frac{m_i}{\rho_w A_o}$$

that represents the "eustatic" solution of Suess.

For a RIGID, non-gravitating Earth (Green functions = 0), the SLE has the eustatic solution. The solution is only determined by the mass variation of the ice sheet and the area of the oceans !

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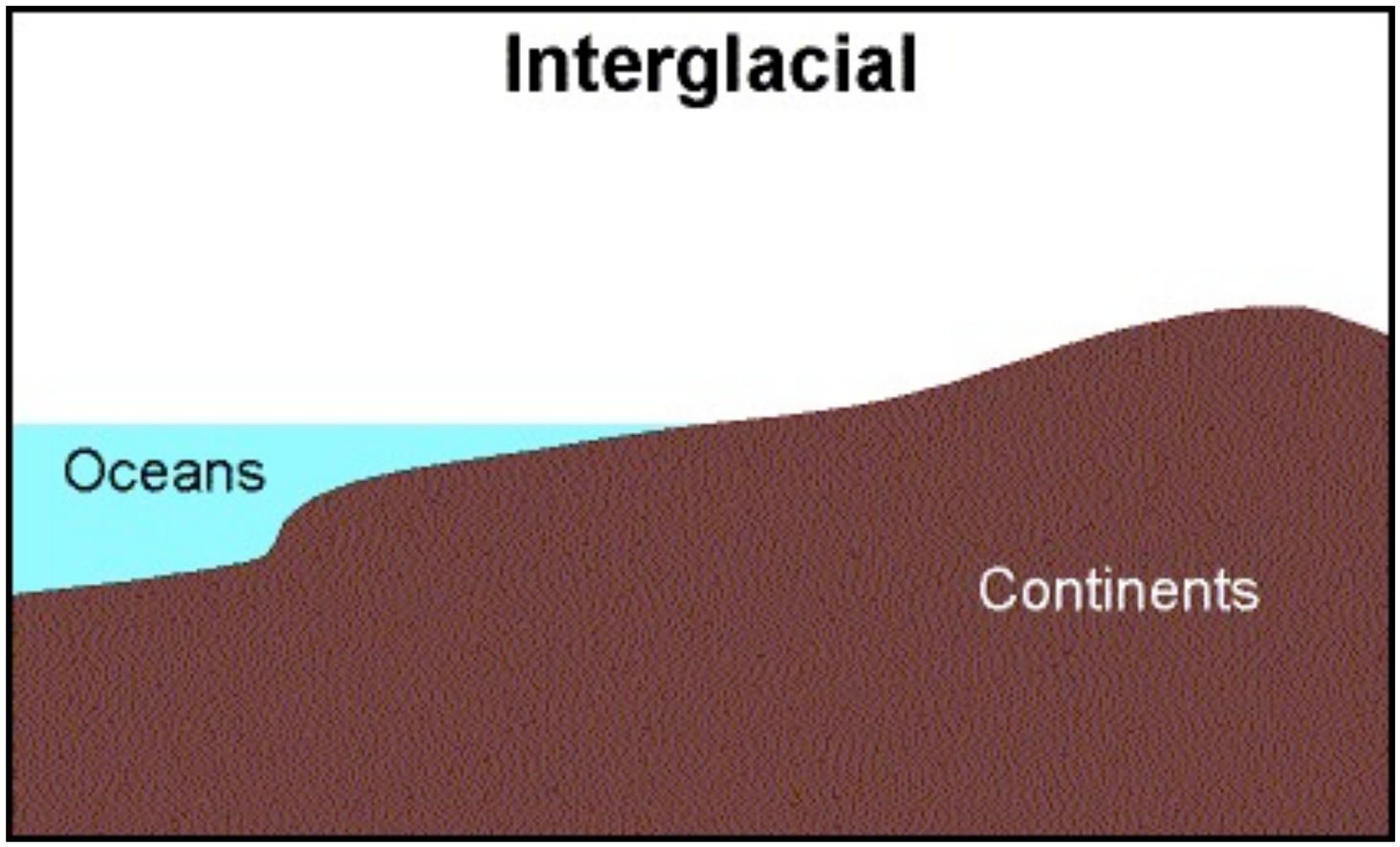
5) Eustasy.



- Eustatic Change -

Because of glacial fluctuations true, or eustatic, sea level (which those of us who live along oceanic coasts may think of as a constant, ignoring waves, tides, and storms) varies across spans of observation ranging from decades to hundreds of thousands of years. This variation, in turn, is superimposed on sea level fluctuations across millions of years caused by changes in the volume of the ocean basins.

$$S = S^E(t) = -\frac{m_i(t)}{\rho_w A_o}$$

is solution of the SLE if:

$$U = 0 \quad \text{and} \quad \Phi = 0$$

that is, if:

$$G_u = G_\phi = 0$$

RIGID EARTH
+
NO GRAVITATION

7) More observations on Eustasy.

Since the real Earth is ***not*** rigid and Newton's constant is ***not*** zero, we must expect that the melting of the ice sheets actually produces spatially non-uniform sea level variations, which may significantly depart from eustasy.

However, the SLE has a peculiar property. We take the ocean-average of it:

$$\overline{S} = \frac{\rho_i}{\gamma}\overline{G_s \otimes_i I} + \frac{\rho_w}{\gamma}\overline{G_s \otimes_o S} + \overline{S^E} - \frac{\rho_i}{\gamma}\overline{\overline{G_s \otimes_i I}} - \frac{\rho_w}{\gamma}\overline{\overline{G_s \otimes_o S}}$$

Hence:

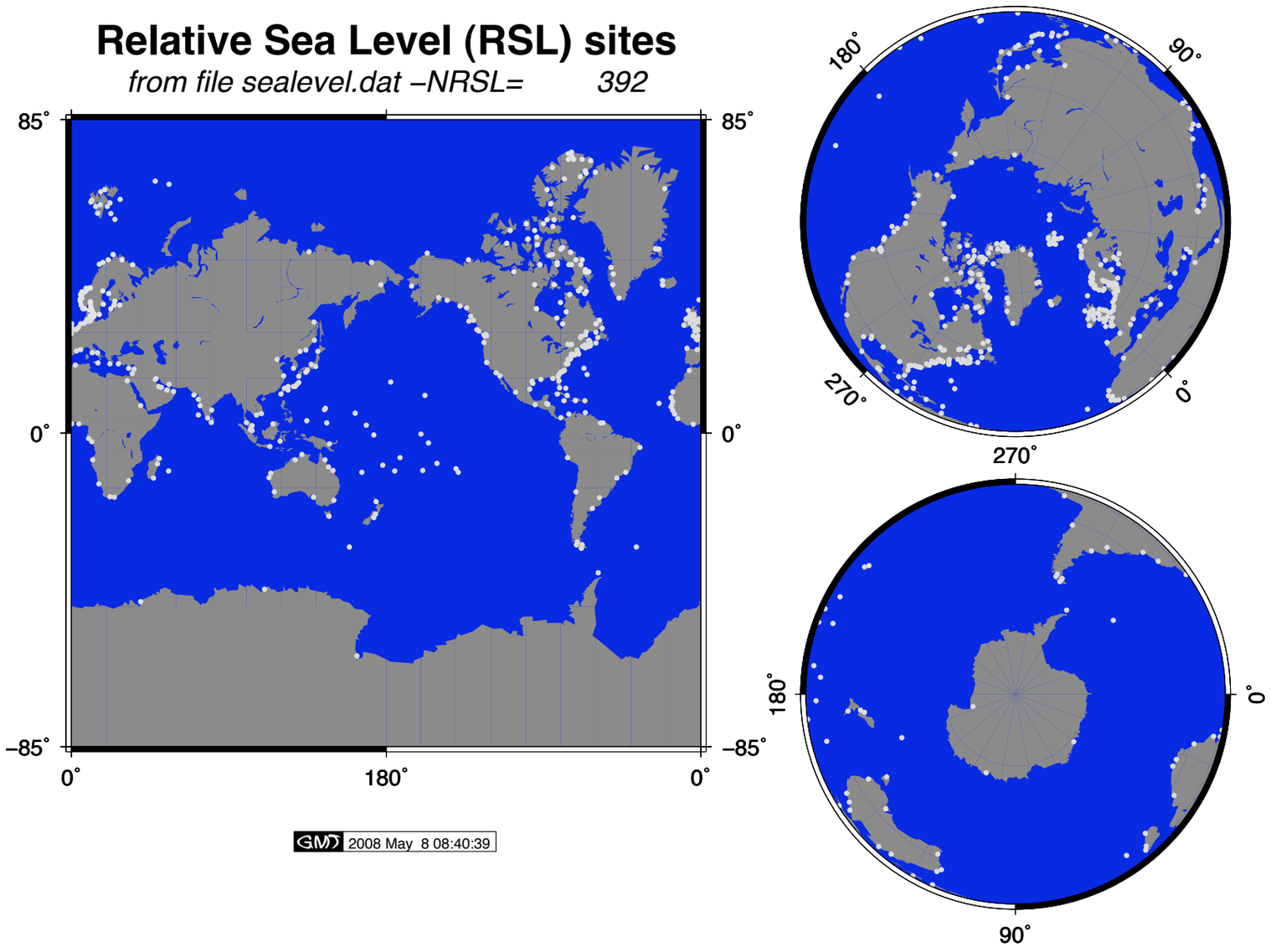
$$\overline{S} = S^E$$

In words: <<the ocean-averaged sea level change coincides with the sea level change we would observe in eustatic conditions ! >>

7) More observations on Eustasy.

if the ocean-averaged sea level change coincides with the sea level change we would observe in eustatic conditions,

it follows that the history of the volume of ice sheets could be estimated by averaging the observed RSL curves over the oceans!



The RSL sites are not uniformly distributed over the oceans...

does an average make sense?

8) About the "Woodward solution".

Woodward (1888)* found an *analytical* solution of the SLE assuming that:

1- *the Earth is rigid,*

2- *the ice sheet is localized and impulsive – so that here we face the instantaneous freezing (or melting) of a "point load",*

3- *the oceans cover uniformly the Earth, except from a point-like "continent",*

4- *variations in the ocean mass distribution do not affect the gravity field – i. e., "self–gravitation of the oceans" is neglected.*

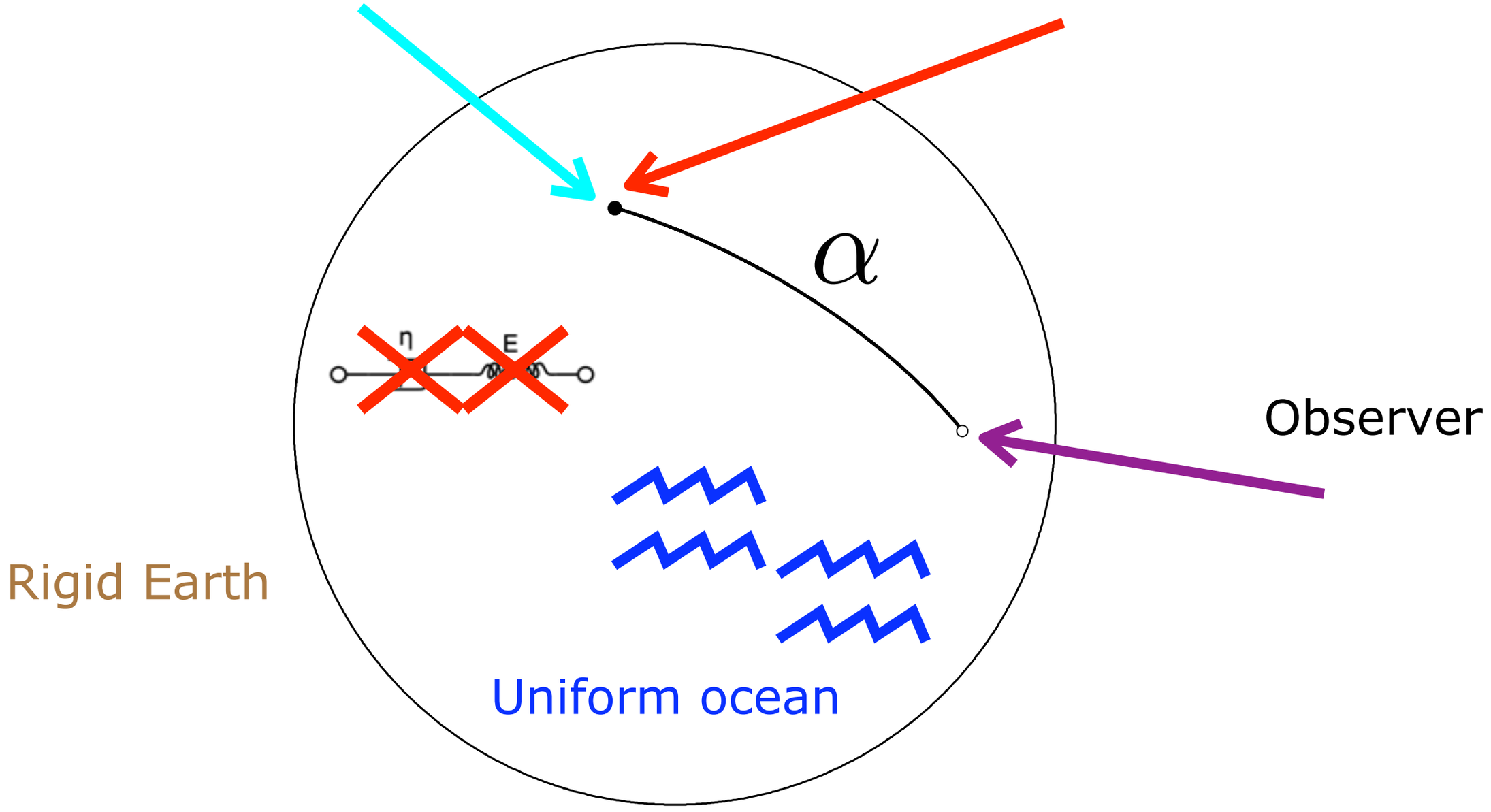
Thus, the ONLY PHYSICAL EFFECT that is accounted for is gravitational attraction between the ICE and the OCEANS ...

Woodward, R. S., On the form and position of mean sea level, U.S. Geol. Survey Bull., 48, 87–170, 1888.

"Woodward problem"

Frozen point mass

Micro-continent ("*Lilliput*")



Seen the strong simplifying assumptions, the Woodward problem has a simple closed-form solution*:

$$S^W(\alpha,t) = \frac{a}{m_e}\left(\frac{1}{2\sin(\alpha/2)} - \frac{\overline{\rho}_e}{3\rho_w} - 1\right)$$

with:

$S^W(\alpha,t)$ Sea level variation

α Colatitude

t time

a Radius of the Earth

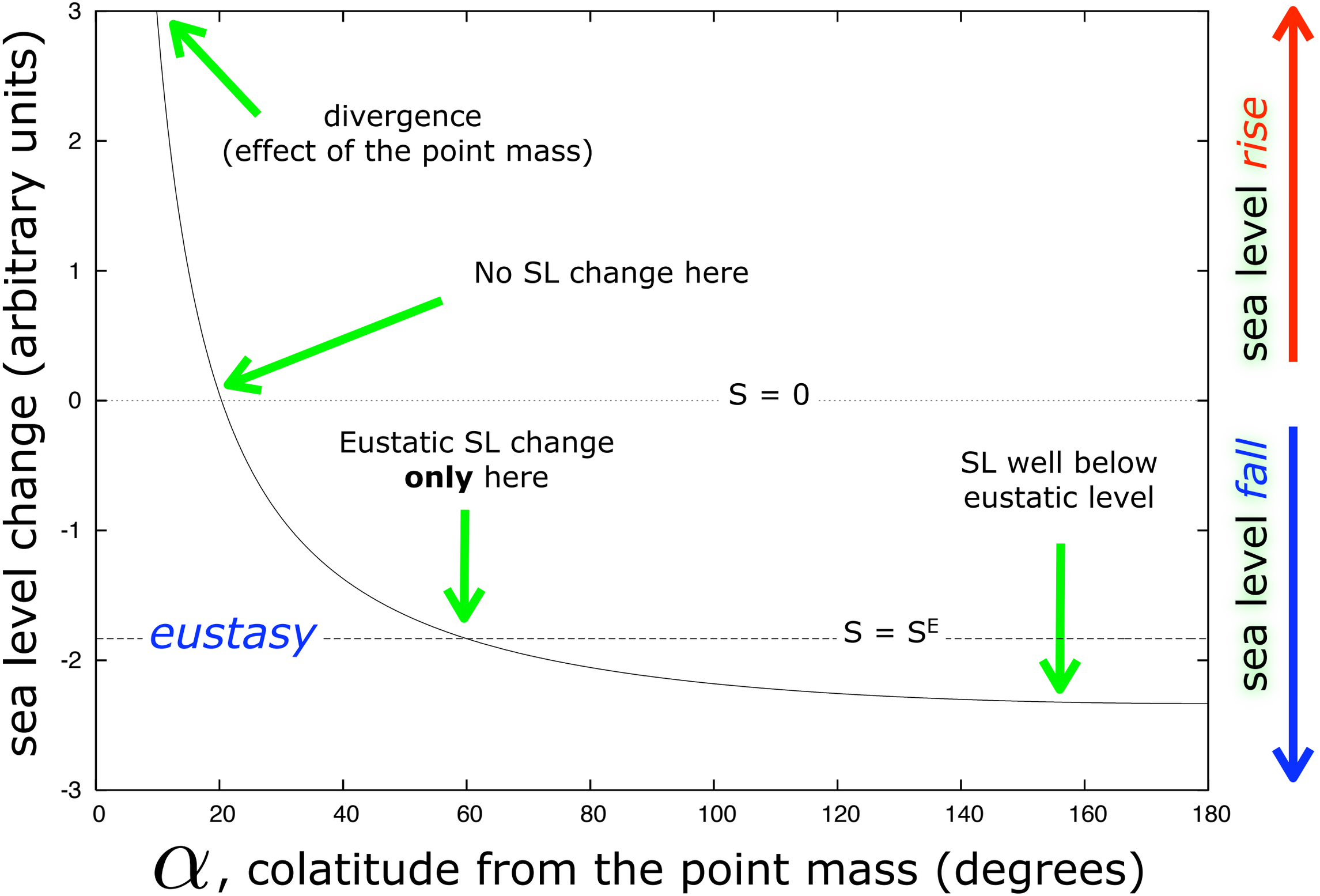
m_e Mass of the Earth

$\overline{\rho}_e$ Average Earth density

ρ_w Water density

* For the detailed derivation of this solution, see: Spada G., & Stocchi, P., 2006. *The Sea Level Equation, Theory and Numerical Examples*, ISBN: 88-548-0384-7, 96 pp., Aracne, Roma.

"Woodward problem": Freezing of a point mass



9) HOW to solve the SLE ? A few ideas here ...

The SLE "looks like" a *"inhomogeneous integral Fredholm equation of the 2nd type"*, where the unknown occurs both inside and outside the integral, and the integration interval is fixed (fixed shorelines, in our language), which in one dimension has the form:

$$\varphi(x) = f(x) + \lambda \int_a^b K(x, x')\varphi(x')dx',$$

The problem can be solved by "iteration" of a first-guess solution:

0. Assume $\varphi(x) = \varphi^{(0)}$

1. This gives $\varphi^{(1)}(x) = f(x) + \lambda \int_a^b K(x, x')\varphi^{(0)}dx'$

... ...

n. At step "n": $\varphi^{(n)}(x) = f(x) + \lambda \int_a^b K(x, x')\varphi^{(n-1)}(x')dx'$

... ...

continue until convergence ...

9) HOW to solve the SLE ? A few ideas here ...

$m_i(t)$ Given the ice chronology, compute the ice mass variation

$S^E = -\frac{m_i}{\rho_w A_o}$ Then, evaluate the eustatic sea level change

$S^0 = S^E$ At step #0, sea level change is eustatic

$$S^1 = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S^0 + S^0 - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S^0} \quad \text{step \#1}$$

... ...

$$S^k = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S^{k-1} + S^0 - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S^{k-1}} \quad \text{step \#k}$$

... ...

$$\int_{sphere} \frac{|S^k - S^{k-1}|}{|S^k|} dA \leq \epsilon$$ Stopping criterion
(~5 iterations are sufficient)

SELEN 3.0

http://www.fis.uniurb.it/spada/SELEN_minipage.html

Numerical approach:

- Pixelization by the Icosahedron method (Tegmark, '96)

- Pseudo-spectral approach,

- Ice models are discretized in space (grid + harmonics) and time (piecewise constant)

- Evolving shorelines and rotational feedbacks

- NEW: HPC applications with INGV Rome & ISCRA (CINECA).

10) On the meaning of some quantities relevant to the SLE:

Symbol	*Meaning*	*Observed by*
S	Sea Level Change	Tide gauges
N	"Absolute" sea surface variation	Altimetry
Φ	Gravity potential variation	Gravimetry (e.g. GRACE)
U	Vertical displacement	GPS
V	Horizontal displacement	GPS
RSL	Relative Sea Level	Geological record

10) Some interconnections between quantities relevant to the SLE

$$S = N - U$$

Sea Level Change, from the solution of the SLE (S)

$$U = U(S, I)$$

$$\Phi = \Phi(S, I)$$

$$V = V(S, I)$$

$$G = \frac{\Phi}{\gamma}$$

From the ice history (I) and the SLE (S)

$$N = \frac{\Phi}{\gamma} + c$$

By the SLE (S) and mass conservation

$$c = -\frac{m_i}{\rho_w A_o} - \overline{\left(\frac{\Phi}{\gamma} - U\right)}$$

Ensures mass conservation

$$RSL$$

RSL derives from S > as we will see in one moment...

Outline

1). Surface loads (“ice” and “water loads”)

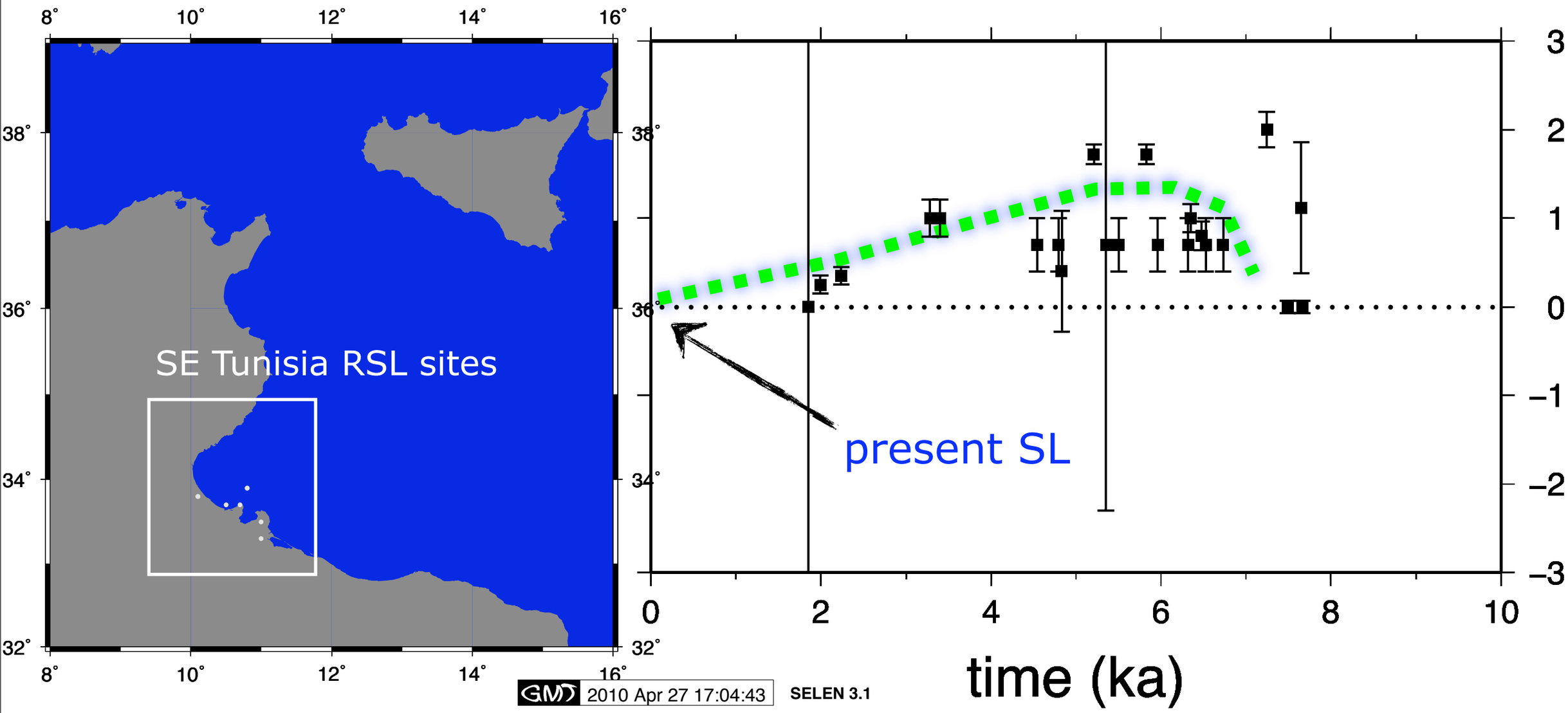
2). Response to surface loads

3). “The Sea Level Equation” (with discussion)

4). Special topics: global RSL variations and a SLE benchmark, and maps of geodetic quantities.

Charles University in Prague, Department of Geophysics
November 7–11, 2011

Special topic (1/3)
Relative Sea Level (RSL)

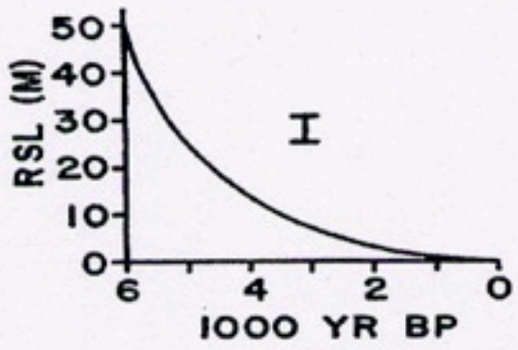


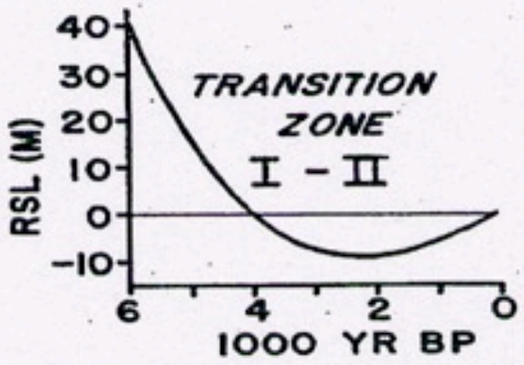
What is a Relative Sea Level (RSL) curve?
How do we construct it from the SLE?
useful for interpretation of geological observations...

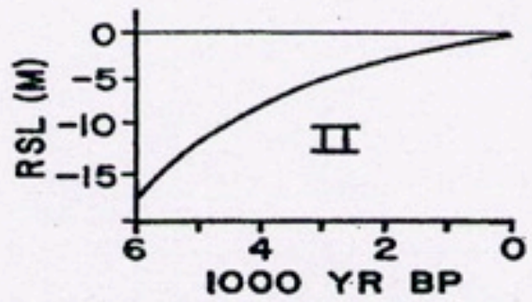
Relative Sea Level (RSL)

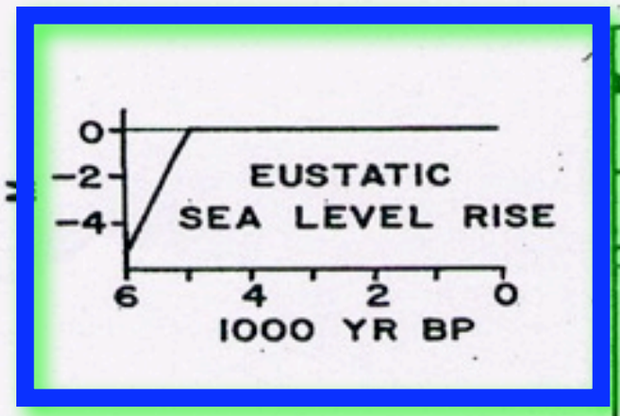
"Clark zones"

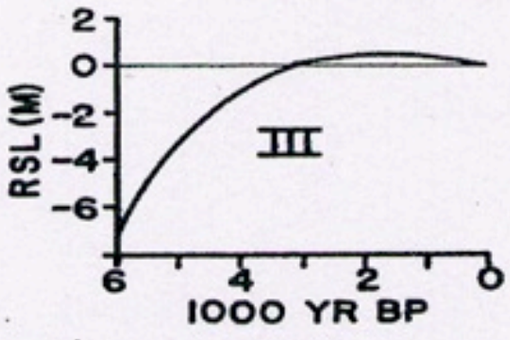
Clark & Lingle, 1979

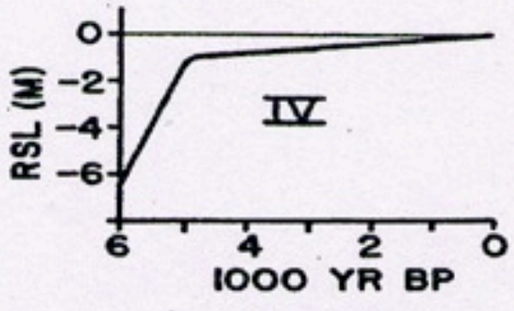


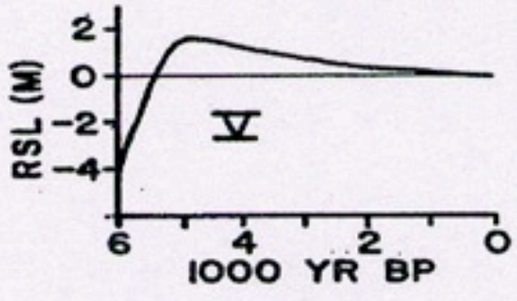












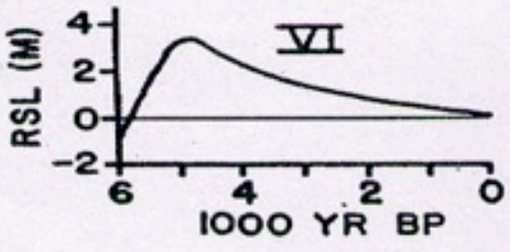


Fig. 1. Distribution of six predicted sea level zones resulting from melting of ice sheets 20,000–7,000 yr ago. Within each zone the form of the sea level response is similar. Typical curves indicating the predicted change in sea level relative to the present geoid for each zone are included (RSL means relative sea level). The curves show the wide variety of sea level expressions possible despite the assumption of no change in ocean volume during the past 5,000 yr (no change in 'eustatic' sea level) [from *Clark and Lingle,* 1979].

or, equivalently:

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

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$$S = N - U$$

or, equivalently:

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}$$

with:

$S =$ sea level change

$m_i =$ ice mass variation

$\rho_i, \rho_w =$ ice and water density

$A_o =$ area of the oceans

$G_s =$ sea level Green function

$\otimes_i, \otimes_o =$ 3(2+1)D convolutions

$I =$ ice thickness variation

$\overline{(\ldots)} =$ ocean average

Relative Sea Level (RSL)

The SLE does **not** provide absolute sea level variations, but rather it allows to compute "sea level change". Solving the SLE gives "*S*":

$$S(\omega, t_{BP}) = SL(\omega, t_{BP}) - SL(\omega, t_r)$$

with:

t_{BP} time Before Present (BP),

t_r a reference "remote" time BP,

SL Sea Level (sea surface-topography offset).

At present time, we can similarly write:

$$S(\omega, t_p) = SL(\omega, t_p) - SL(\omega, t_r)$$

t_p present time.

Relative Sea Level (RSL)

The relative sea level (RSL) "relative" to a given epoch BP is **defined** as the past sea level referred to the present–day level:

$$RSL(\omega, t_{BP}) = SL(\omega, t_{BP}) - SL(\omega, t_p)$$

that can be rephrased as follows:

$$\begin{aligned} RSL(\omega, t_{BP}) &= SL(\omega, t_{BP}) - SL(\omega, t_r) \\ &\quad -(SL(\omega, t_p) - SL(\omega, t_r)) = \\ &= S(\omega, t_{BP}) - S(\omega, t_p) \end{aligned}$$

which is directly related to the sea level change *S*, i.e., the solution of the SLE.

What is the shape of the RSL curve for a RIGID, non-gravitating Earth?

In this case, the SLE has the "Eustatic" solution:

$$S = S^E(t) = -\frac{m_i(t)}{\rho_w A_o}$$

This gives:

$$S(t_{BP}) = -\frac{m_i(t_{BP})}{\rho_w A_o} = -\frac{M_i(t_{BP}) - M_i(t_r)}{\rho_w A_o}$$

$$S(t_p) = -\frac{m_i(t_p)}{\rho_w A_o} = -\frac{M_i(t_p) - M_i(t_r)}{\rho_w A_o}$$

Hence, following the definition, RSL is:

$$RSL(\omega, t_{BP}) = SL(\omega, t_{BP}) - SL(\omega, t_p) = -\frac{M_i(t_{BP}) - M_i(t_p)}{\rho_w A_o}$$

A case study:

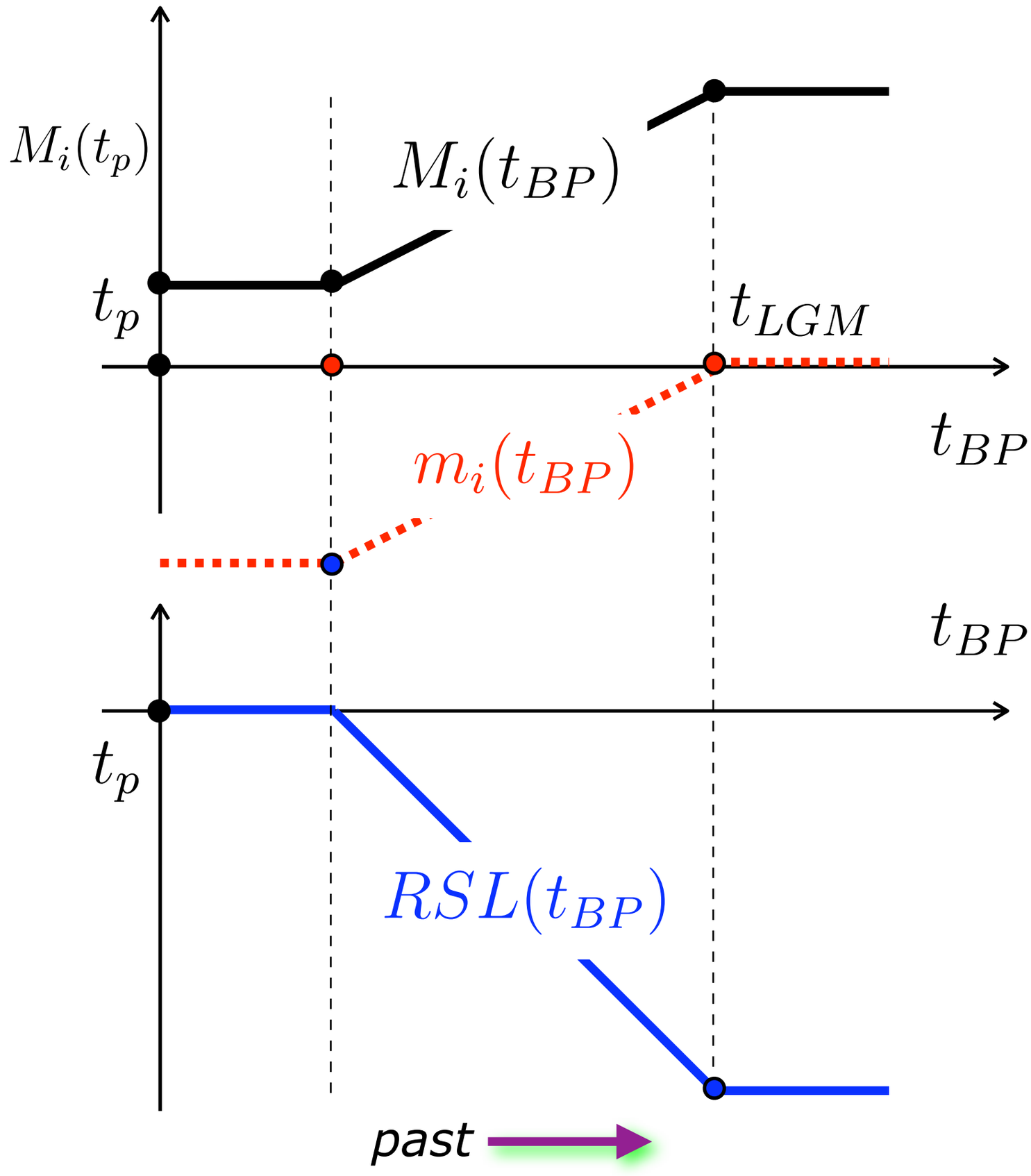
"melting at a constant rate".

The mass is constant until the "LGM" (Last Glacial Maximum).

After a phase with constant melting rate, the mass remains constant.

The RSL curve (blue, solid) can be derived by the mass variation curve (red, dashed) by reflection and rescaling.

The RSL curve is "Eustatic", in this case.



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Record Search Query:

Global Relative Sea Level Histories

Entry ID: EARTH_LAND_NGDC_PALEO_SEALEVEL1

[View Brief Record] [Get Data] [Update this Record]

Summary

The Relative Sea Level Database covers sea level data from the present time back to 14,000 years before the present. The authors compiled the database from numerous published sources in addition to their own field work. Methodology for establishing relative sea level histories varies between sites and investigators, but typically entails geomorphologic ... Click to View Full Summary

Geographic Coverage

An example of HOLOCENE RSL global database: The Peltier dataset is hosted by NOAA (National Oceanographic and Atmospheric Administration).

Data Set Citation

Dataset Creator: Peltier, W.R. and A.M. Tushingham
Dataset Title: Relative Sea Level Database
Dataset Series Name: IGBP PAGES/World Data Center for Paleoclimatology Data Contribution
Dataset Release Date: 1993
Dataset Release Place: Boulder, CO
Dataset Publisher: NOAA/NCDC Paleoclimatology Program
Issue Identification: 93-016
Online Resource: ftp://ftp.ncdc.noaa.gov/pub/data/paleo/paleocean/relative_sea_level/

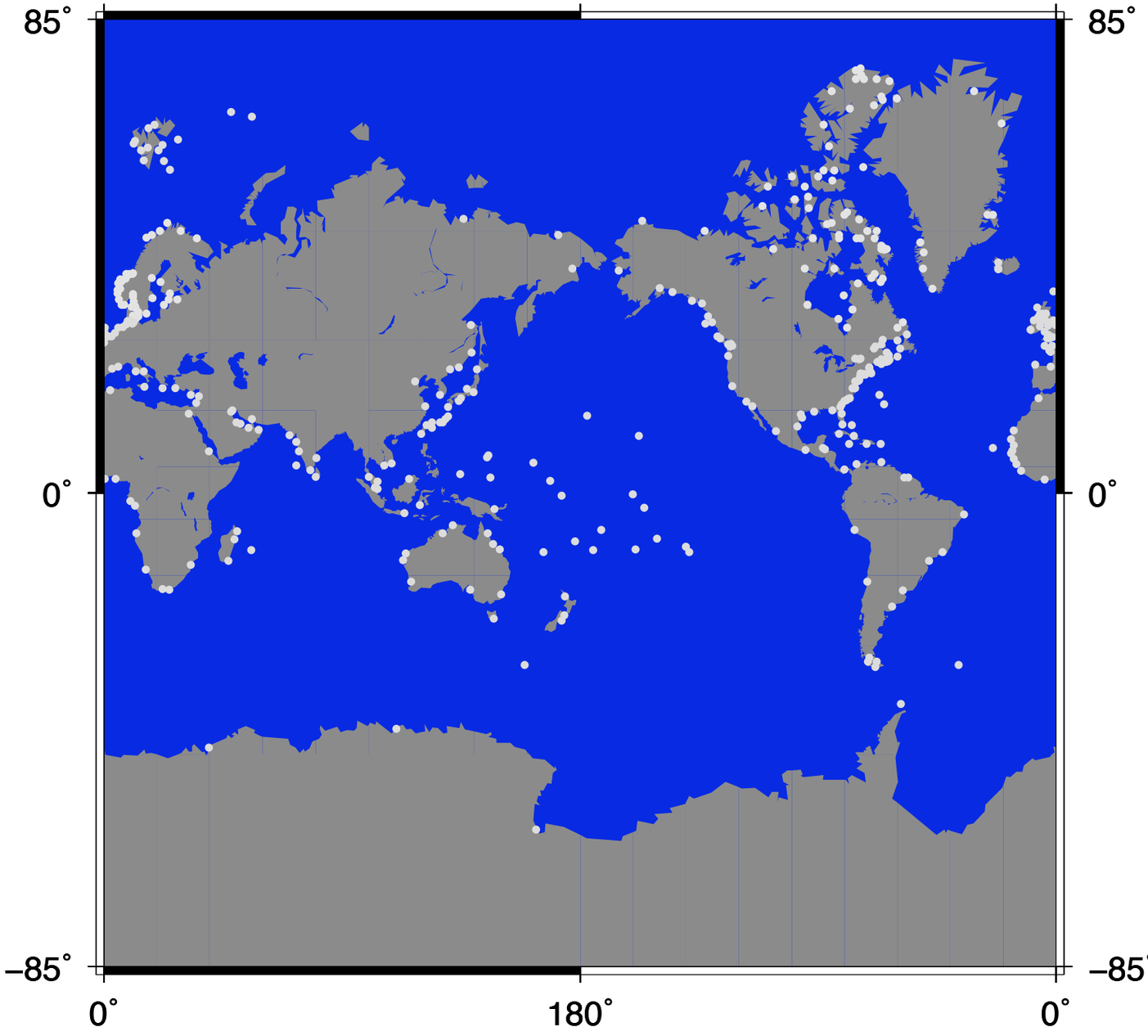
Paleo Temporal Coverage

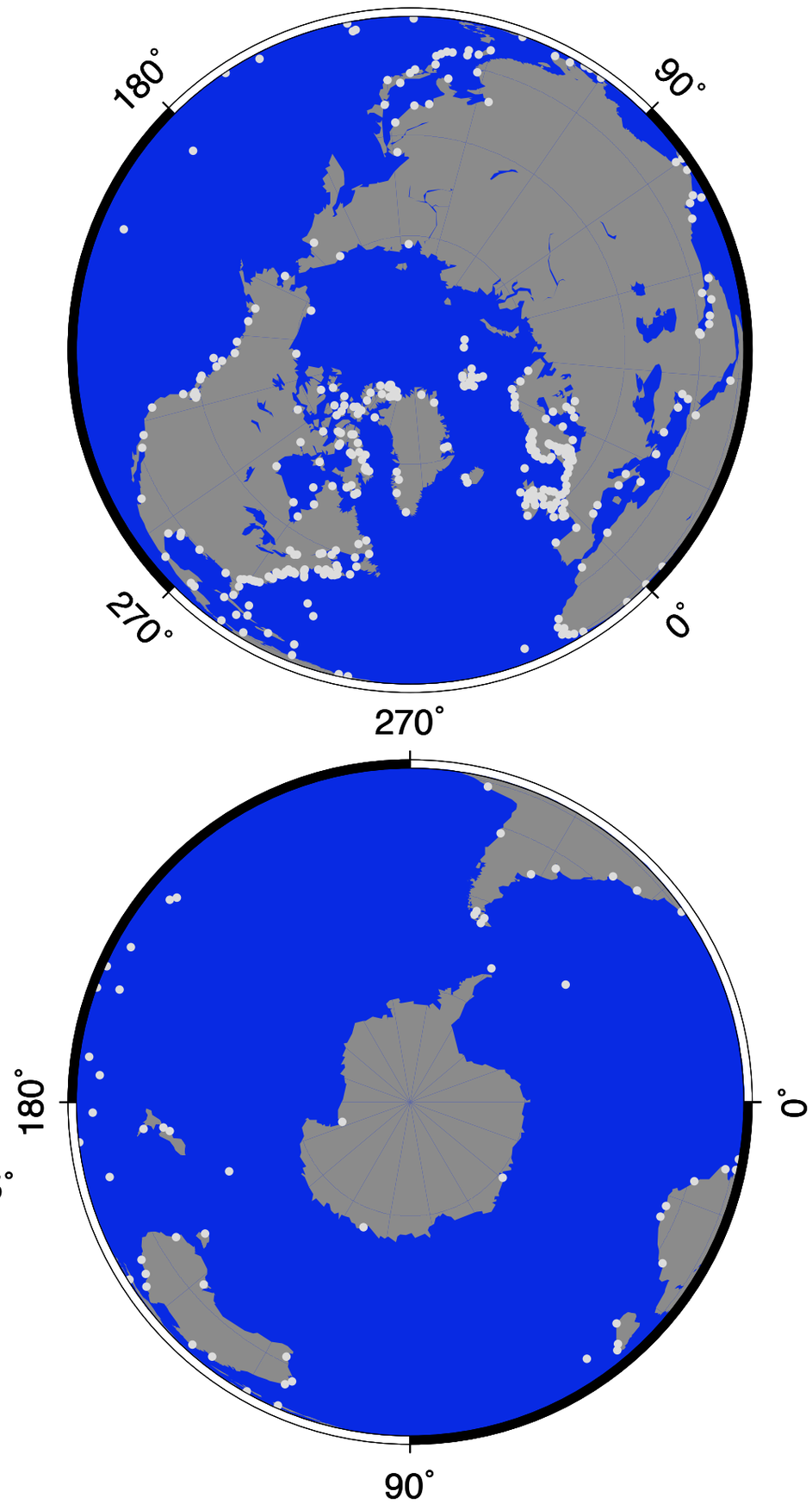
Paleo Start Date: 14,000 ybp
Paleo Stop Date: 0 ybp

http://gcmd.nasa.gov/records/GCMD_EARTH_LAND_NGDC_PALEO_SEALEVEL1.html

Relative Sea Level (RSL) sites

from file sealevel.dat –NRSL= 392





GMT 2008 May 8 08:40:39

Sites in the RSL NOOA database

Summary

The Relative Sea Level Database covers sea level data from the present time back to 14,000 years before the present.

The authors compiled the database from numerous published sources in addition to their own field work.

Methodology for establishing relative sea level histories varies between sites and investigators, but typically entails **geomorphologic methods to identify relict shoreline deposits, which are then dated radiometrically from associated carbonate or organic materials**.

Final results are presented as **relative height in meters versus modern mean sea level.**

Publications/References

Original Reference:

Tushingham, A.M. and W.R.Peltier, Validation of the ICE-3G Model of Wurm-Wisconsin Deglaciation Using a Global Data Base of Relative Sea Level Histories, Journal of Geophysical Research, v97, B3, 3285-3304, 1992.

The microfiche appendix of the Original Reference (Tushingham and Peltier, 1992) contains the complete bibliography, and is available from AGU, 2000 Florida Ave. NW, Washington, D.C., 20009 USA.

NOAA Product Information Catalog. 1988. Washington, DC: US Dept. of Commerce, 171 pp.

http://gcmd.nasa.gov/records/GCMD_EARTH_LAND_NGDC_PALEO_SEALEVEL1.html

The following files are available from the FTP server:

ftp://ftp.ncdc.noaa.gov/pub/data/paleo/paleocean/relative_sea_level/

---

rsl.readme - readme file

sealevel.* - 392 individual Relative Sea Level data files;
There are 392 files named sealevel.*, each containing data from one site. These files are located in the subdirectory sitefiles.

sealevel.dat - All Relative Sea Level data files, concatenated;
The file sealevel.dat contains radiocarbon-controlled relative sea level histories from 392 sites.

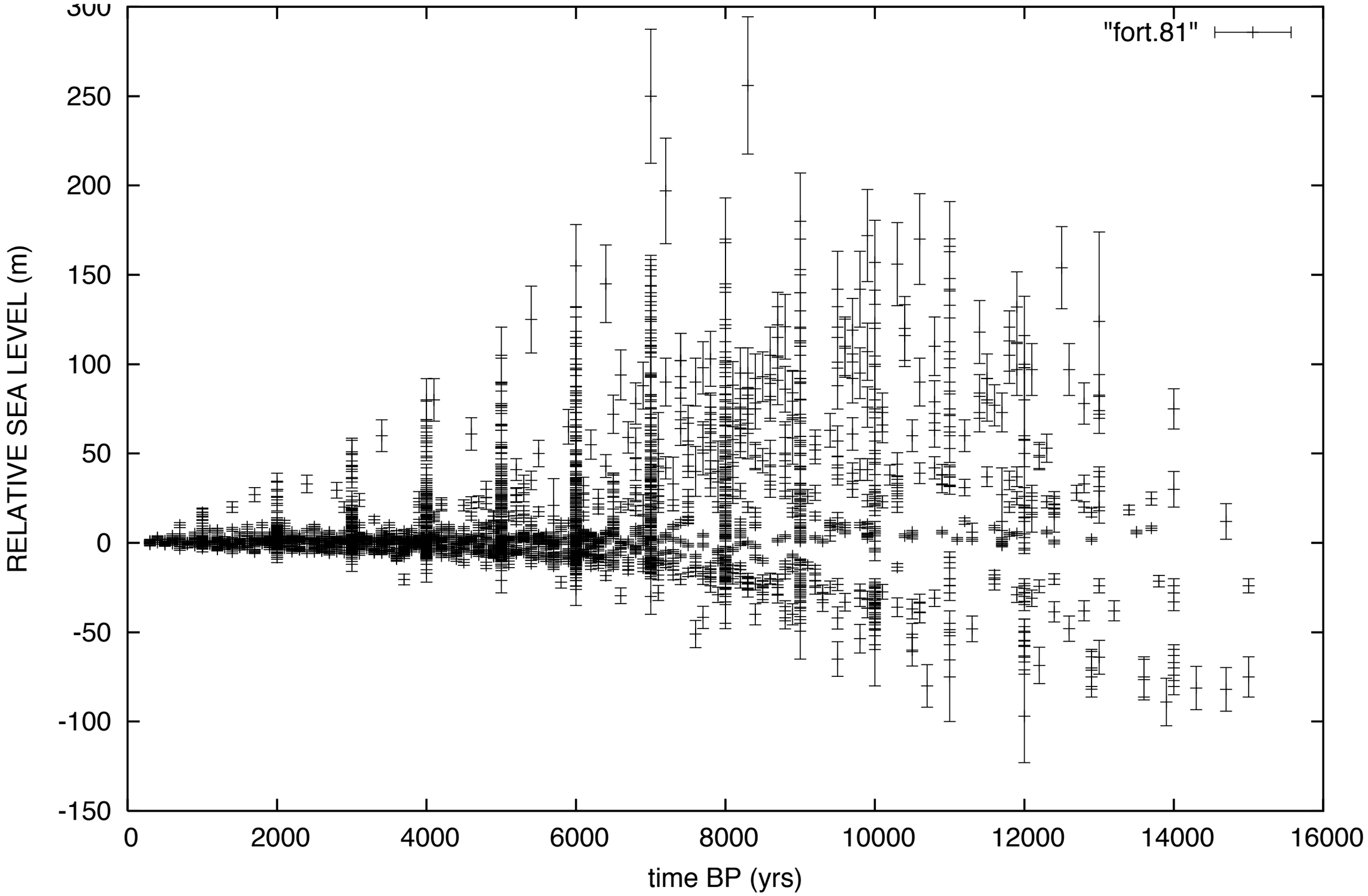
rslsite.lst - list of sites and file names in rsl database;
A list of site names and locations can be found in the file rslsite.lst.

rslform.lst - FORTRAN code used to write sealevel.* data files

http://gcmd.nasa.gov/records/GCMD_EARTH_LAND_NGDC_PALEO_SEALEVEL1.html

a Relative Sea Level database

http://www1.ncdc.noaa.gov/pub/data/paleo/paleocean/relative\sea\level/



a Relative Sea Level database

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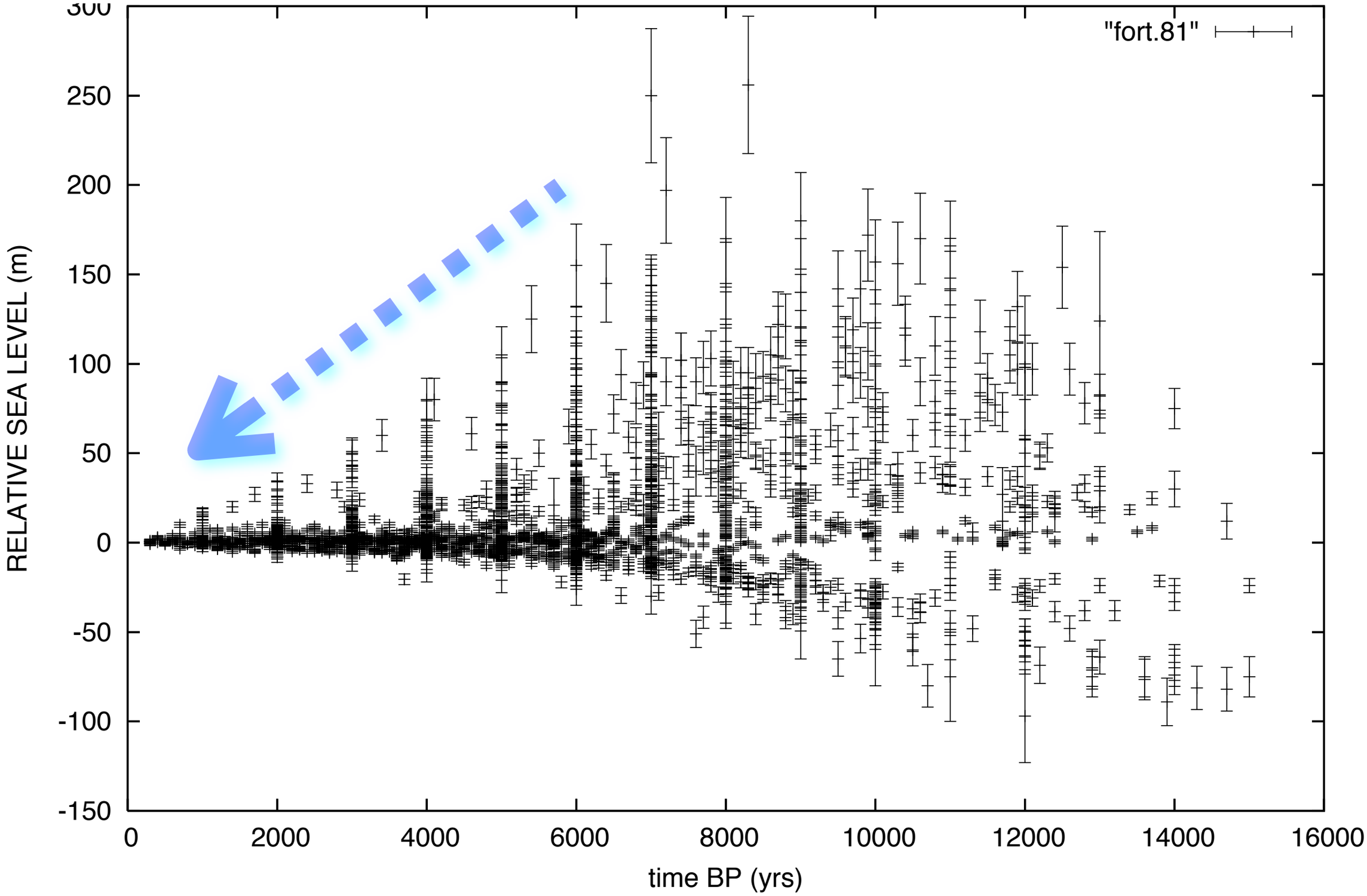

a Relative Sea Level database

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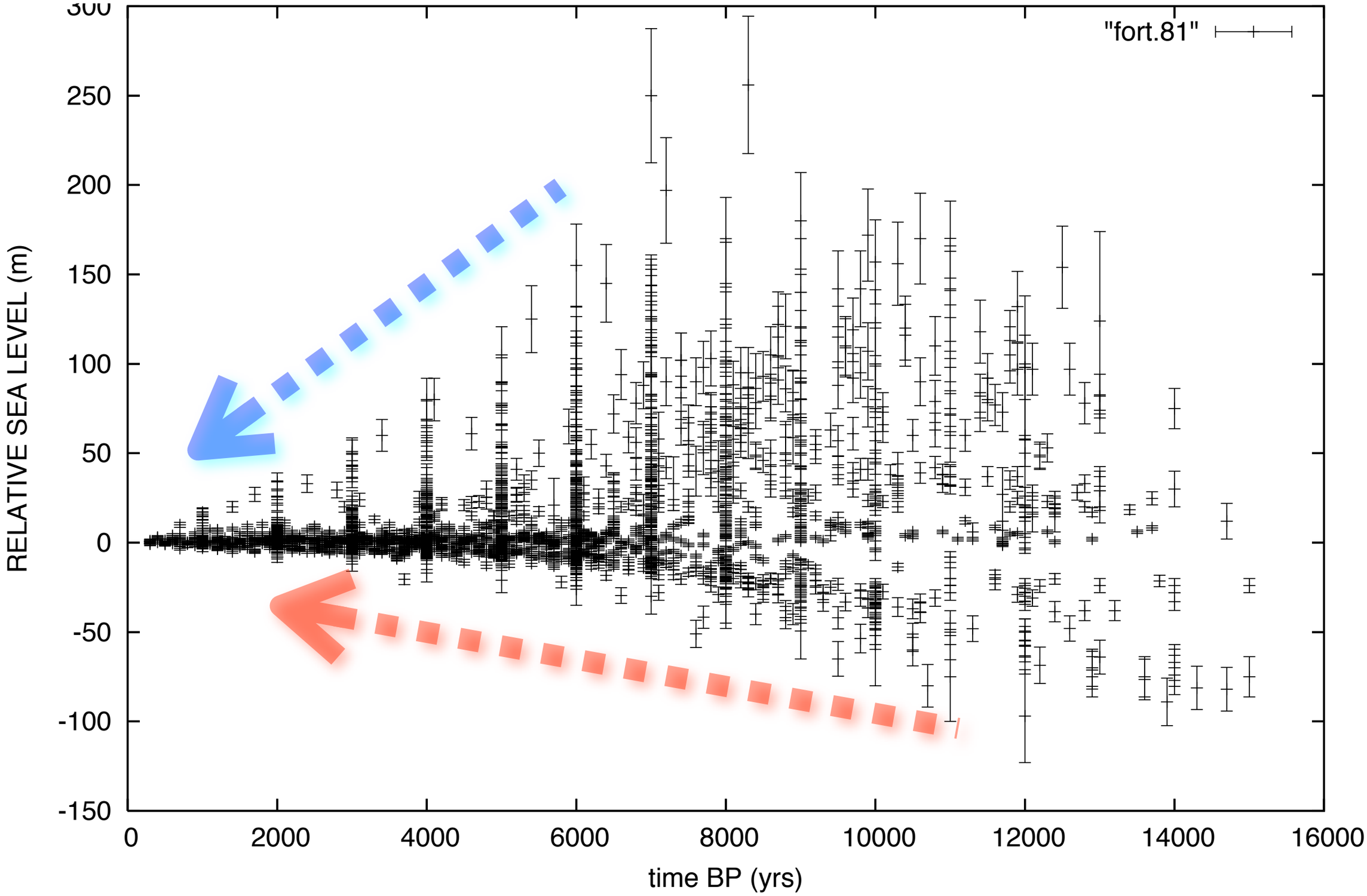

a Relative Sea Level database

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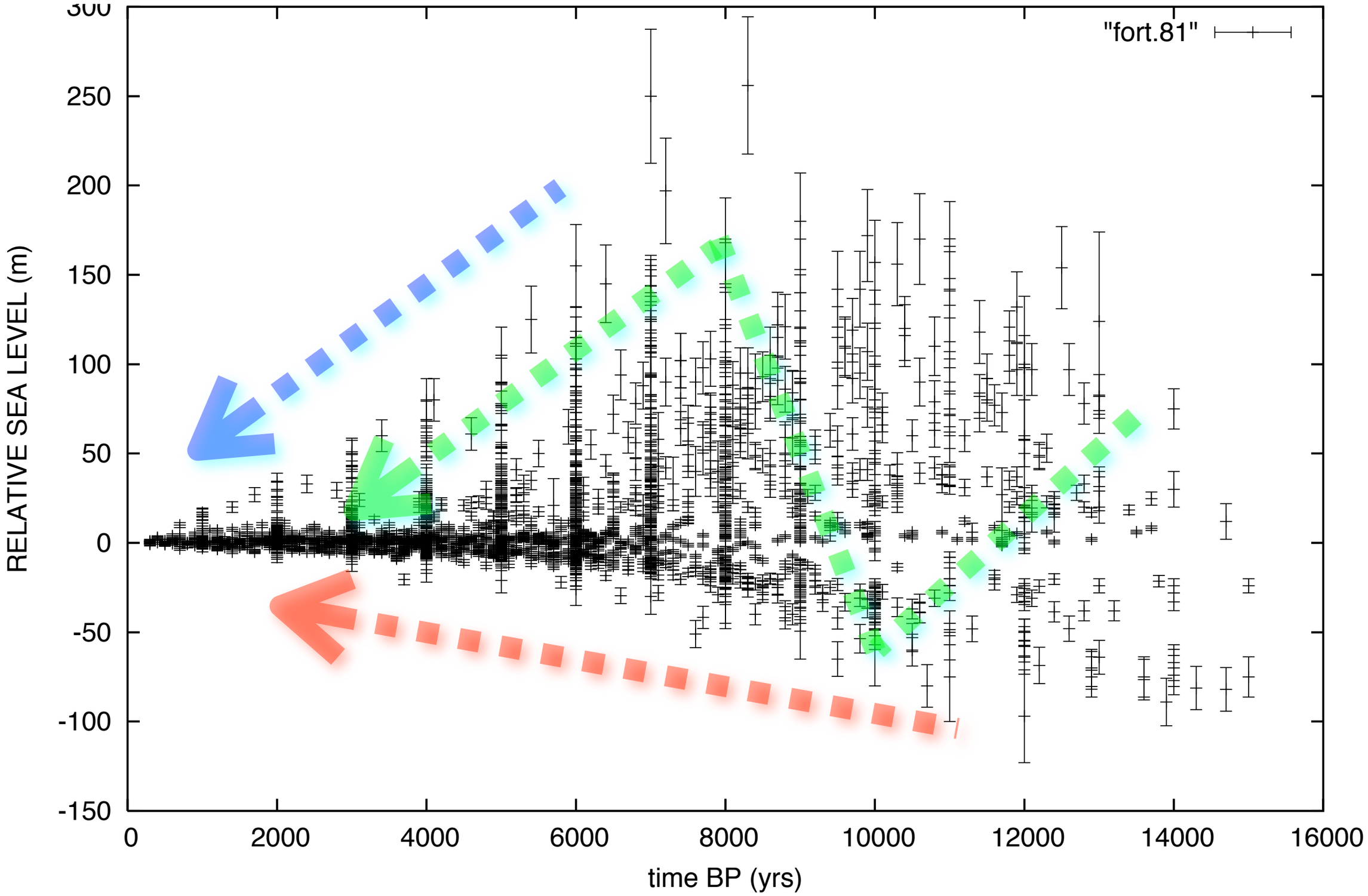
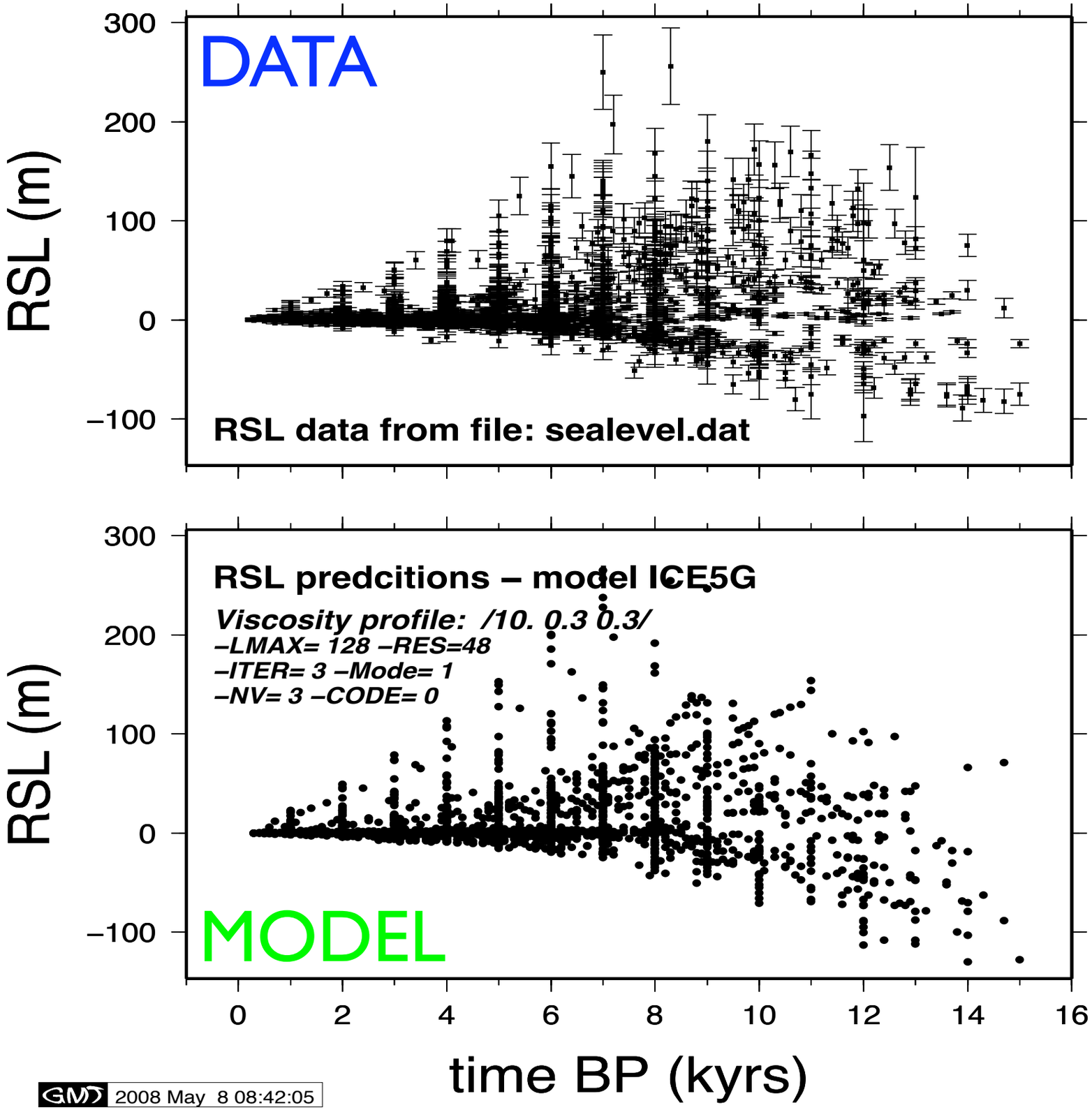


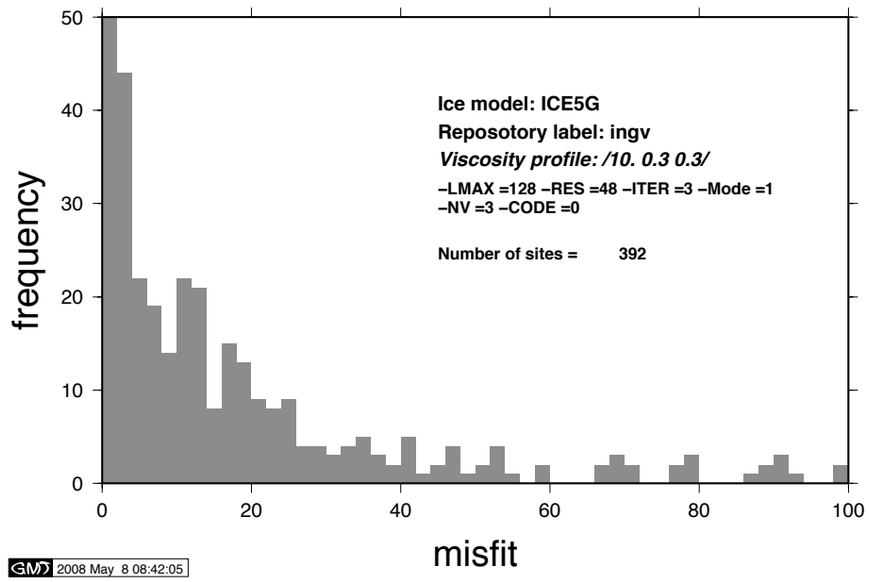
Relative Sea Level - global view



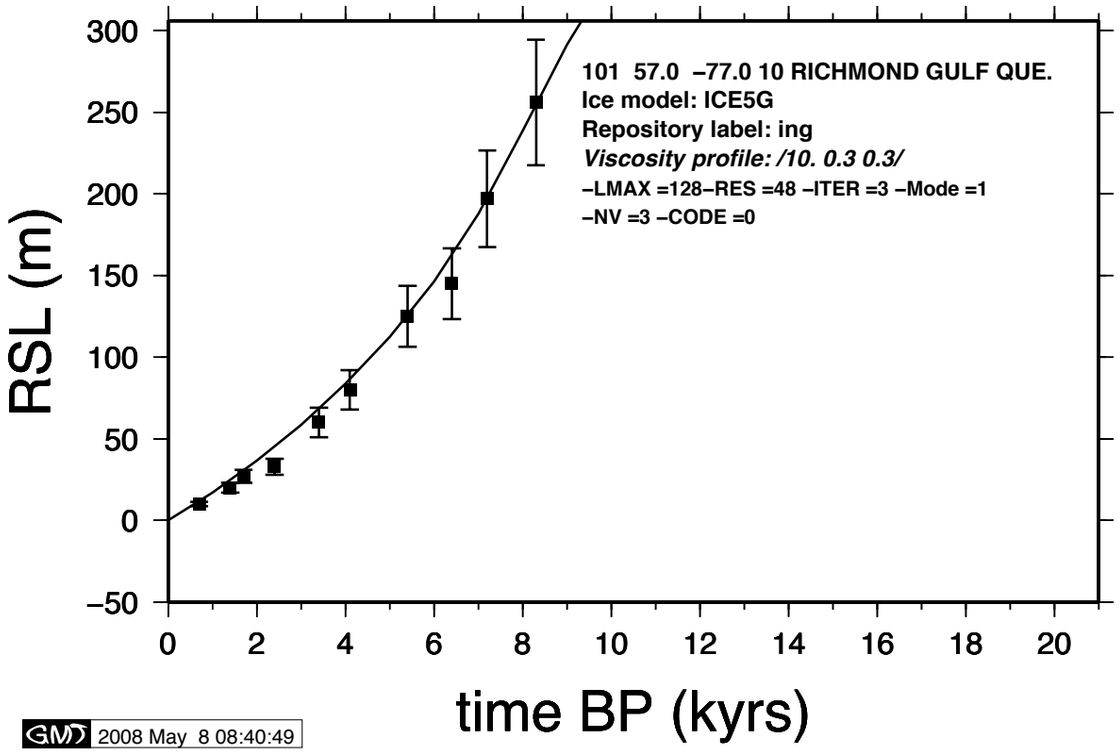
Can the RSL data be explained GLOBALLY ?

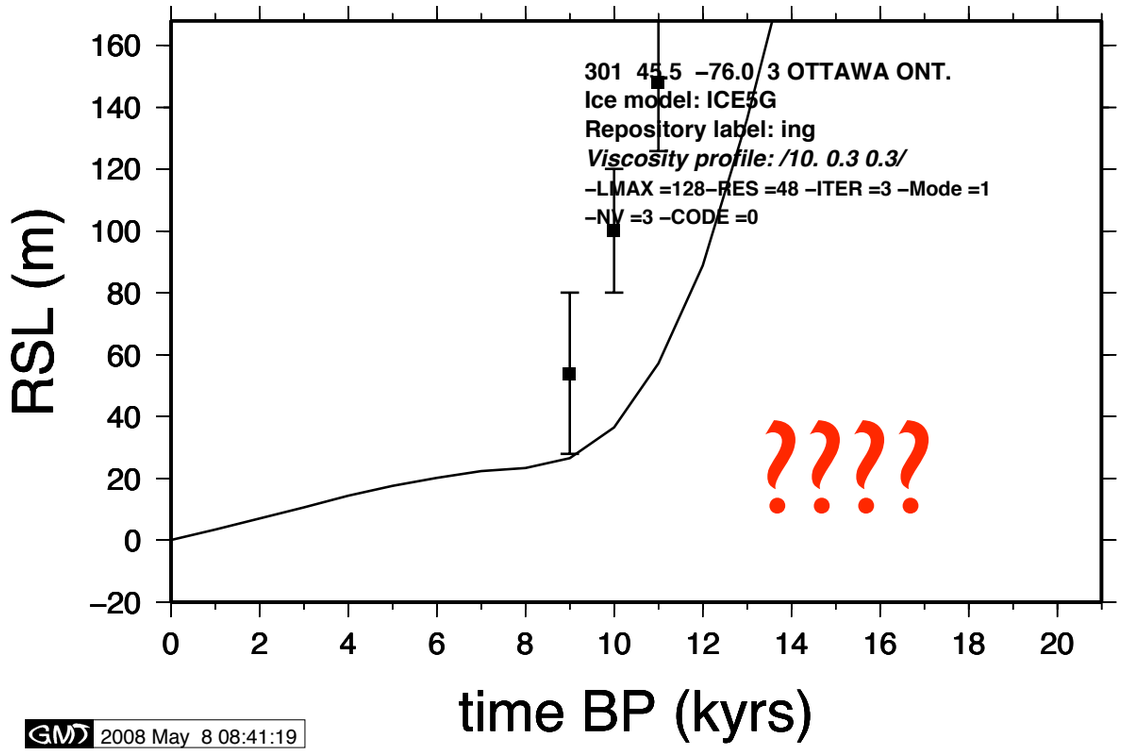
Misfit distribution

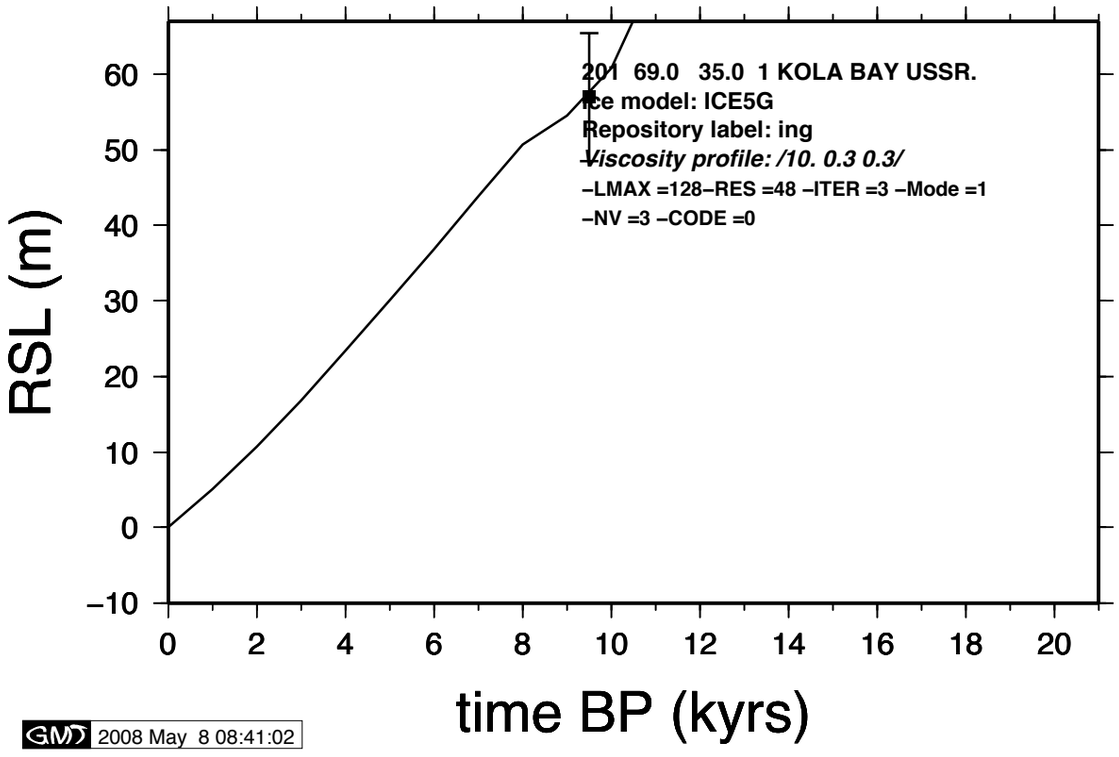
$$\mu = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\mathrm{OBS_i} - \mathrm{PRED_i}}{\sigma_i}\right)^2$$

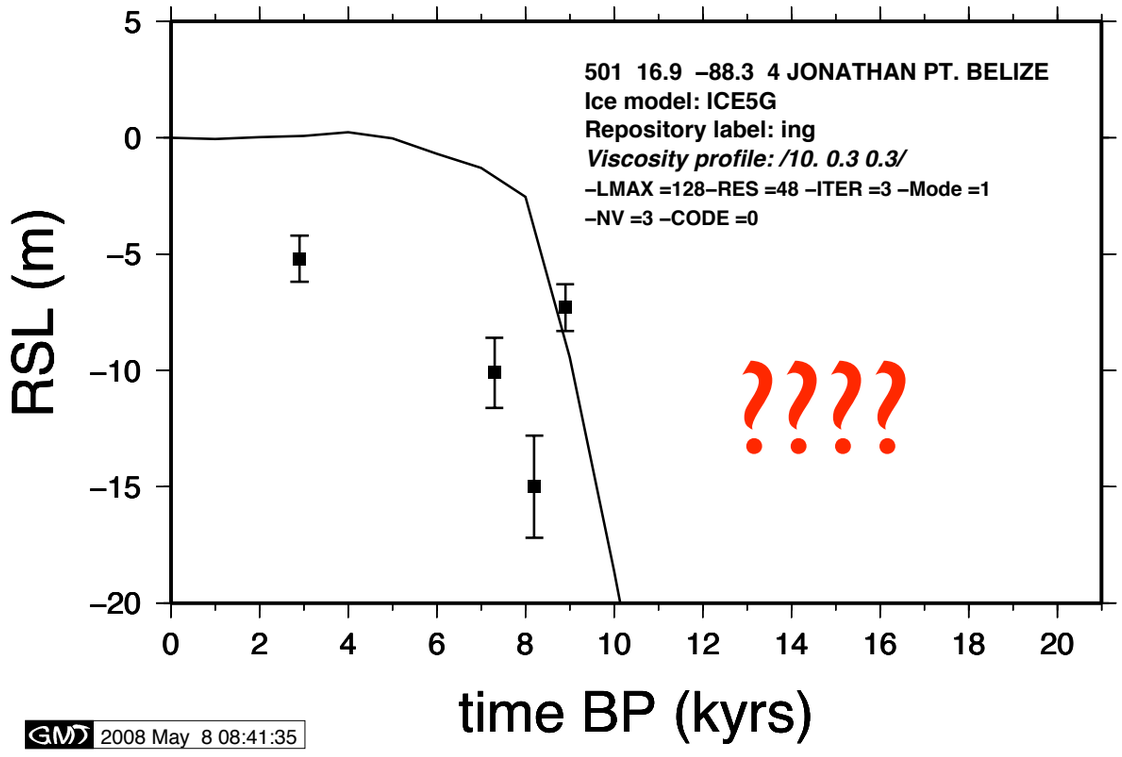


Regional and local fit (or misfit)

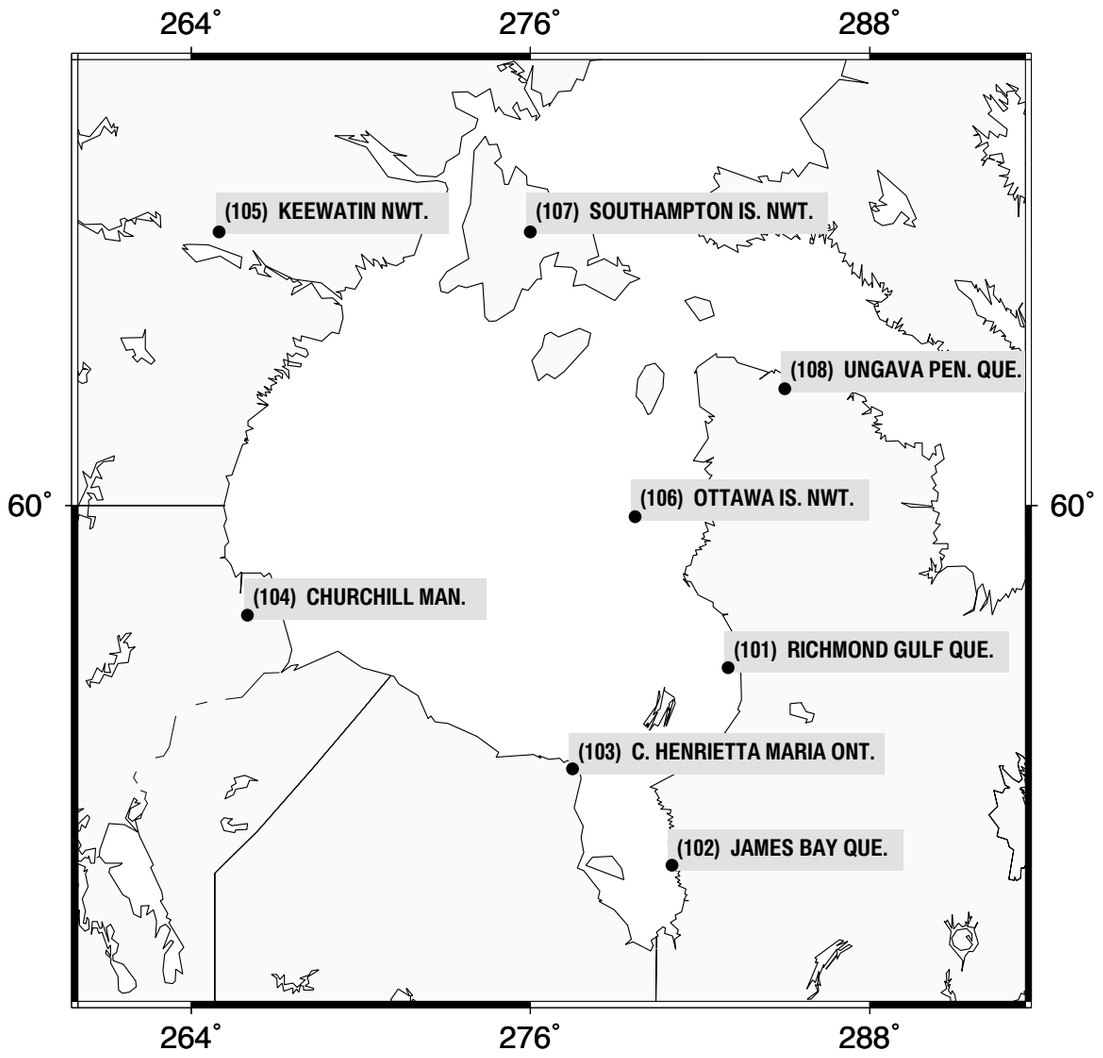




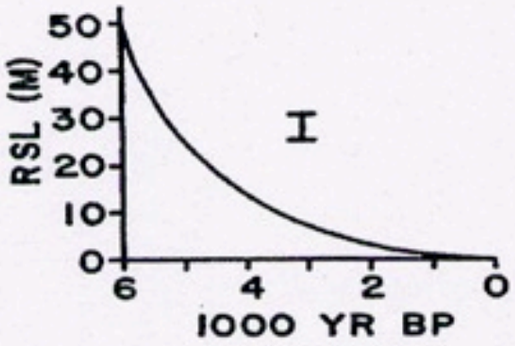


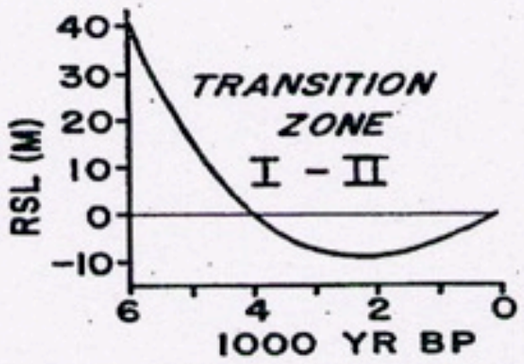


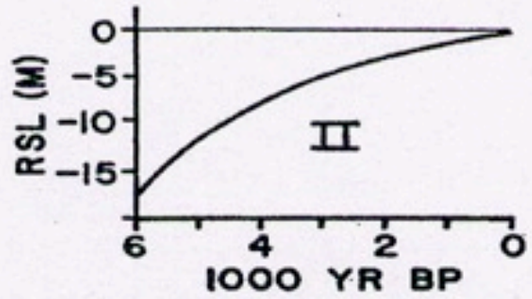
Example: history of RSL across the Hudson Bay

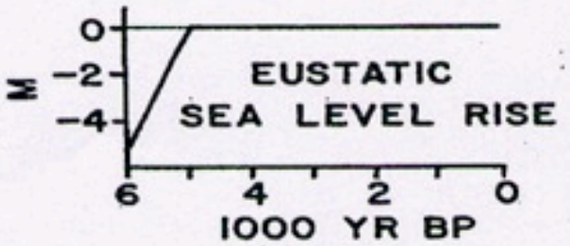


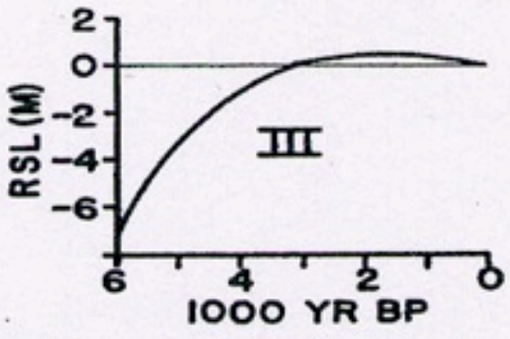
From: Clark & Lingle, 1979

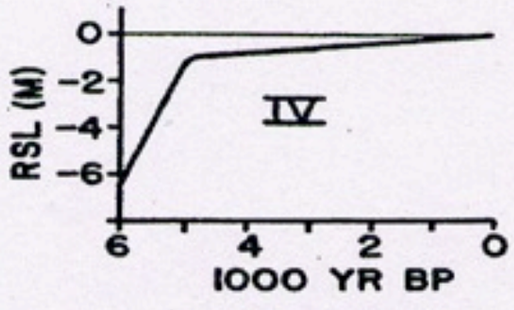


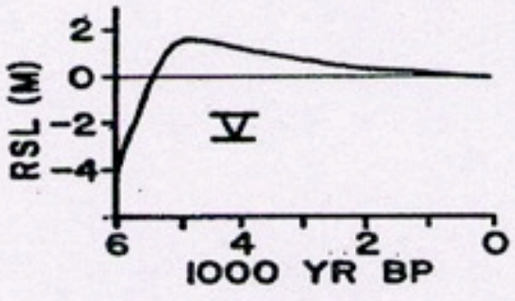












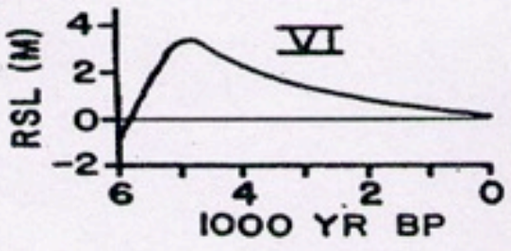
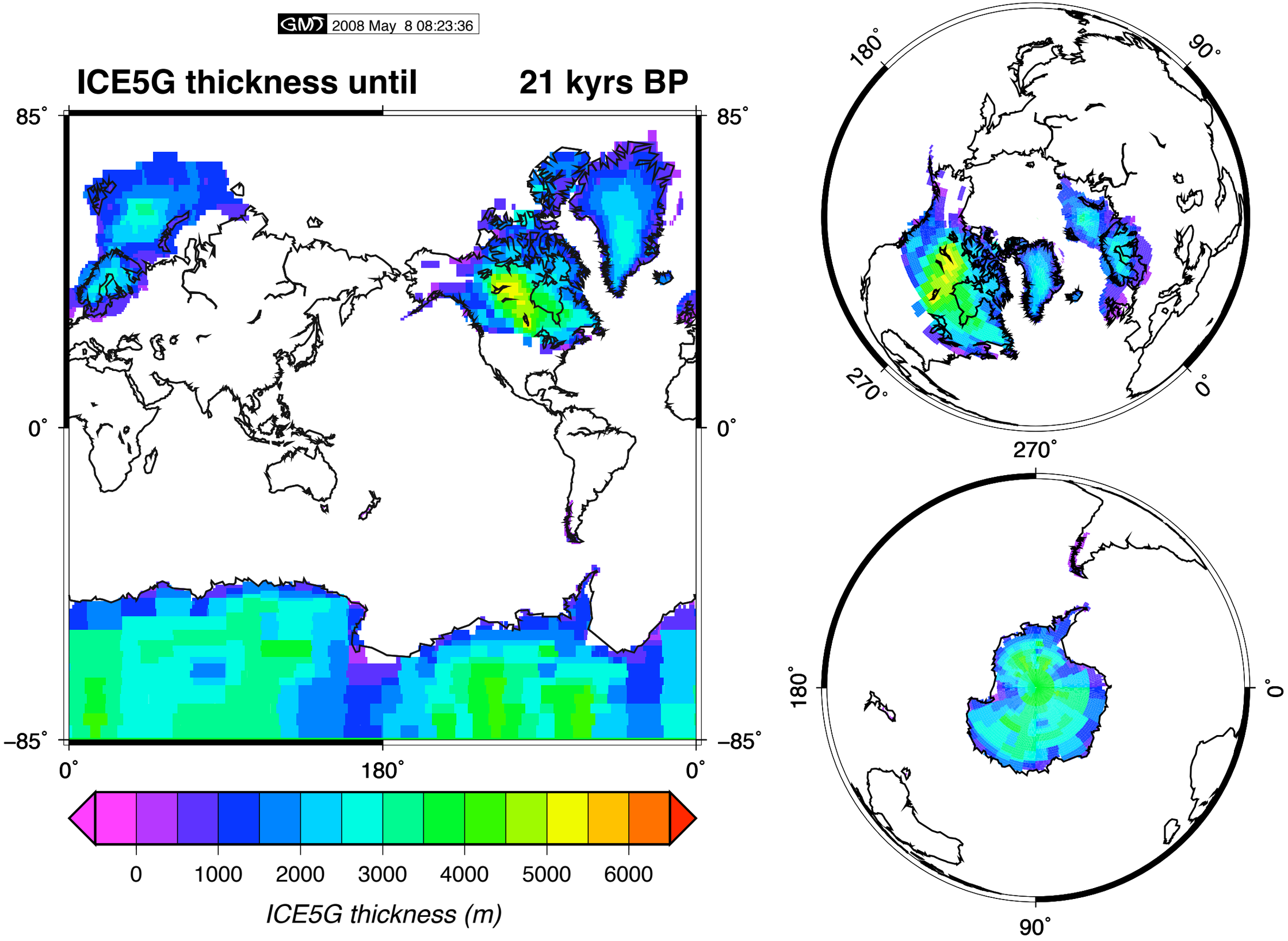


Fig. 1. Distribution of six predicted sea level zones resulting from melting of ice sheets 20,000–7,000 yr ago. Within each zone the form of the sea level response is similar. Typical curves indicating the predicted change in sea level relative to the present geoid for each zone are included (RSL means relative sea level). The curves show the wide variety of sea level expressions possible despite the assumption of no change in ocean volume during the past 5,000 yr (no change in 'eustatic' sea level) [from *Clark and Lingle,* 1979].

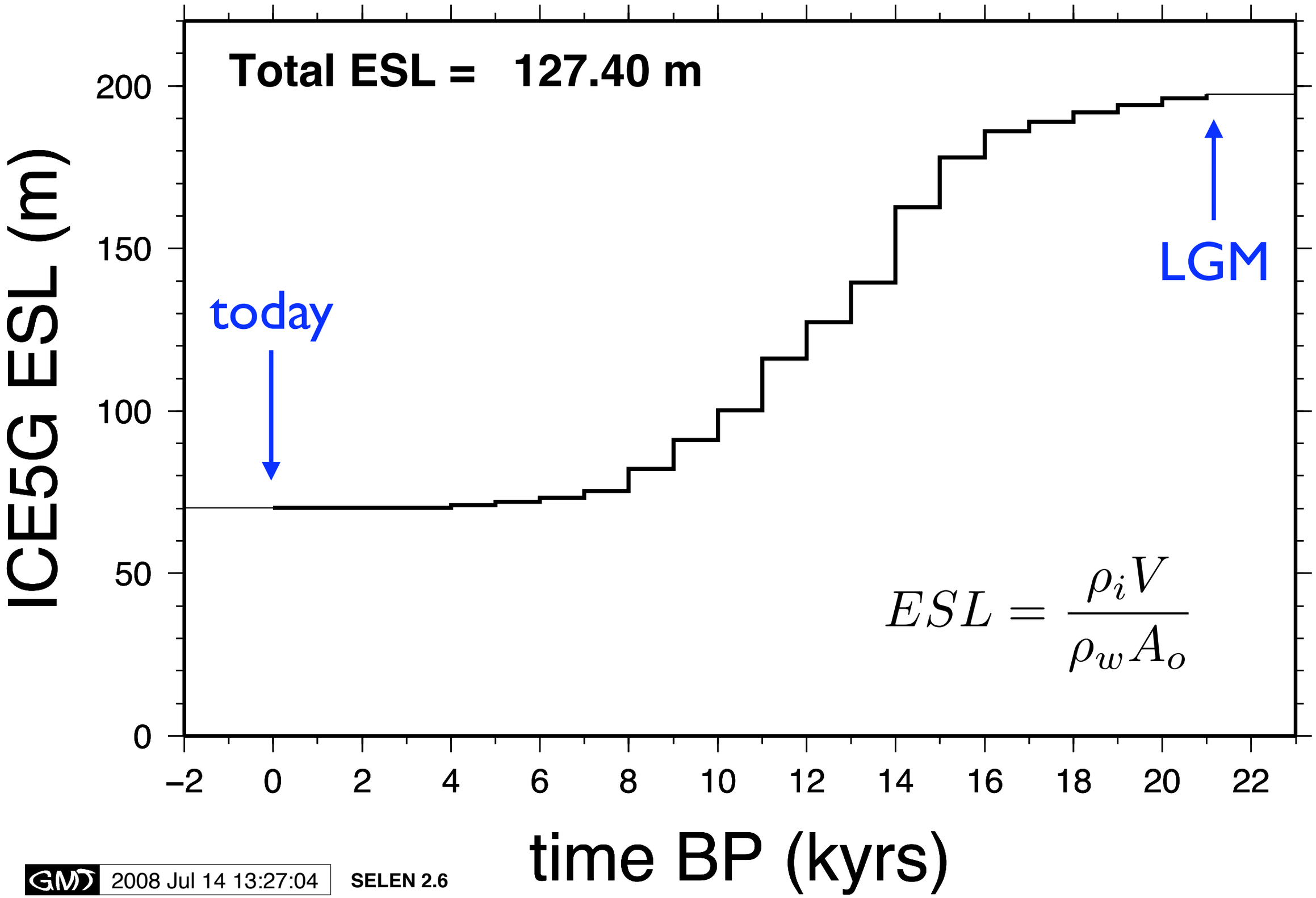
A case study

the Hudson Bay

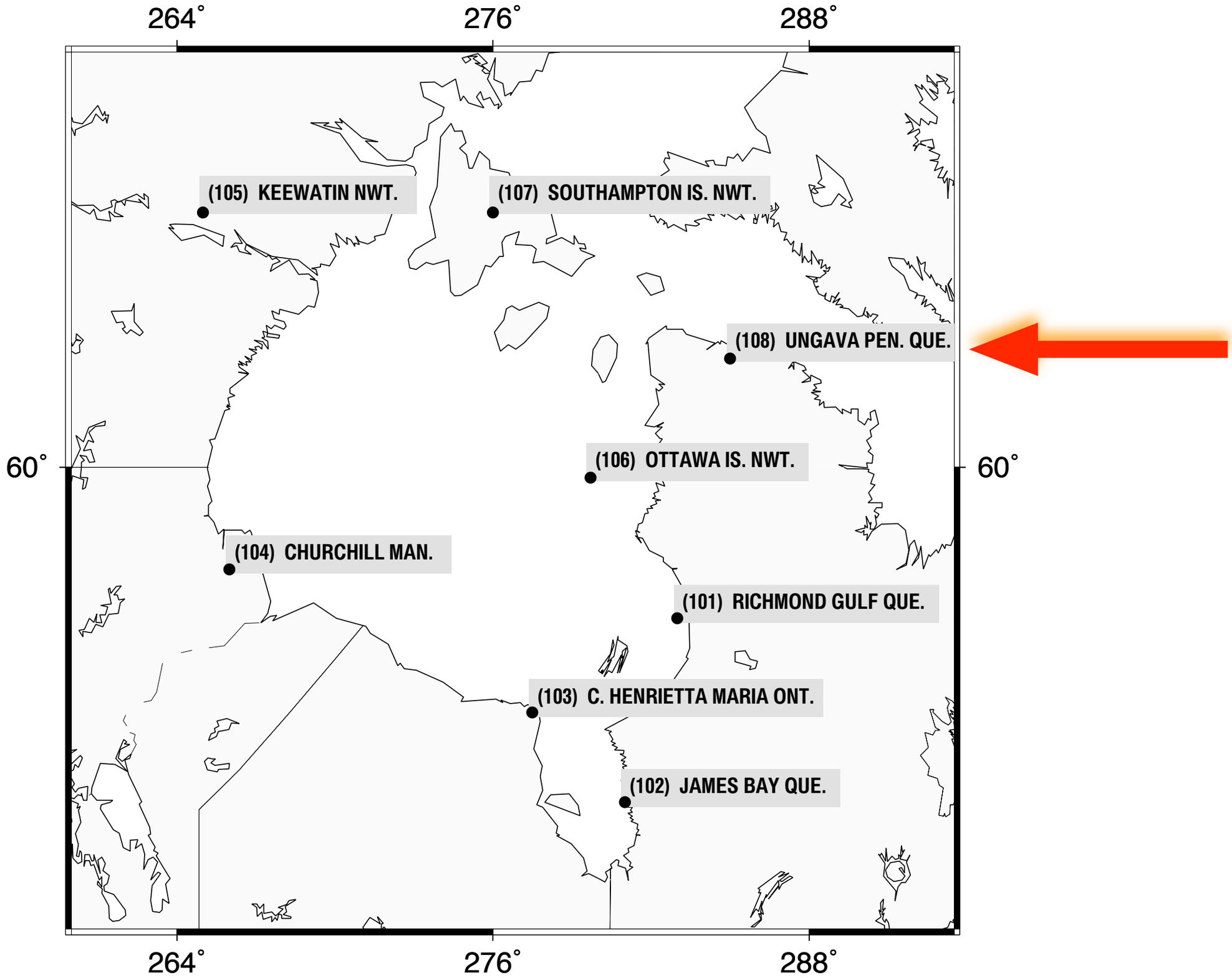
A case study: the Hudson Bay



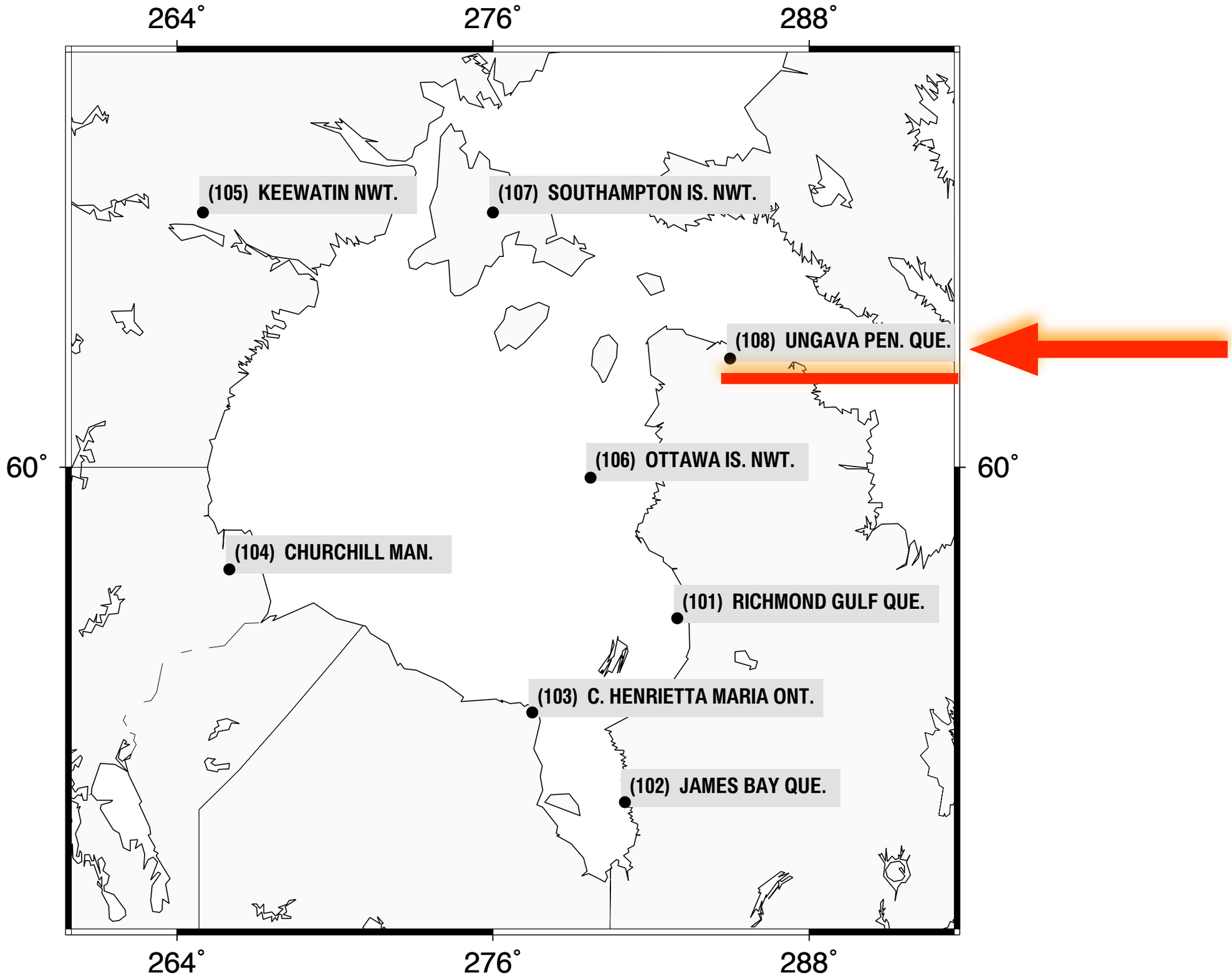
EQUIVALENT SEA LEVEL

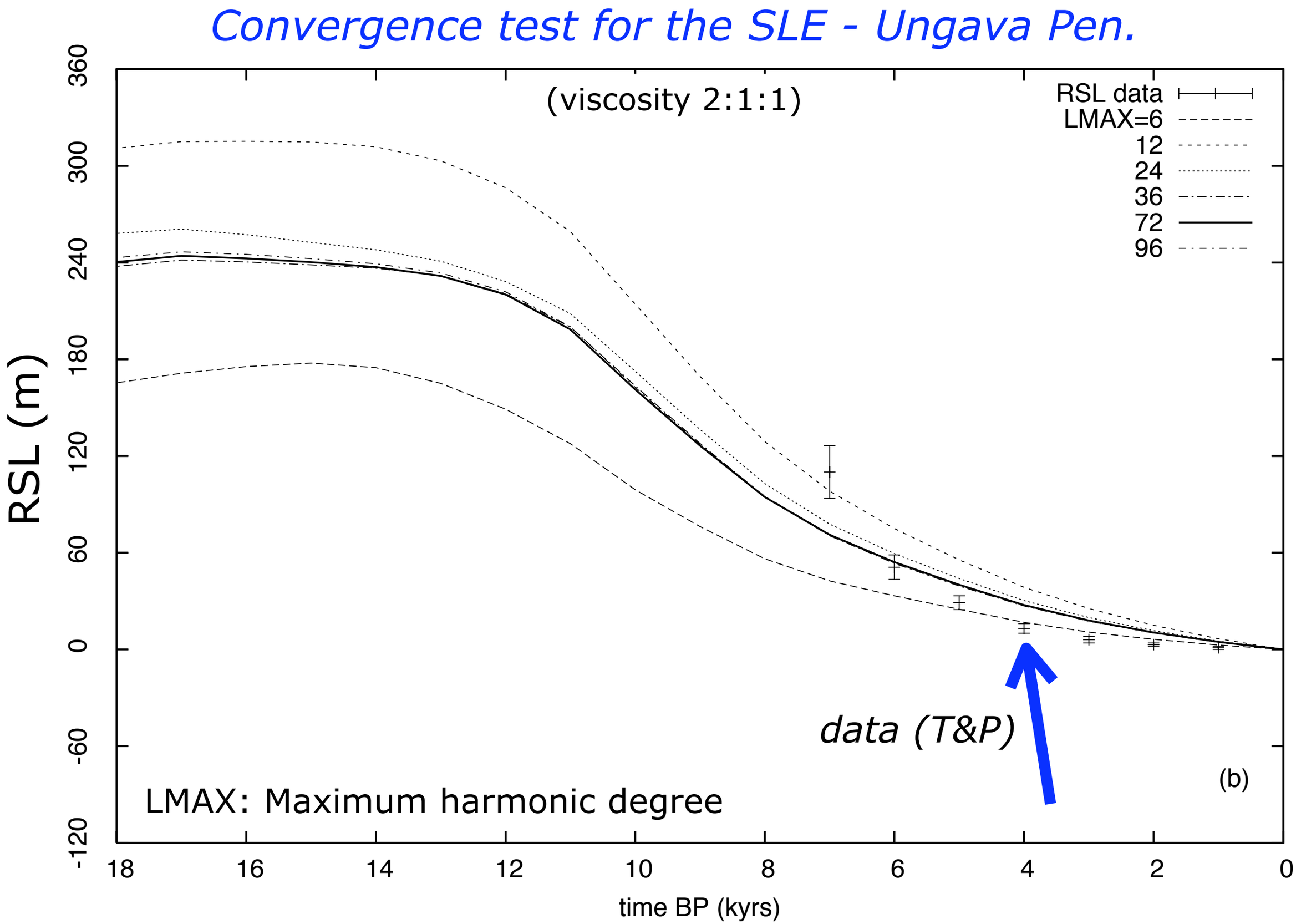


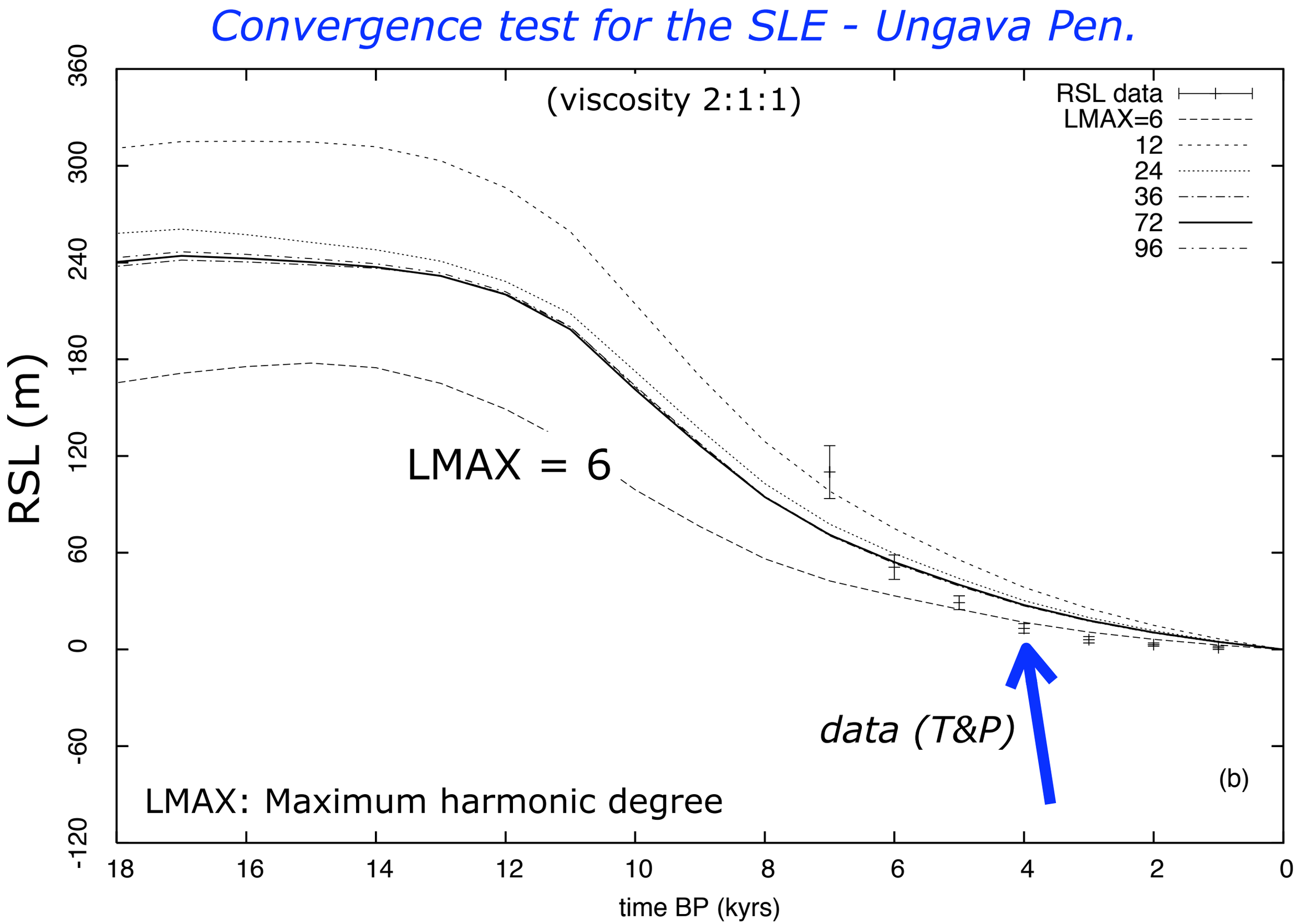
A case study the Hudson Bay



A case study the Hudson Bay




Convergence test for the SLE - Ungava Pen.
(viscosity 2:1:1)
RSL data
LMAX=6
12
24
36
72
96
RSL (m)
360
300
240
180
120
60
0
-60
-120
18
16
14
12
10
8
6
4
2
0
time BP (kyrs)
data (T&P)
(b)
LMAX: Maximum harmonic degree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(viscosity 2:1:1)
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-120
LMAX = 6
data (T&P)
(b)
LMAX: Maximum harmonic degree
18
16
14
12
10
8
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4
2
0
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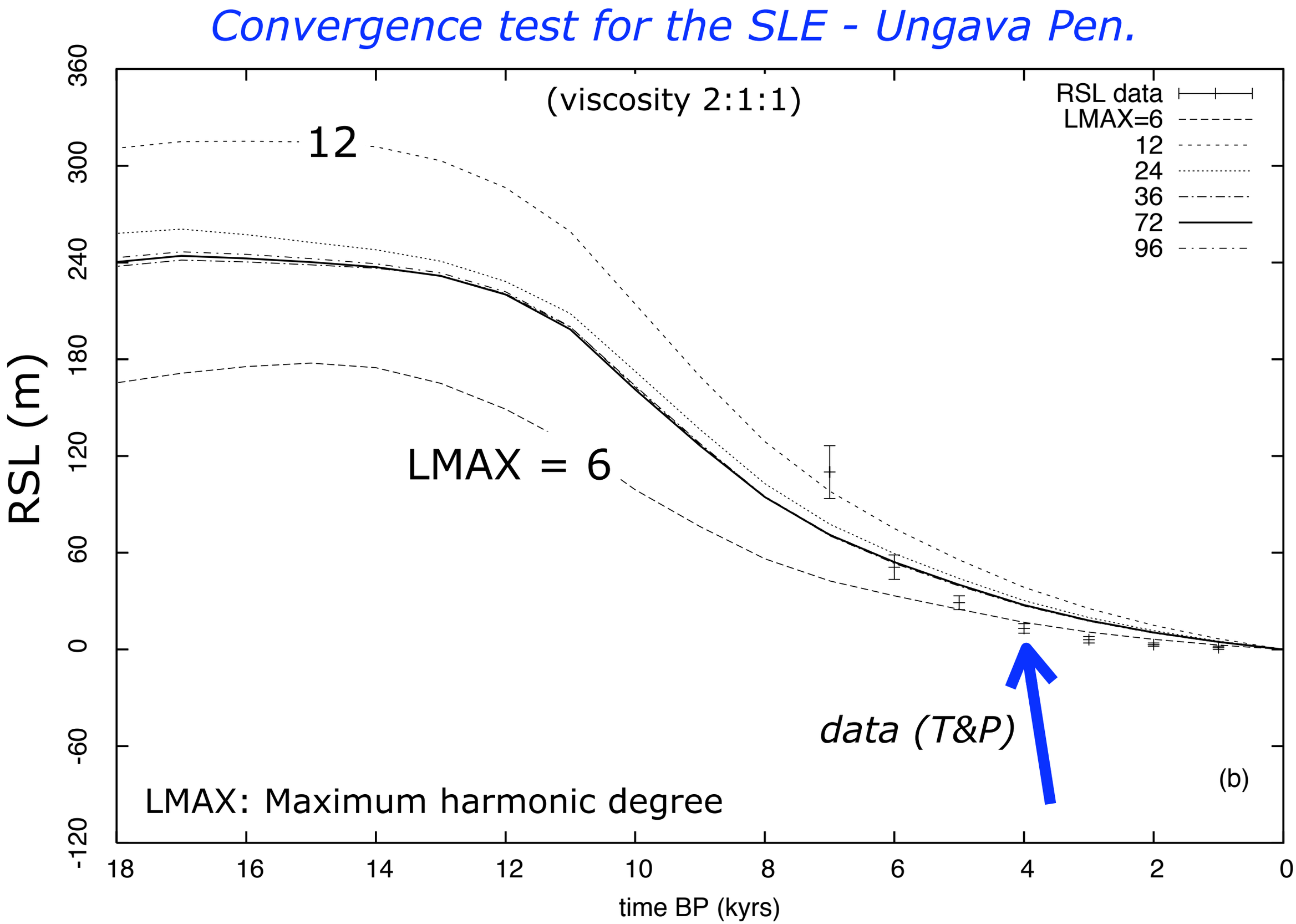
Convergence test for the SLE - Ungava Pen.
(viscosity 2:1:1)
12
LMAX = 6
RSL data
LMAX=6
12
24
36
72
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RSL (m)
360
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180
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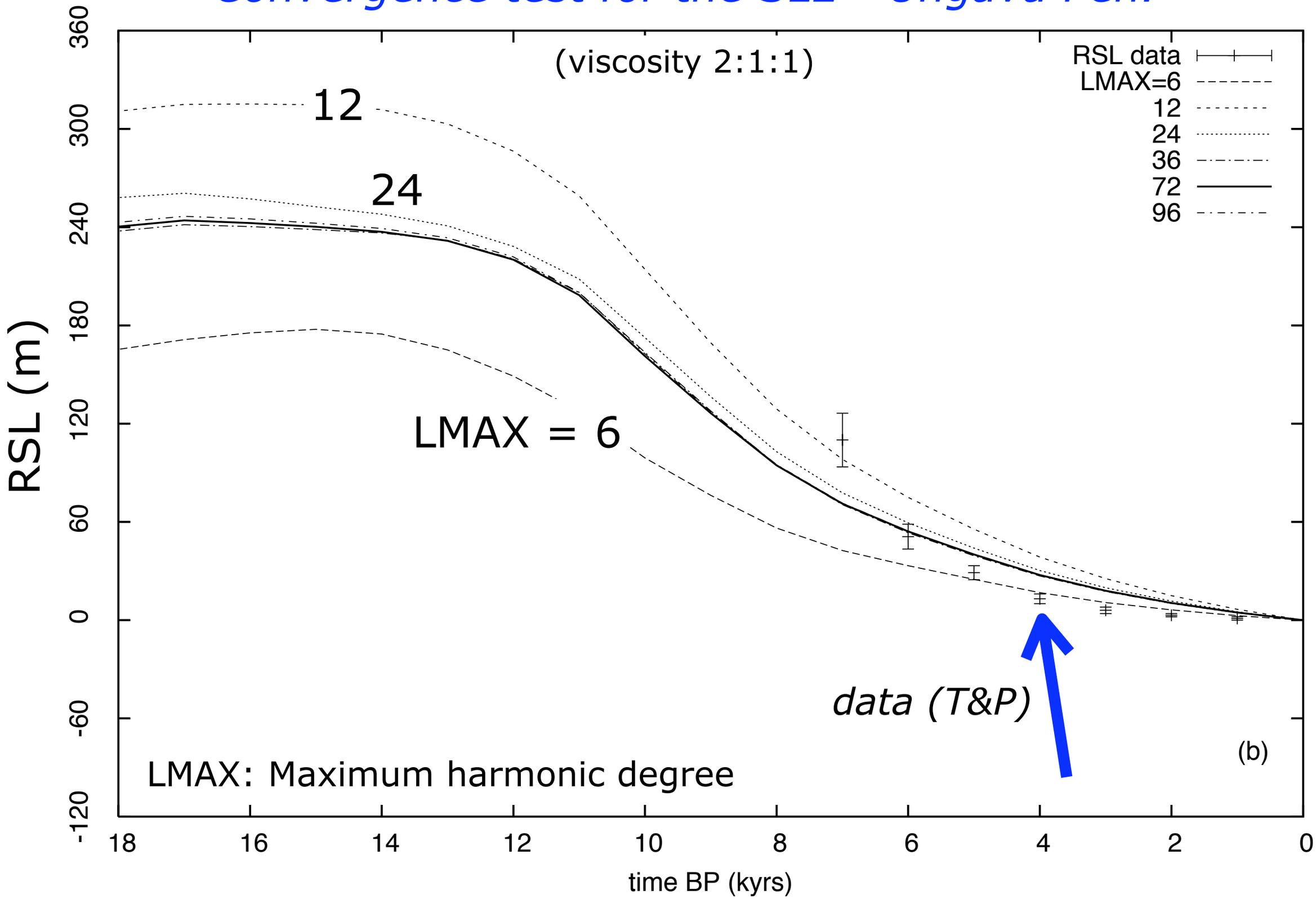
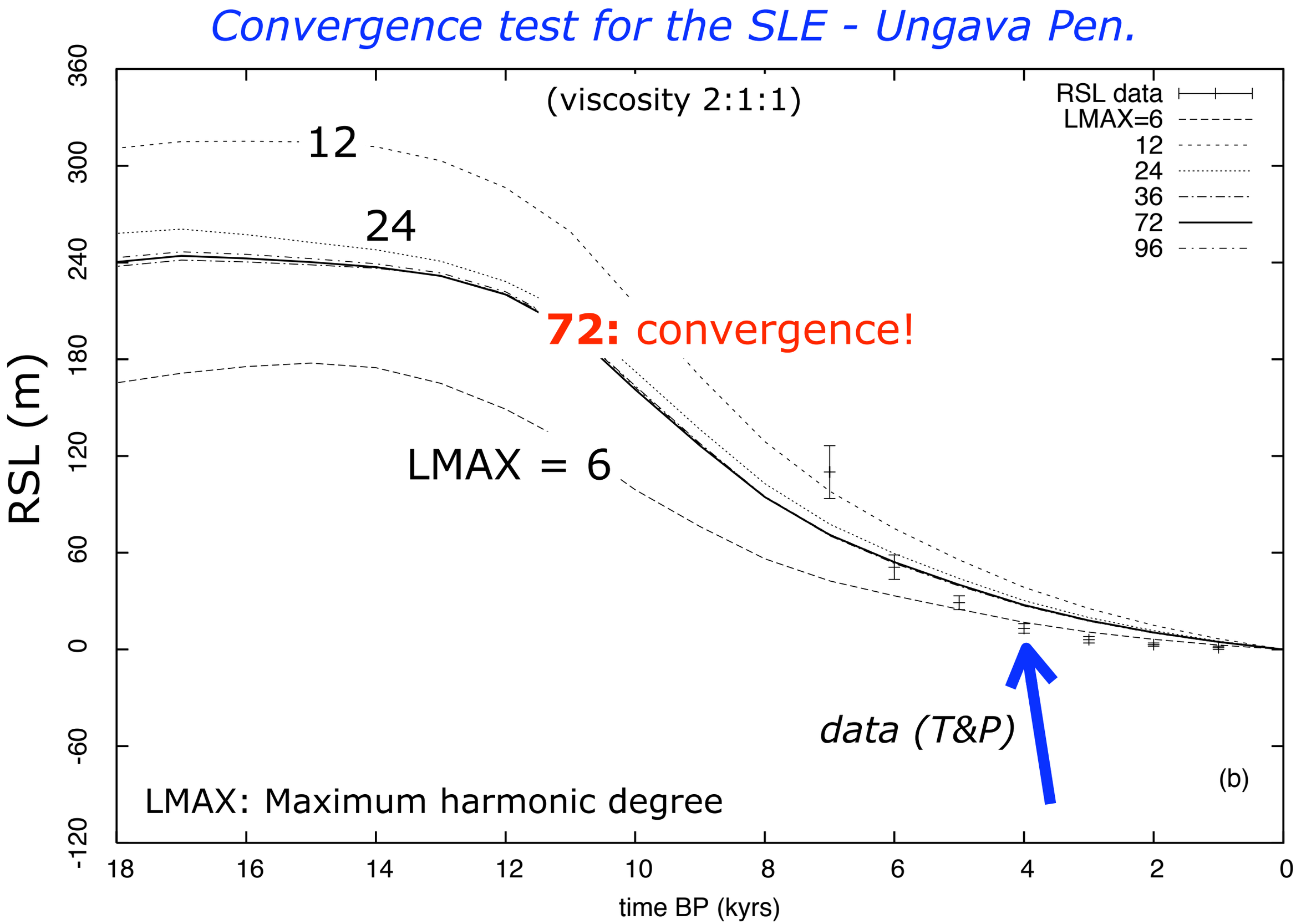
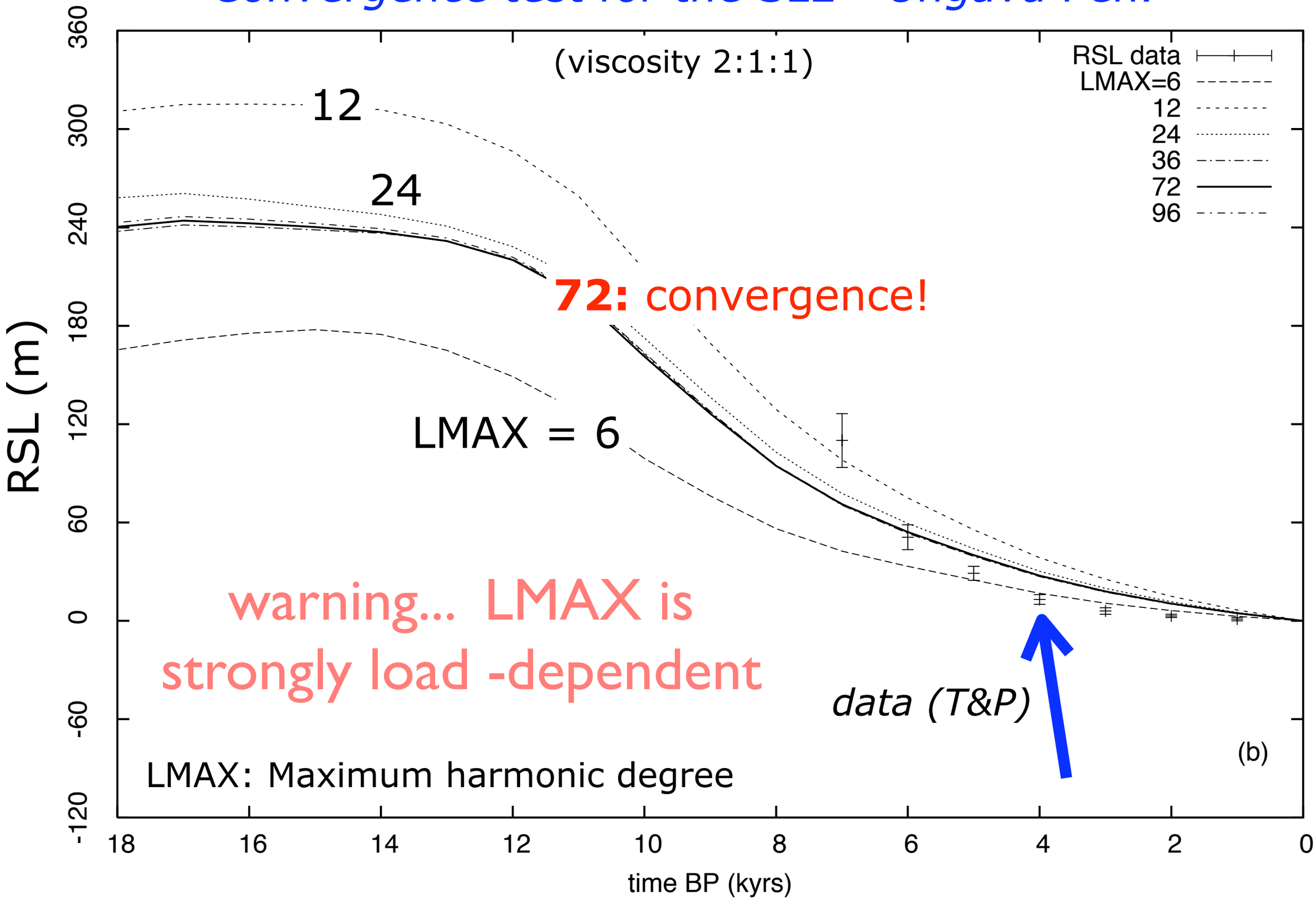
60
0
-60
-120
18
16
14
12
10
8
6
4
2
0
time BP (kyrs)

Convergence test for the SLE - Ungava Pen.

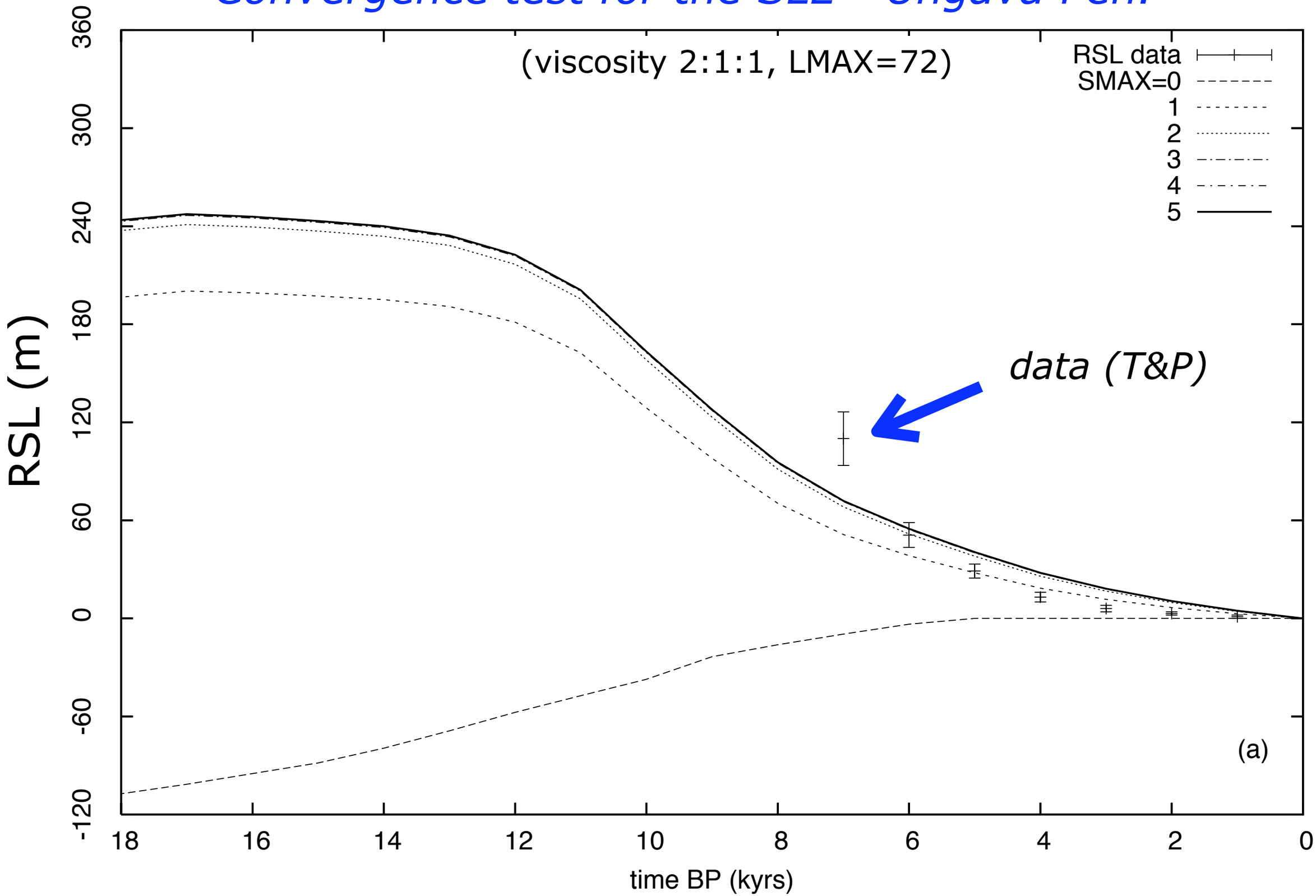



Convergence test for the SLE - Ungava Pen.
(viscosity 2:1:1)
RSL data
LMAX=6
12
24
36
72
96
12
24
72: convergence!
LMAX = 6
data (T&P)
(b)
LMAX: Maximum harmonic degree
RSL (m)
360
300
240
180
120
60
0
-60
-120
18
16
14
12
10
8
6
4
2
0
time BP (kyrs)

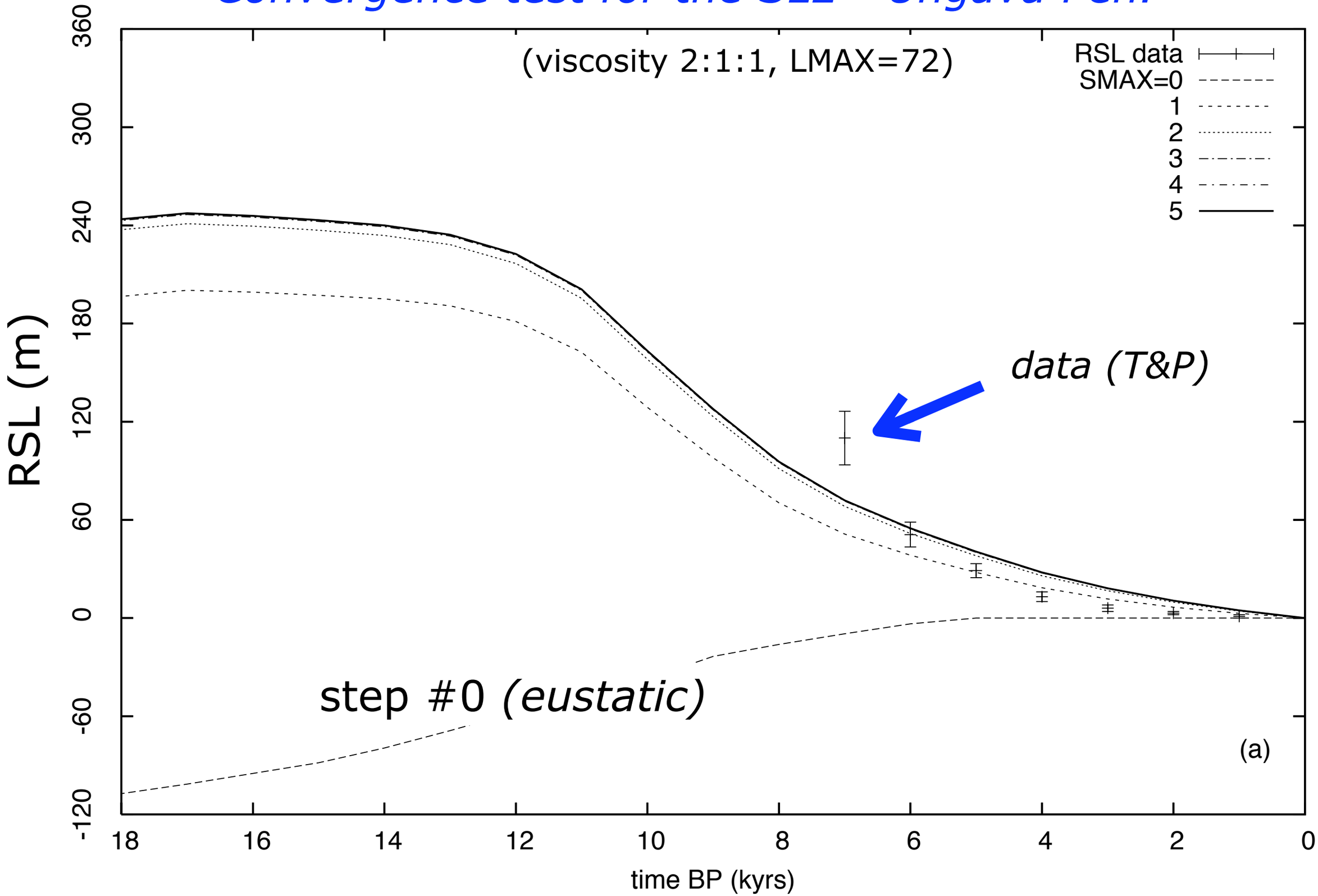
Convergence test for the SLE - Ungava Pen.

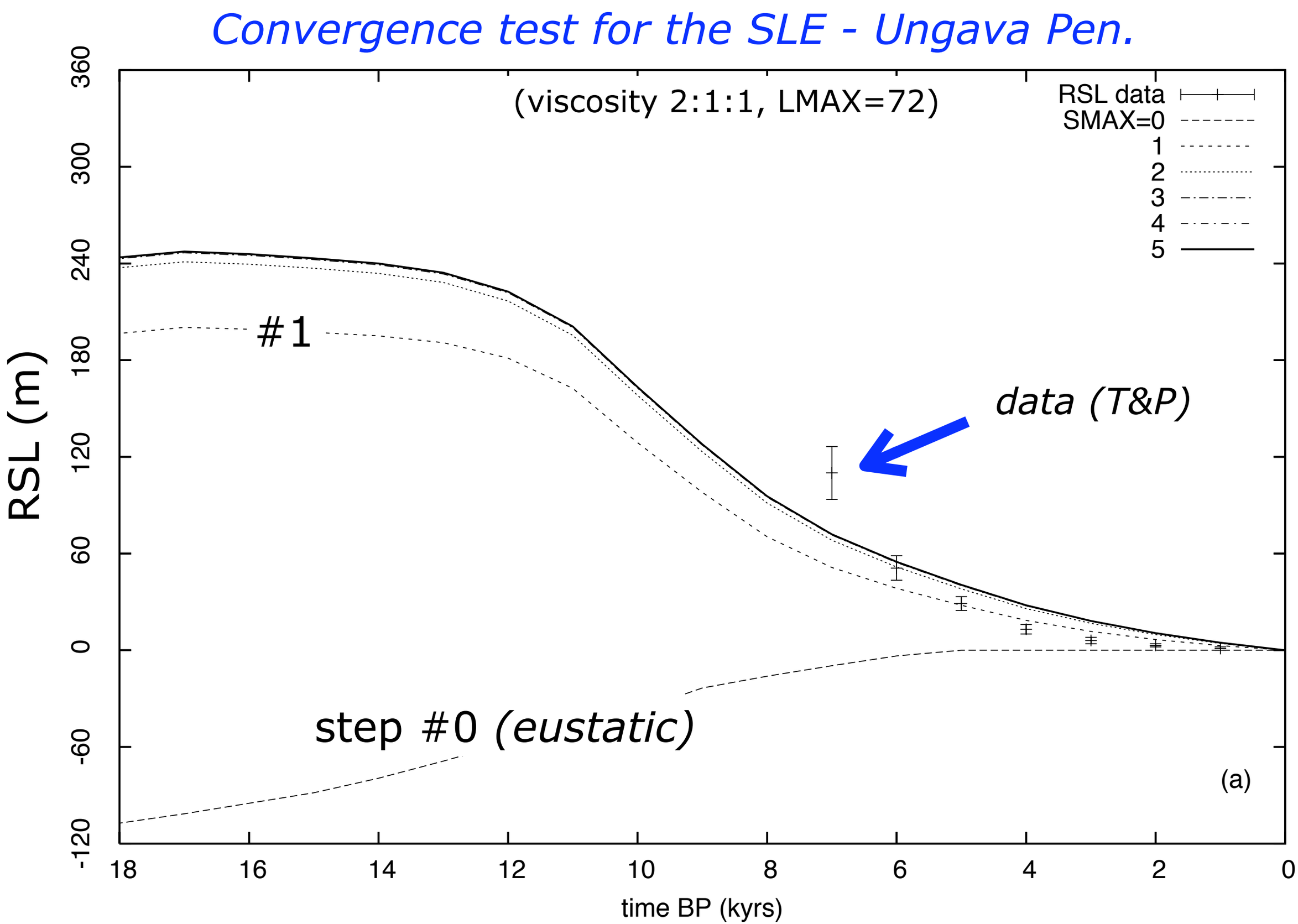


Convergence test for the SLE - Ungava Pen.

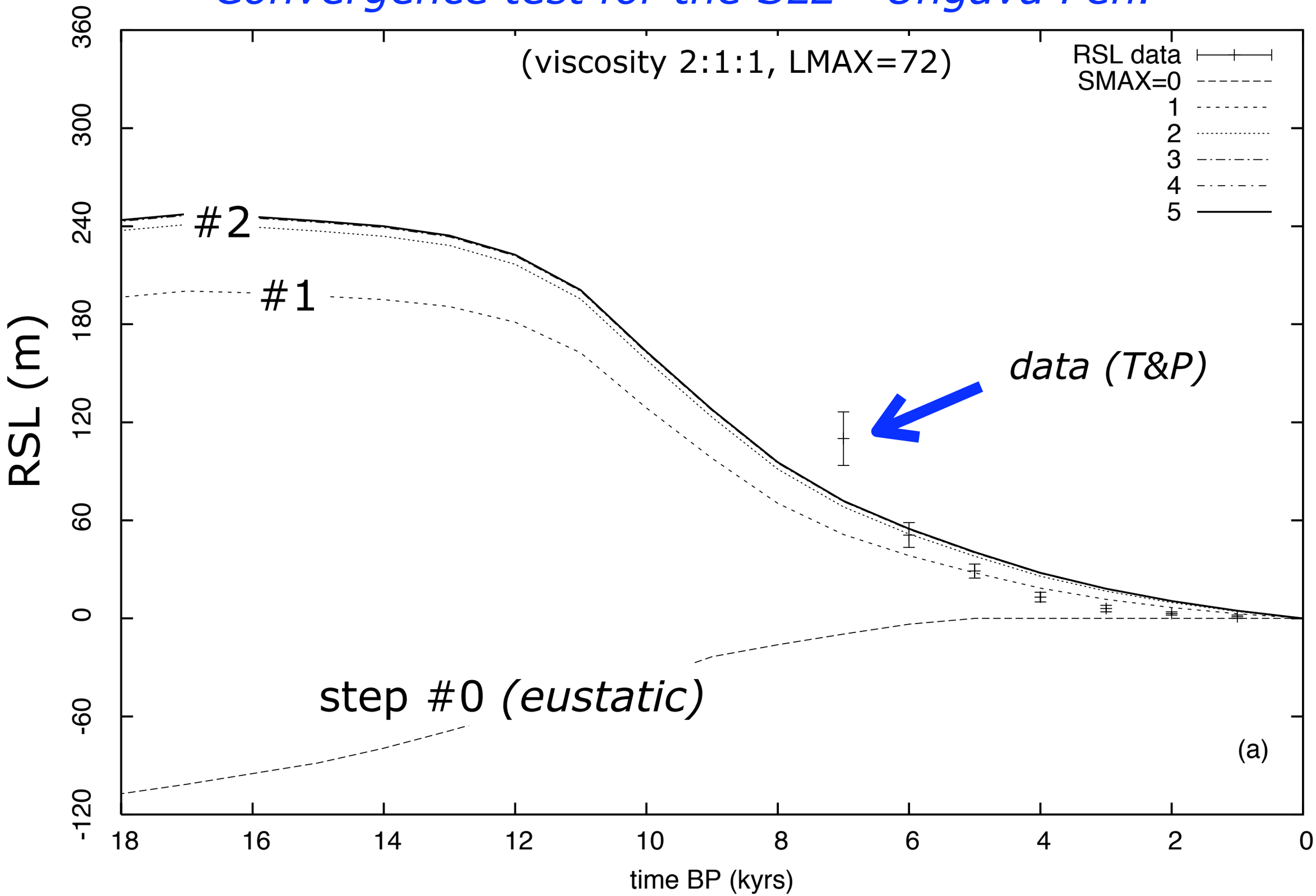


Convergence test for the SLE - Ungava Pen.

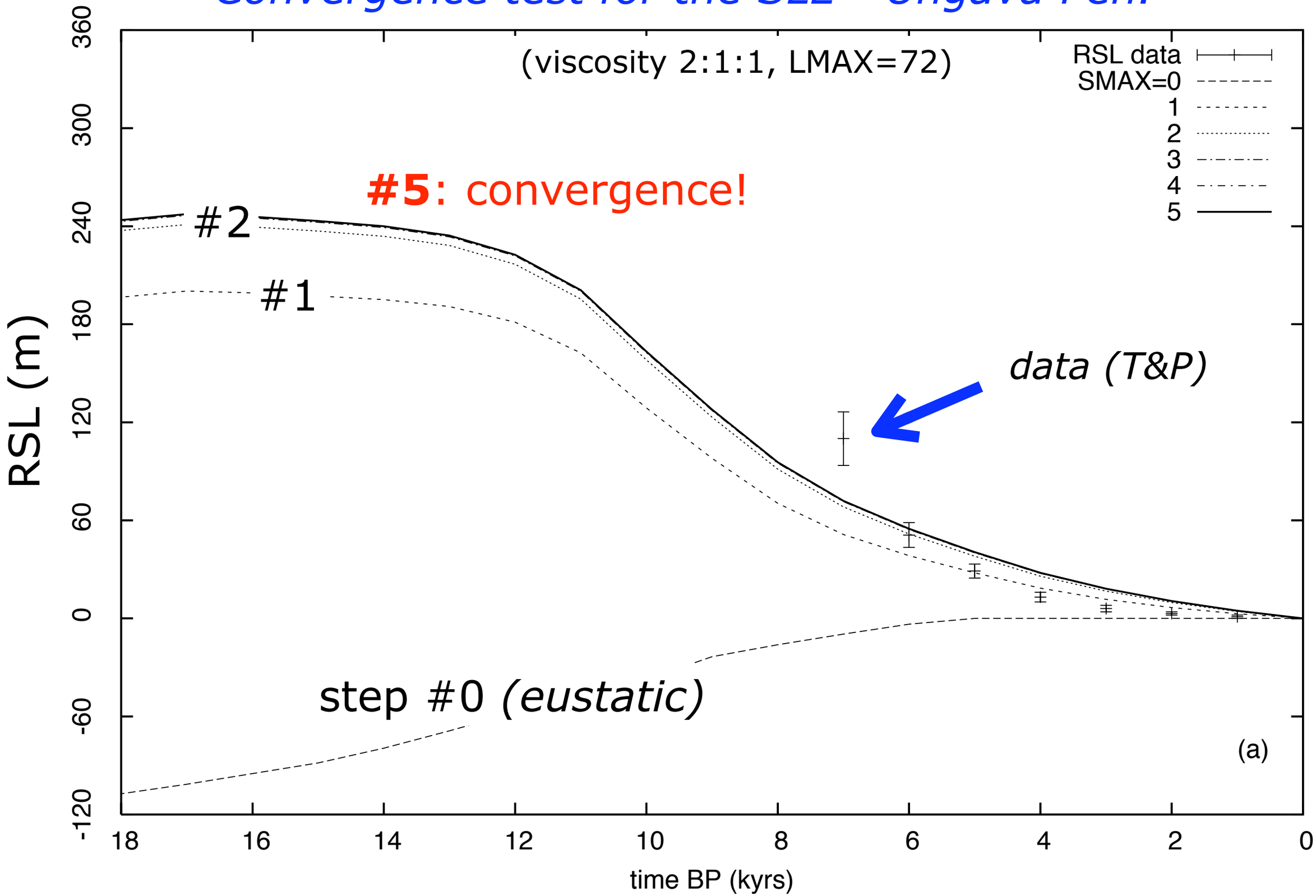



Convergence test for the SLE - Ungava Pen.
(viscosity 2:1:1, LMAX=72)
RSL data
SMAX=0
1
2
3
4
5
#1
data (T&P)
step #0 (eustatic)
(a)
RSL (m)
360
300
240
180
120
60
0
-60
-120
18
16
14
12
10
8
6
4
2
0
time BP (kyrs)

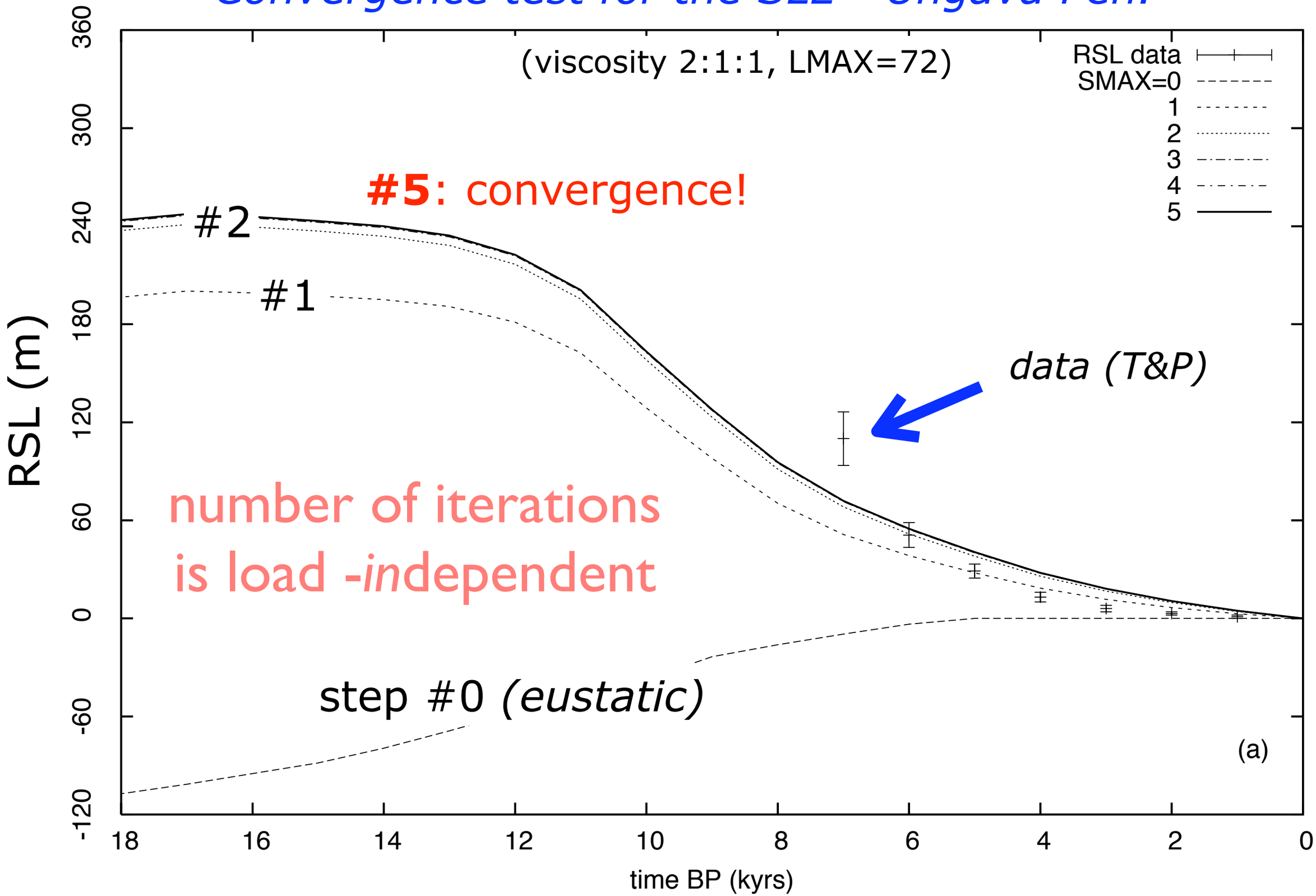
Convergence test for the SLE - Ungava Pen.



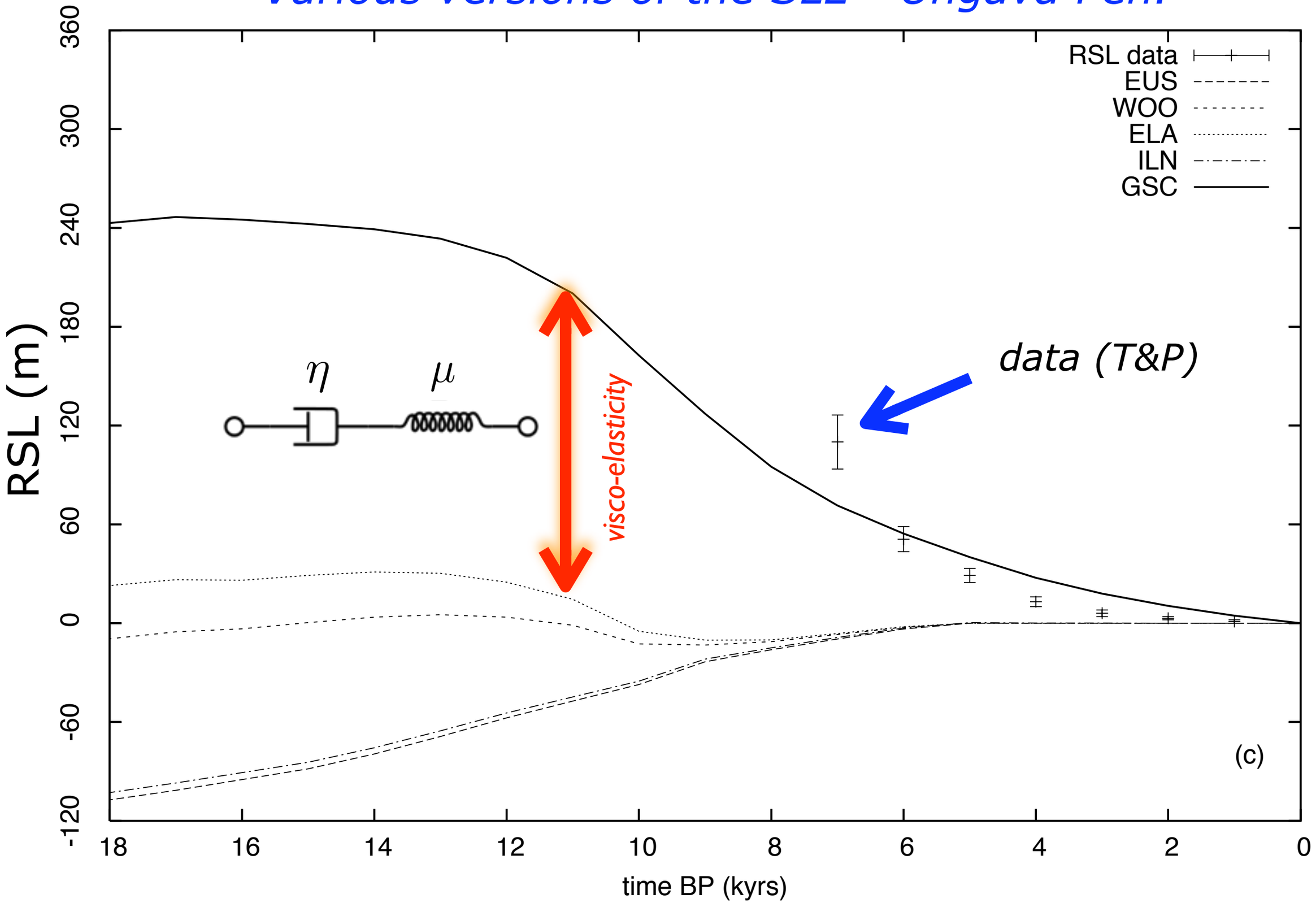
Convergence test for the SLE - Ungava Pen.



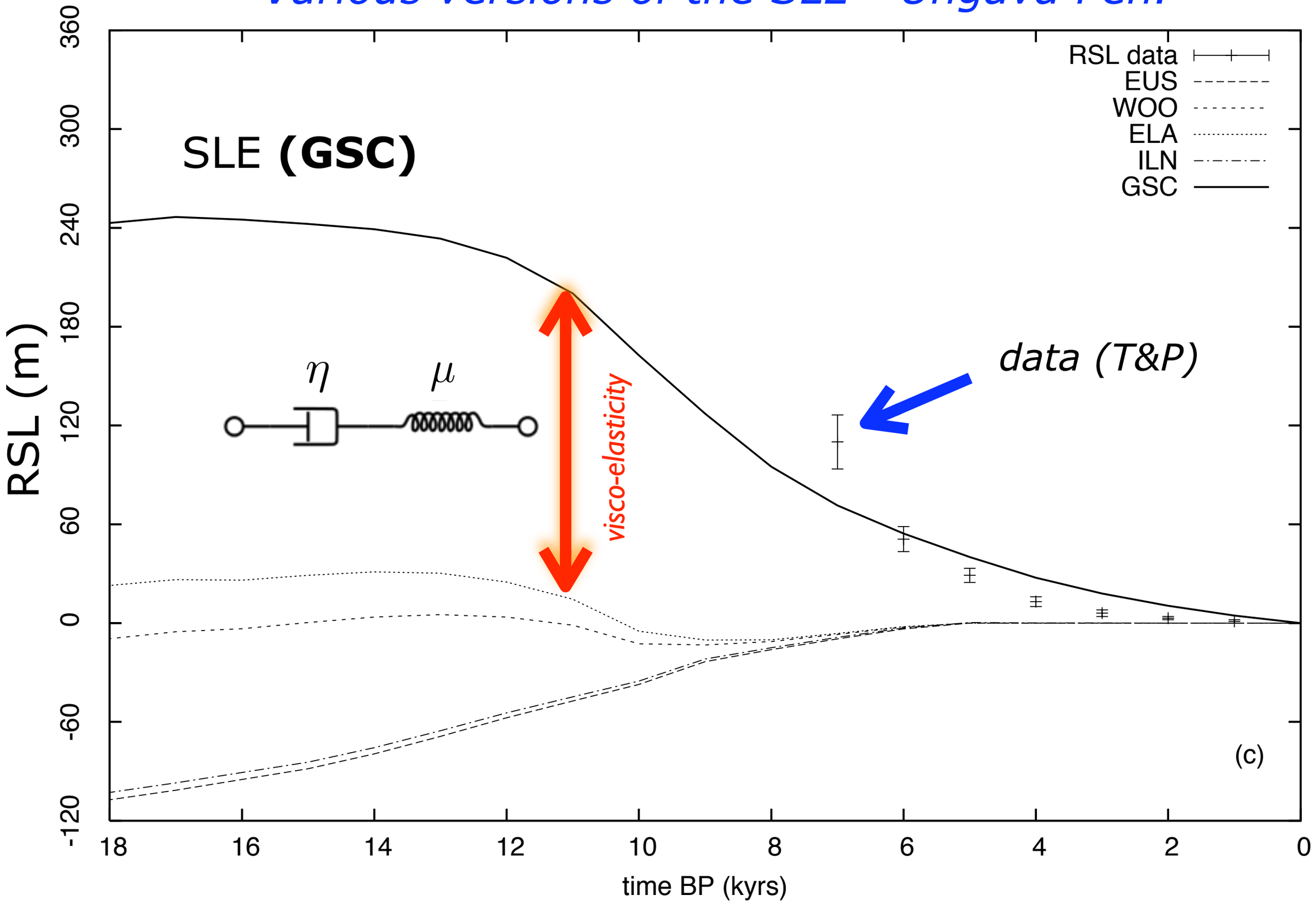
Convergence test for the SLE - Ungava Pen.

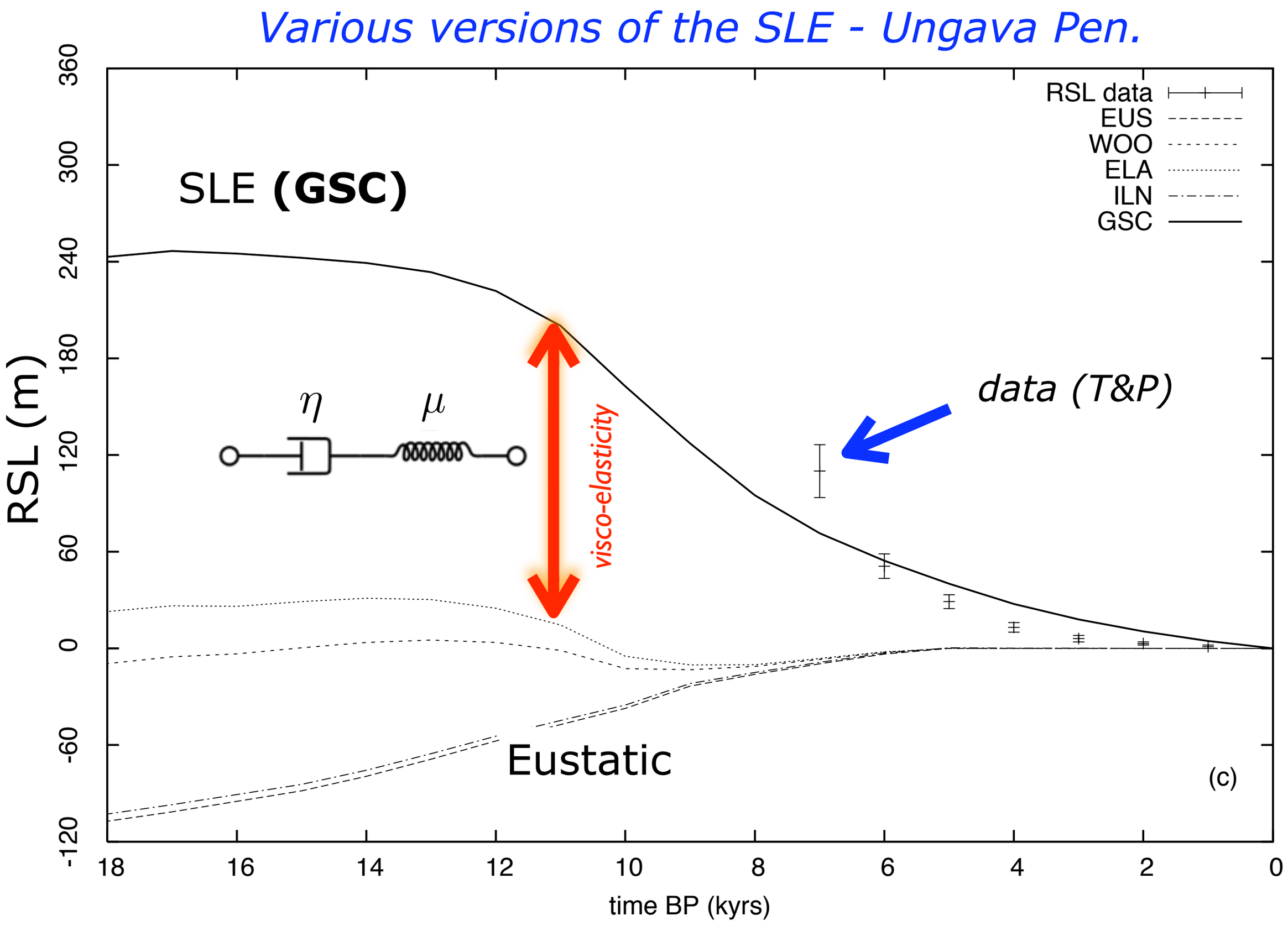


Various versions of the SLE - Ungava Pen.

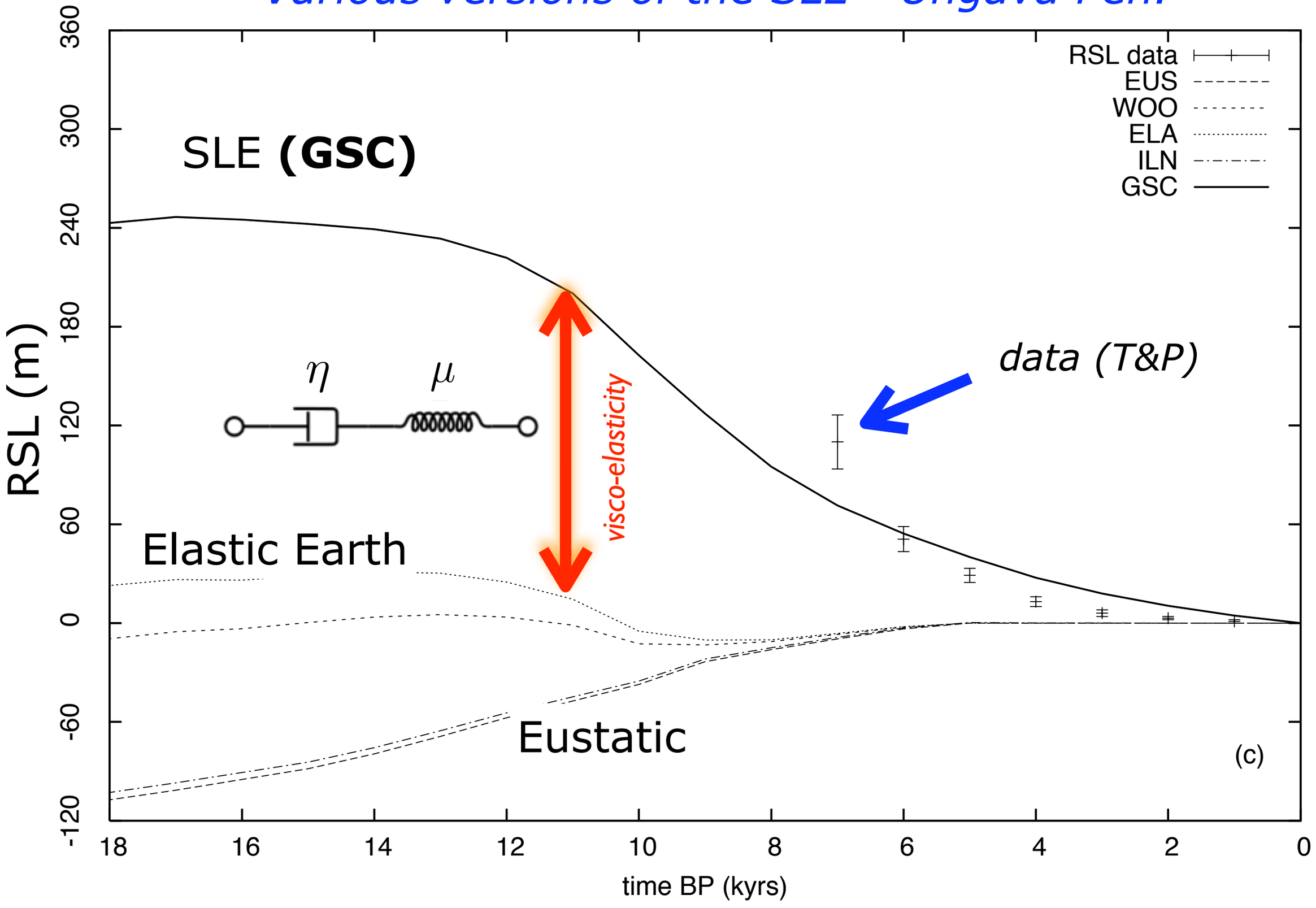


Various versions of the SLE - Ungava Pen.

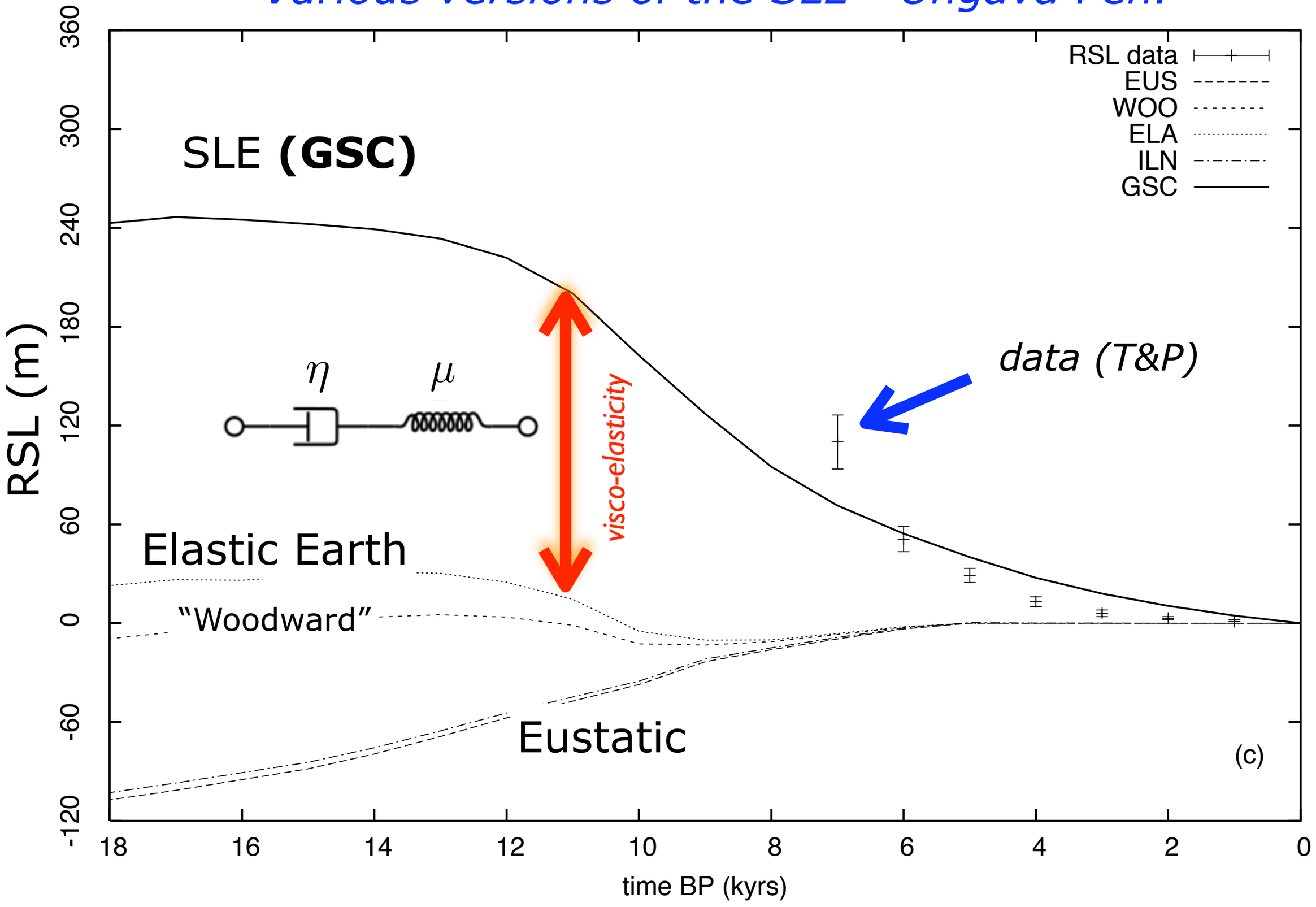



Various versions of the SLE - Ungava Pen.
SLE (GSC)
RSL data
EUS
WOO
ELA
ILN
GSC
RSL (m)
360
300
240
180
120
60
0
-60
-120
18
16
14
12
10
8
6
4
2
0
time BP (kyrs)
η
μ
visco-elasticity
data (T&P)
Eustatic
(c)

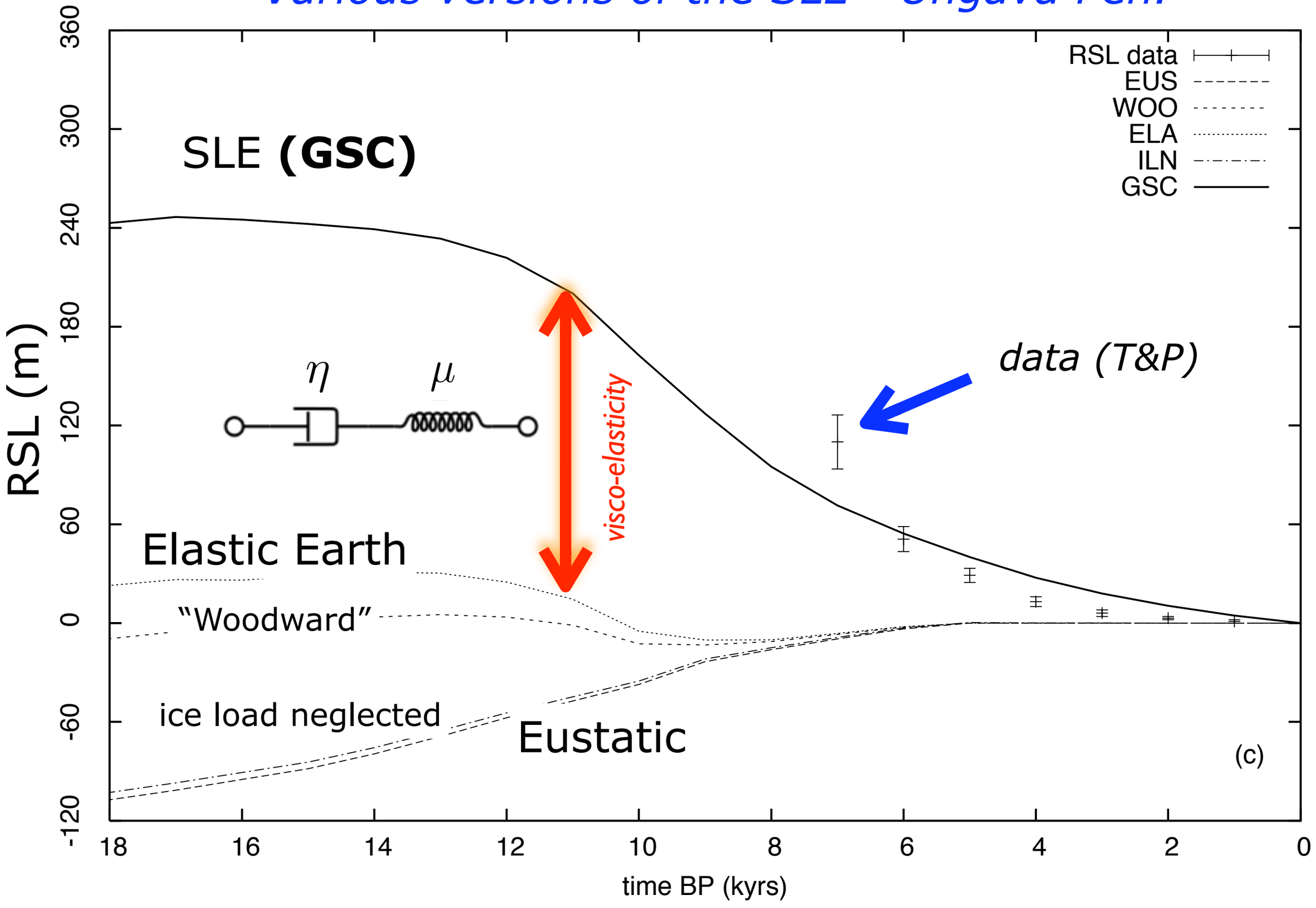
Various versions of the SLE - Ungava Pen.

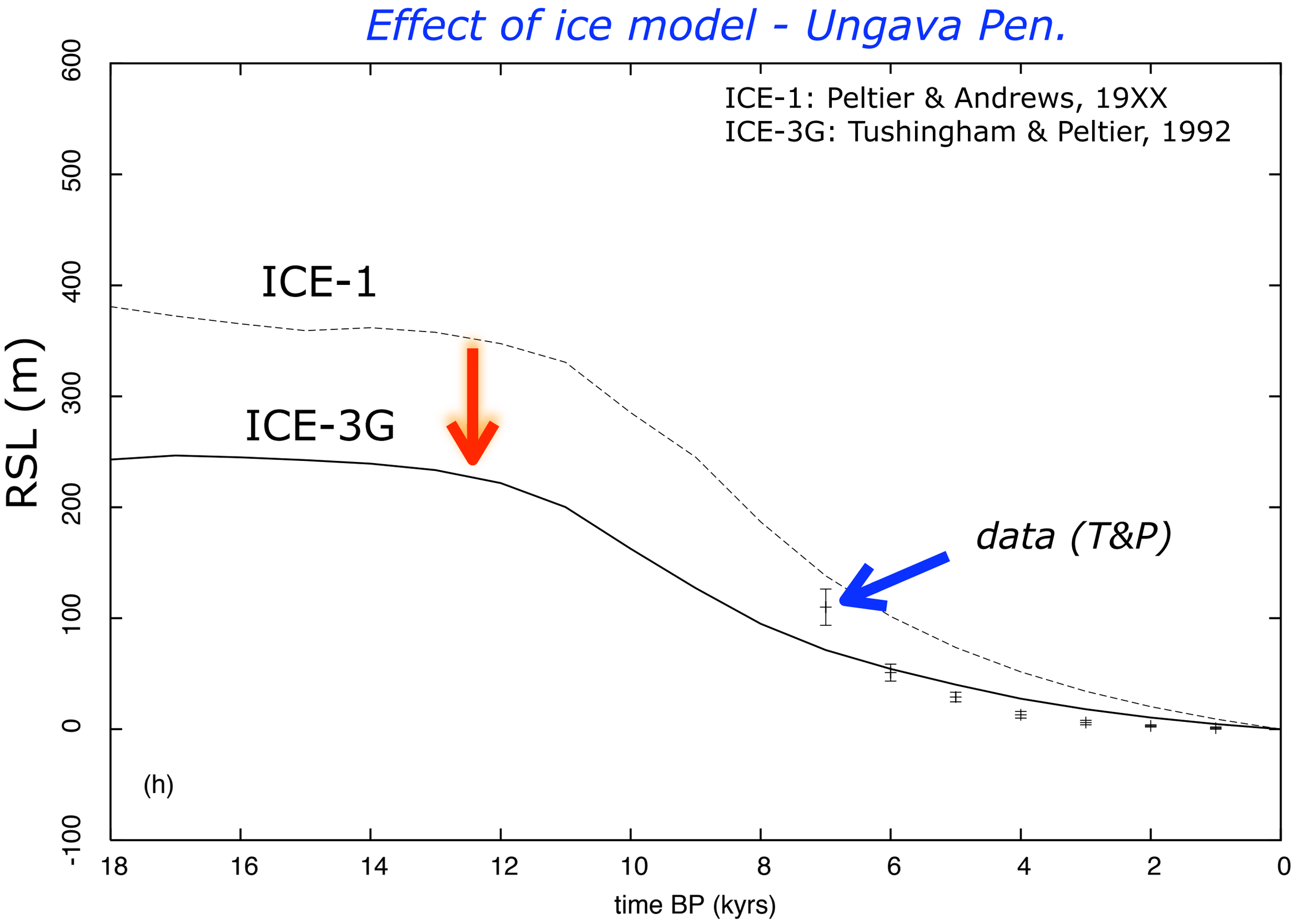


Various versions of the SLE - Ungava Pen.



Various versions of the SLE - Ungava Pen.




Effect of ice model - Ungava Pen.
ICE-1: Peltier & Andrews, 19XX
ICE-3G: Tushingham & Peltier, 1992
ICE-1
ICE-3G
data (T&P)
(h)
RSL (m)
time BP (kyrs)
600
500
400
300
200
100
0
-100
18
16
14
12
10
8
6
4
2
0

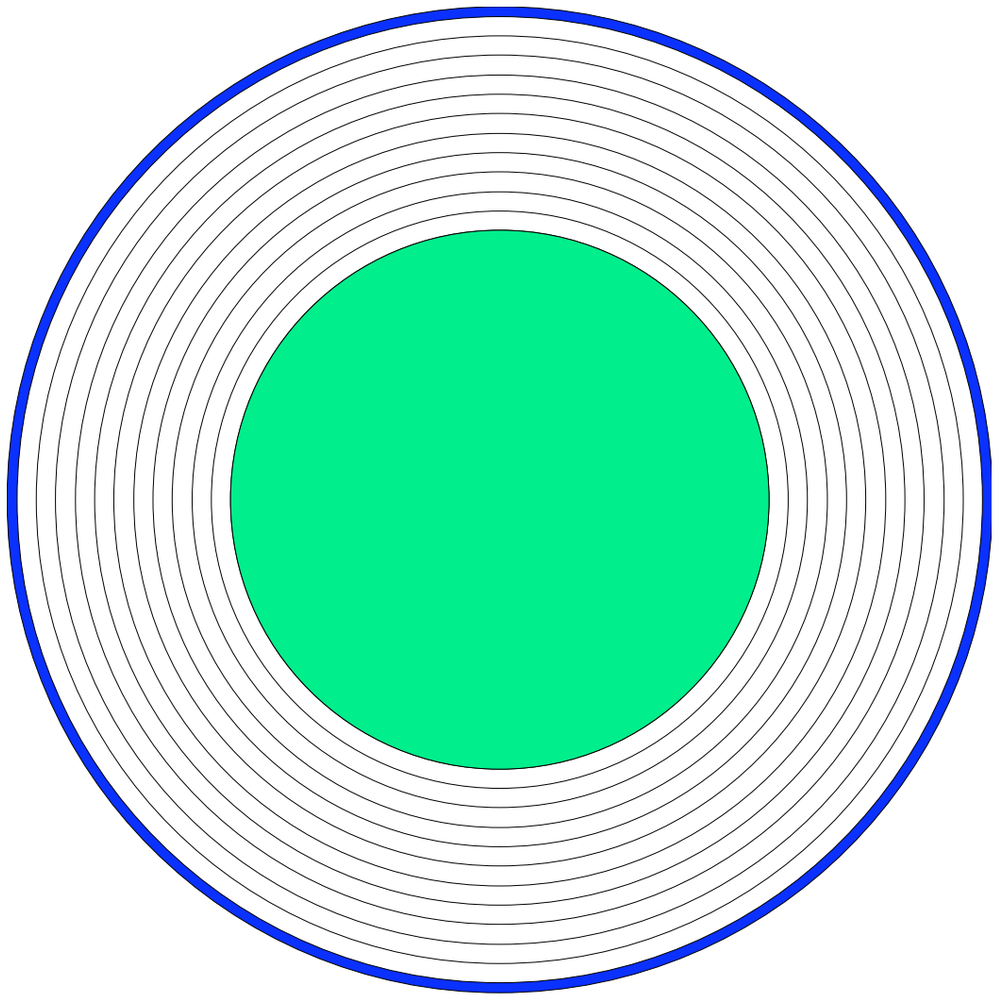
3. Test #0 of the SLE benchmark

a few results

Special topic (2/3)

TEST#0 Earth model description

M3-L70-V01



Spherically symmetric Earth

$$\mu = \mu(r) \qquad \lambda = \lambda(r) \quad \text{Lamè constants}$$

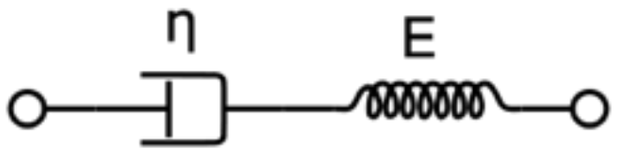
$$\rho = \rho(r) \quad \text{Density}$$

$$\eta = \eta(r) \quad \text{Viscosity}$$

✓(elastic) Lithosphere

✓Fluid inviscid core

✓Maxwell Linear viscoelastic rheology



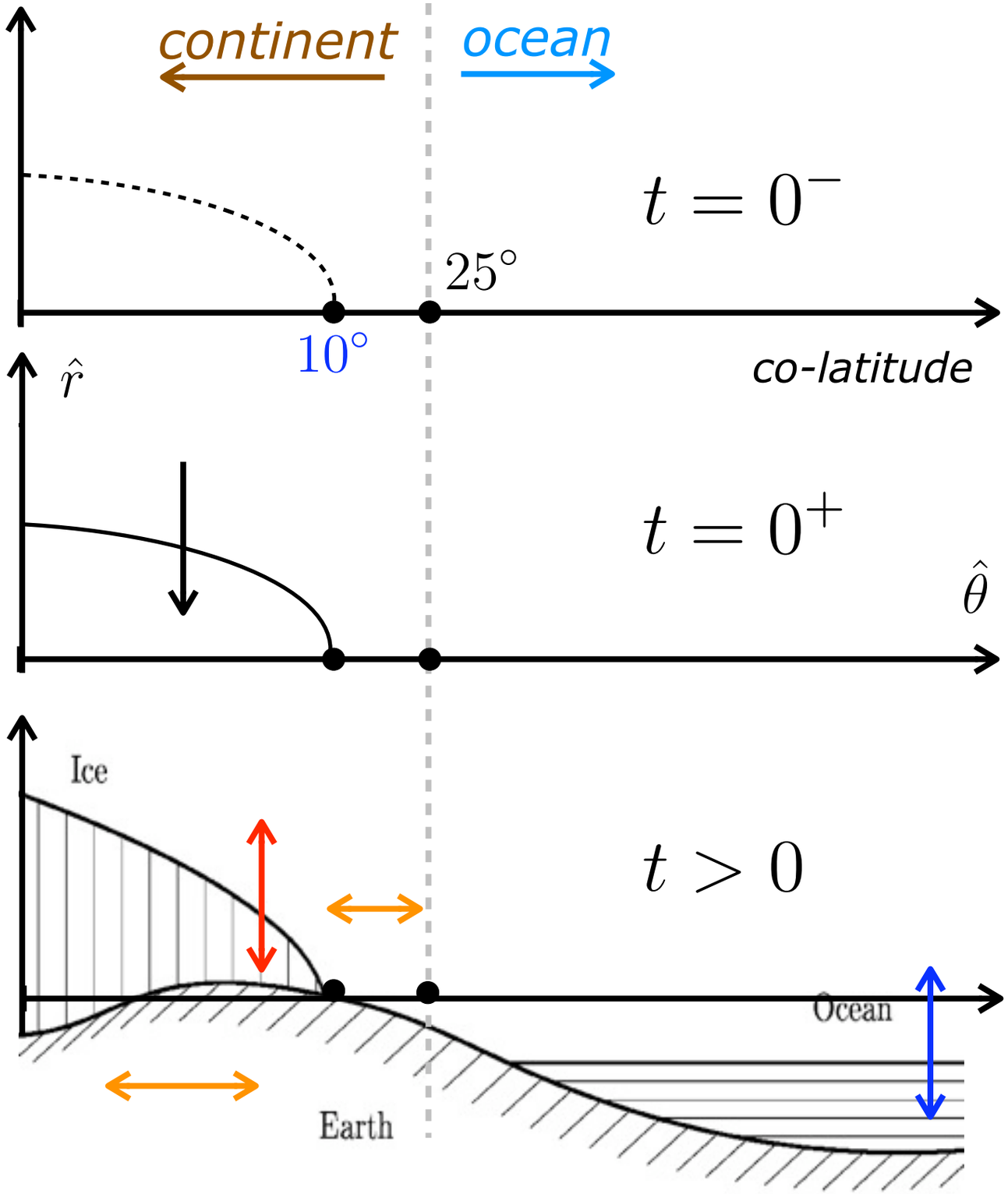
Model M3-L70-V01

(In)compressible, (self)-gravitating

Lithosphere + 3-layer mantle + Fluid Core

Table 1: Model parameters of M3-L70-V01. This is the reference model.

Radius (km)	Density (kg/m^3)	Rigidity (Pa)	Viscosity $(Pa\ s)$
6371	3037	5.0605×10^{10}	∞
6301	3438	7.0363×10^{10}	1×10^{21}
5951	3871	1.0549×10^{11}	1×10^{21}
5701	4978	2.2834×10^{11}	2×10^{21}
3480	10750	0	0



TEST#0 - description

No load is in place.
stress-free surface

A cap load is imposed
surface vertical stress

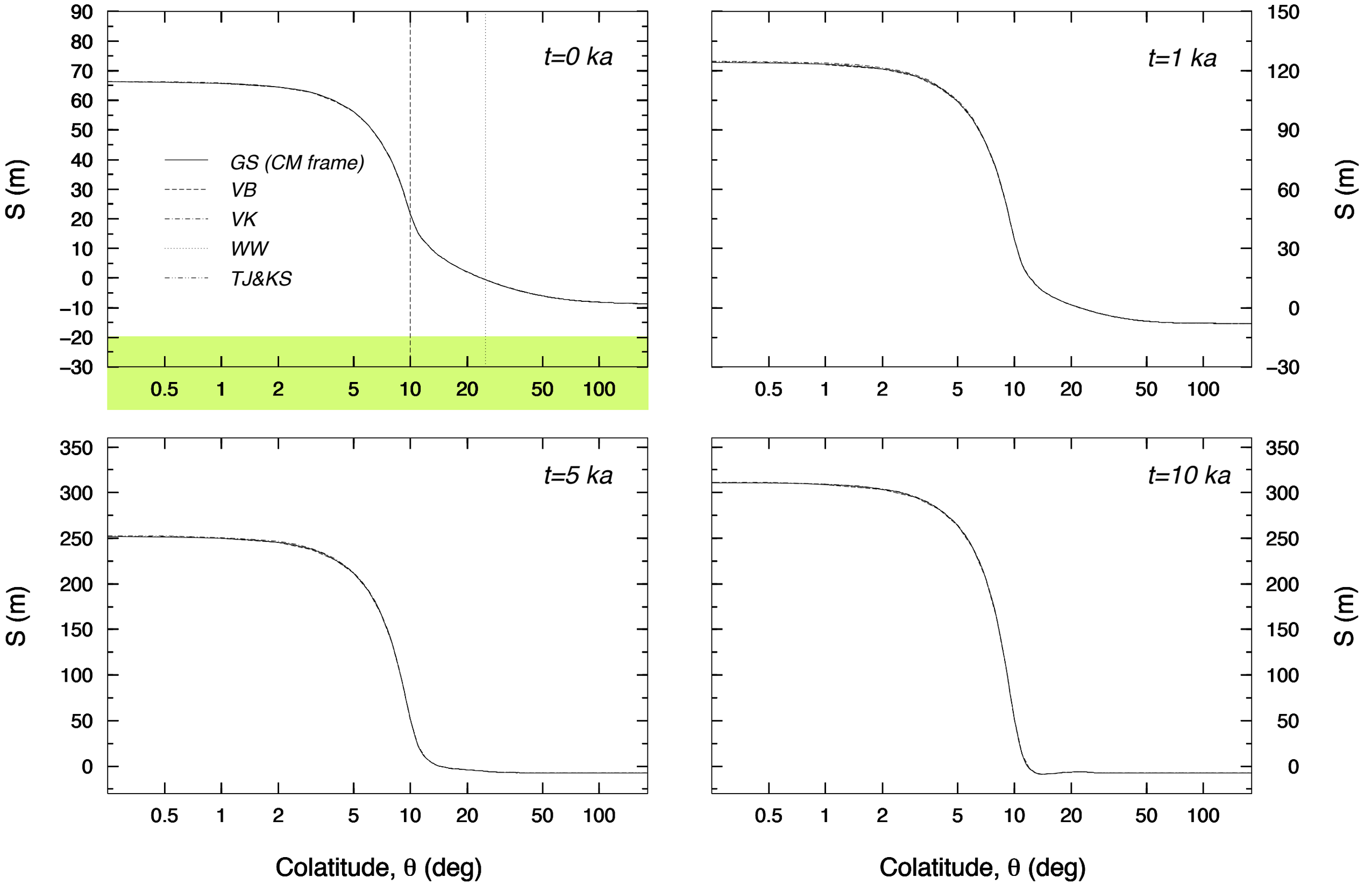
Goal:
computing displacements,
and sea level change...

List of SLE benchmark contributors / Codes / Methods / *not updated / in progress*

WG4 participant	aka	code name	method
Riccardo Riva	RR		
Valentina Barletta	VB		…
Volker Klemann	VK		
Zdenek Martinec	ZM		
Paolo Gasperini (*?)	PG	ABAQUS	FE
Giorgio Spada	GS	SELEN 3	Pseudo-spectral
Wouter van der Wal	WW		
Thomas S. James (*)	TJ	…	

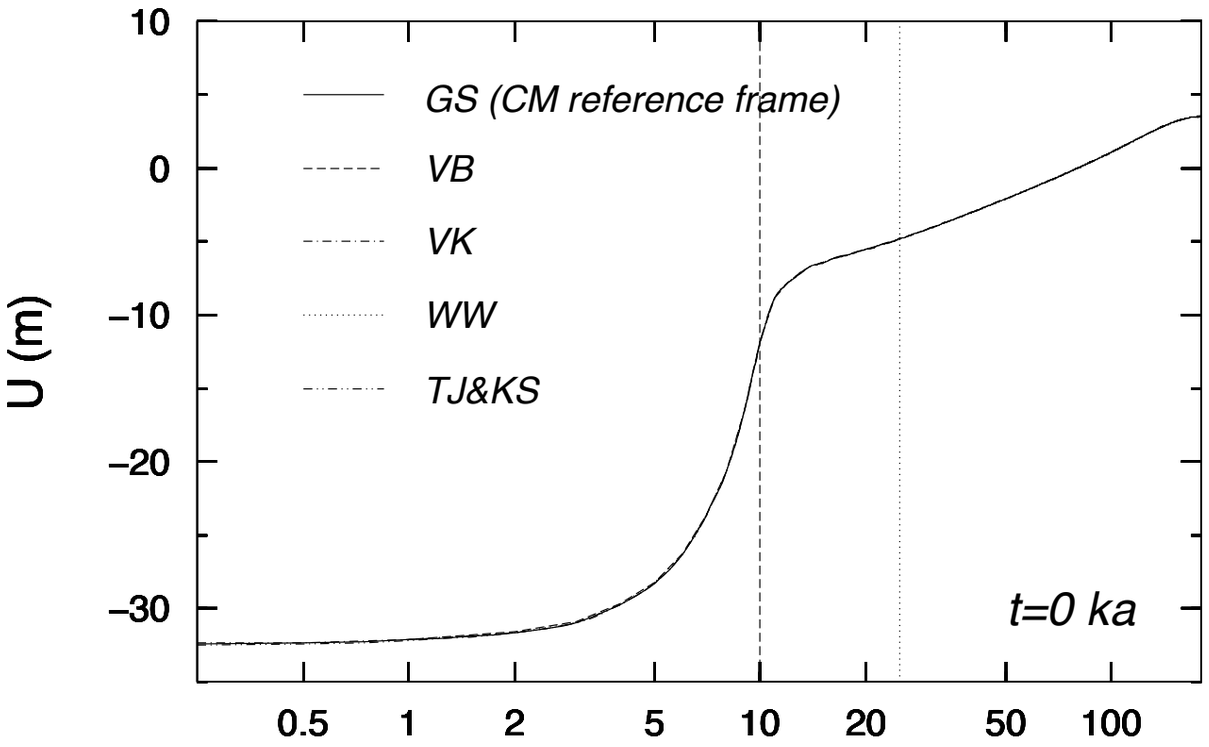
(*) not in WG4

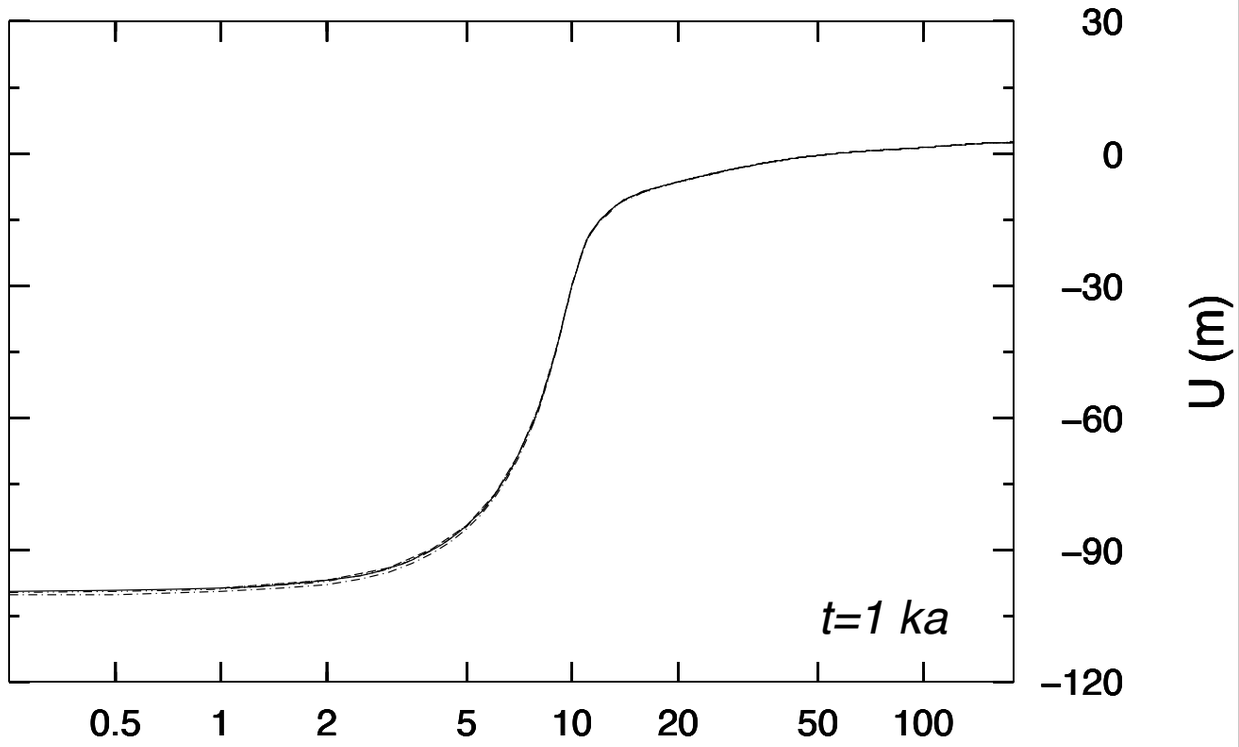
Sea level change (S)

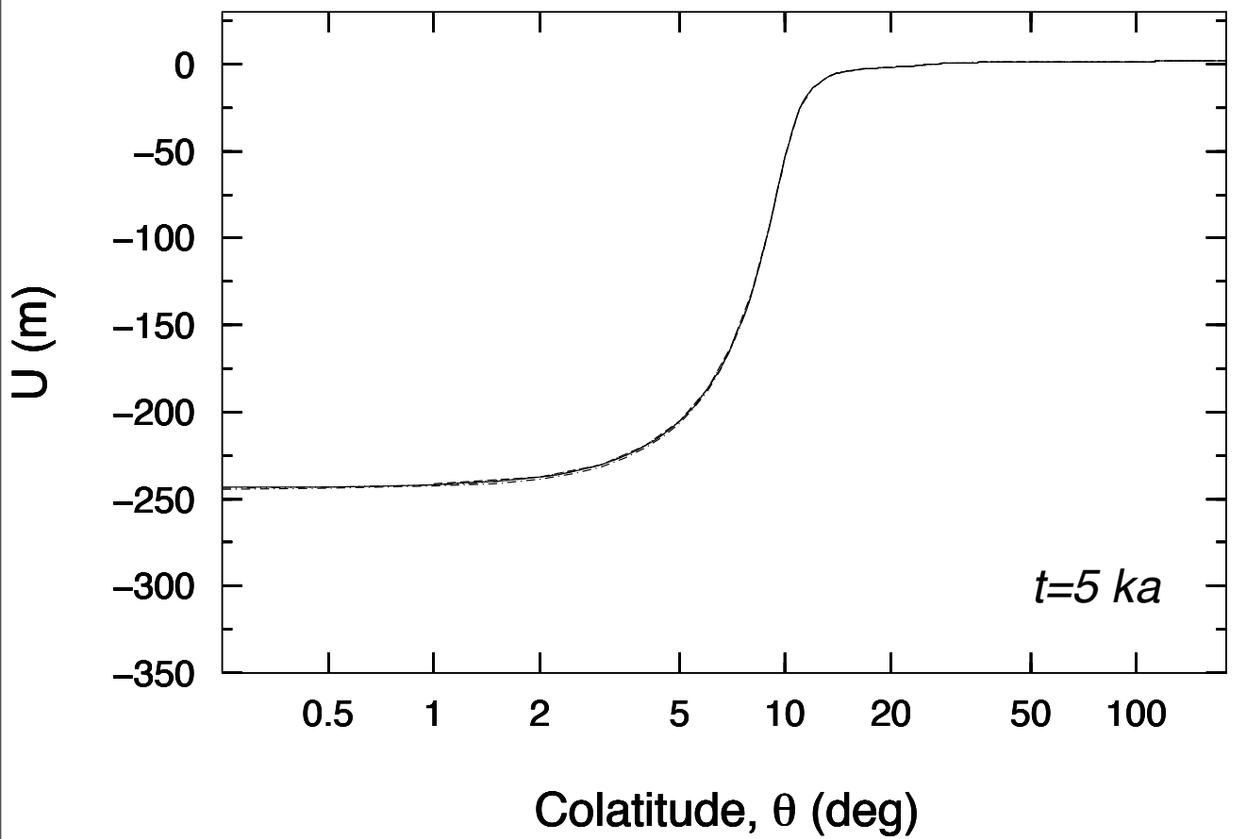


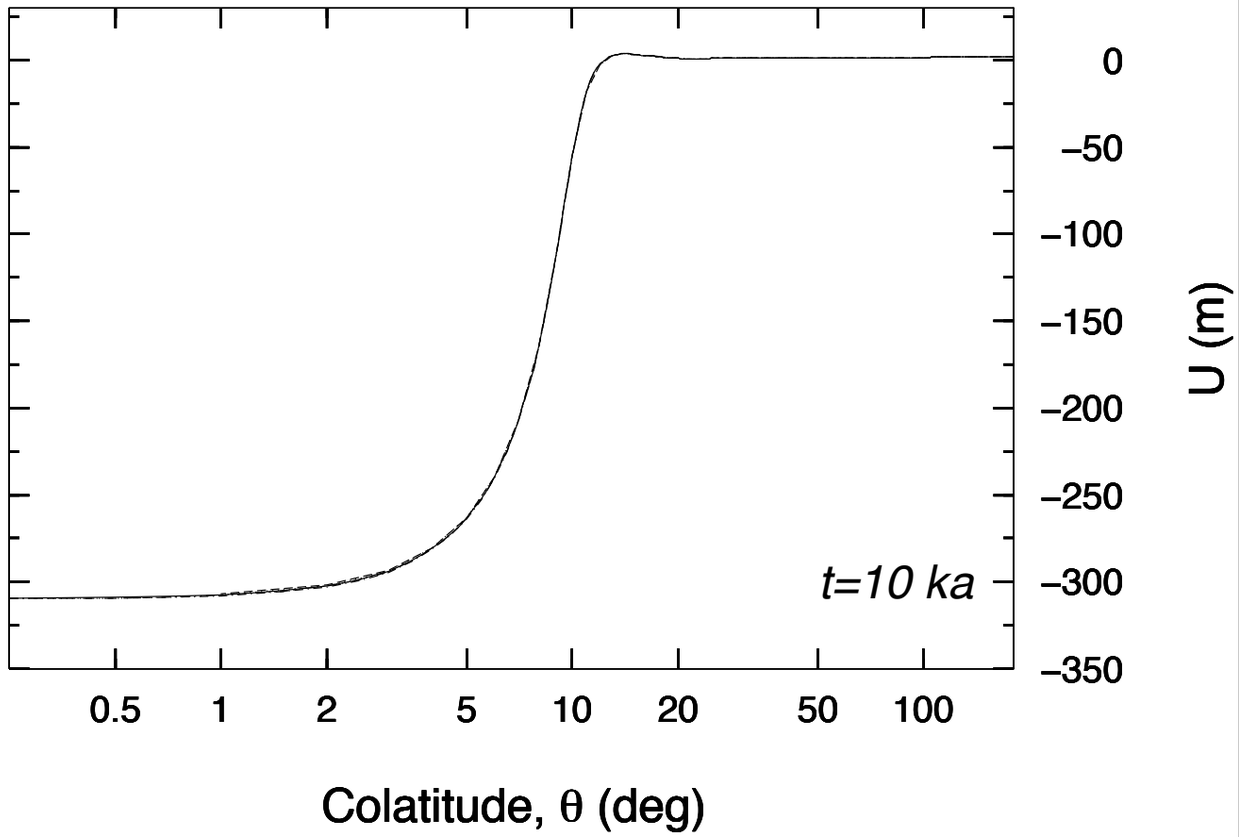
2011 Sep 20 19:24:03 **Compiled_by_GS_SEP2011**

Vertical displacement (U)



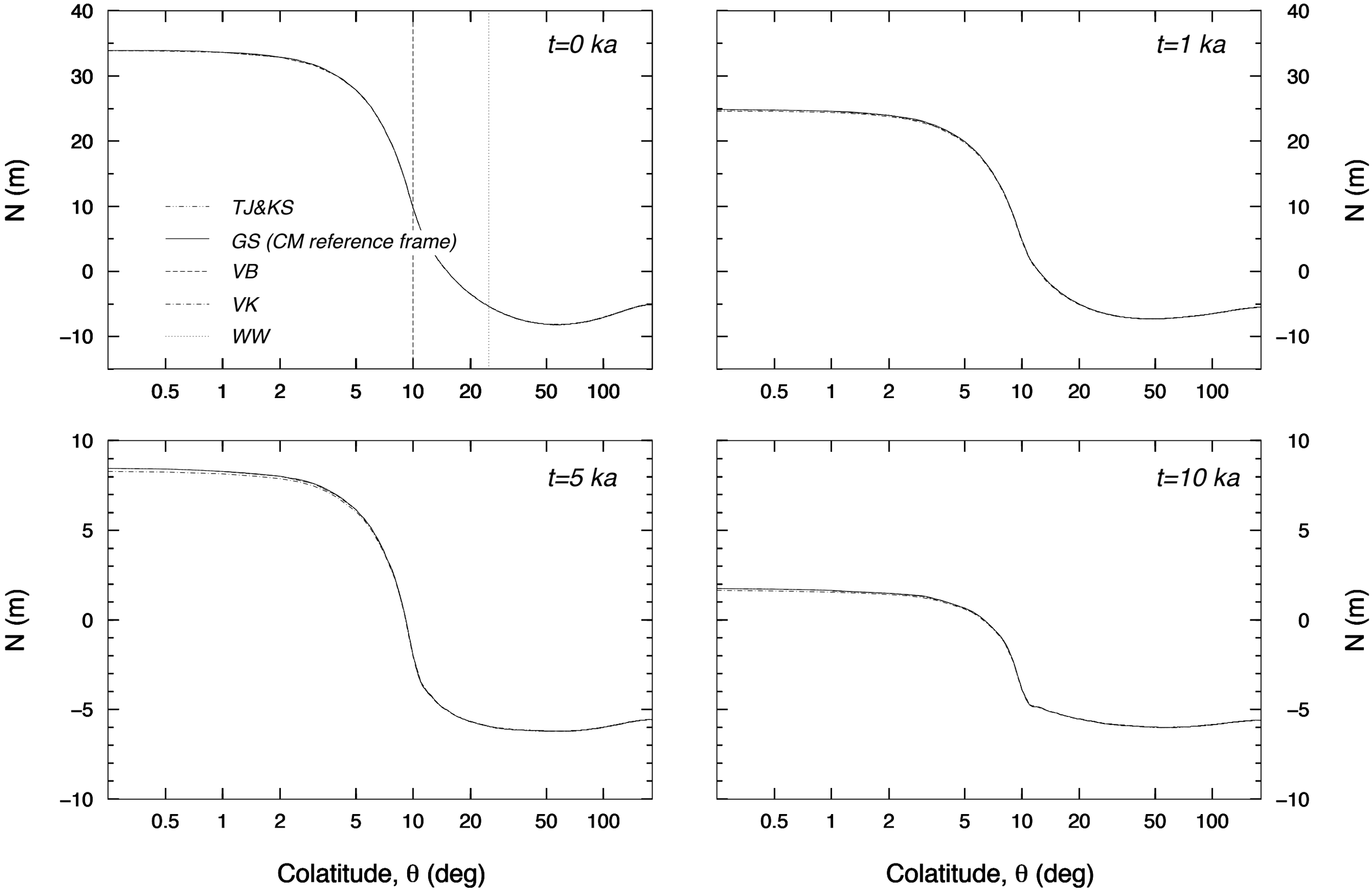






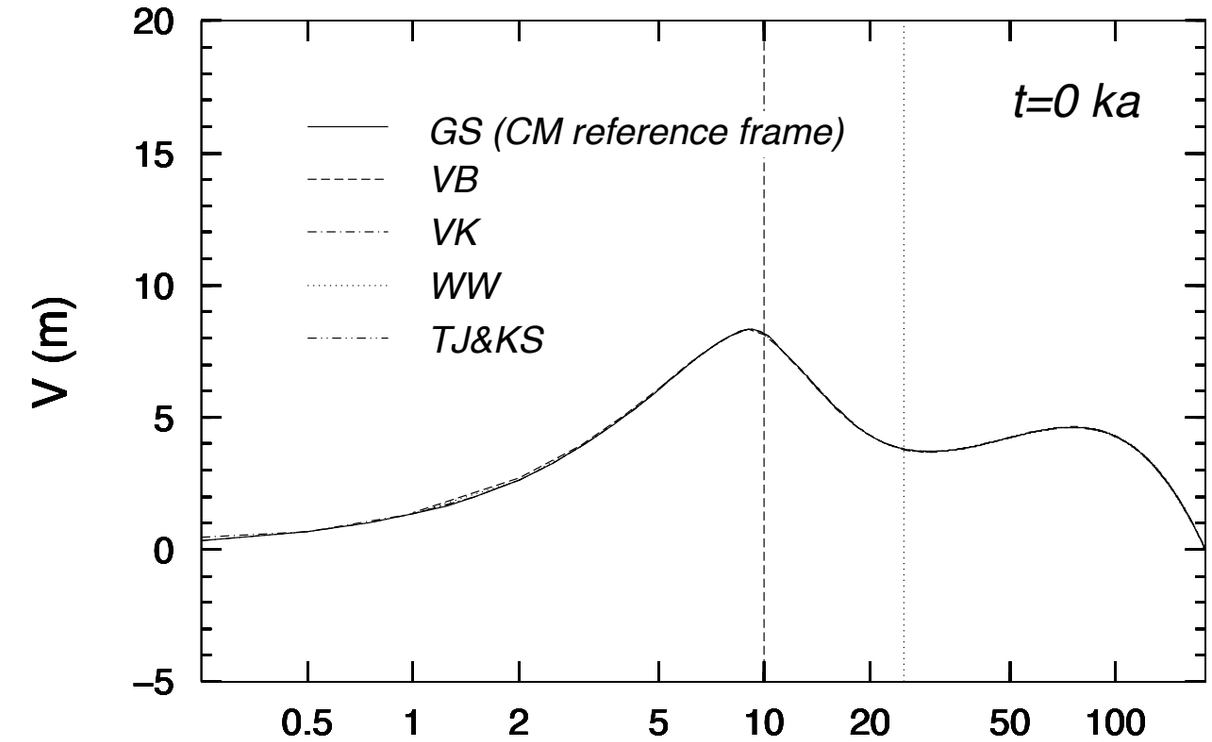
2011 Sep 20 19:17:39 **Compiled_by_GS_SEP2011**

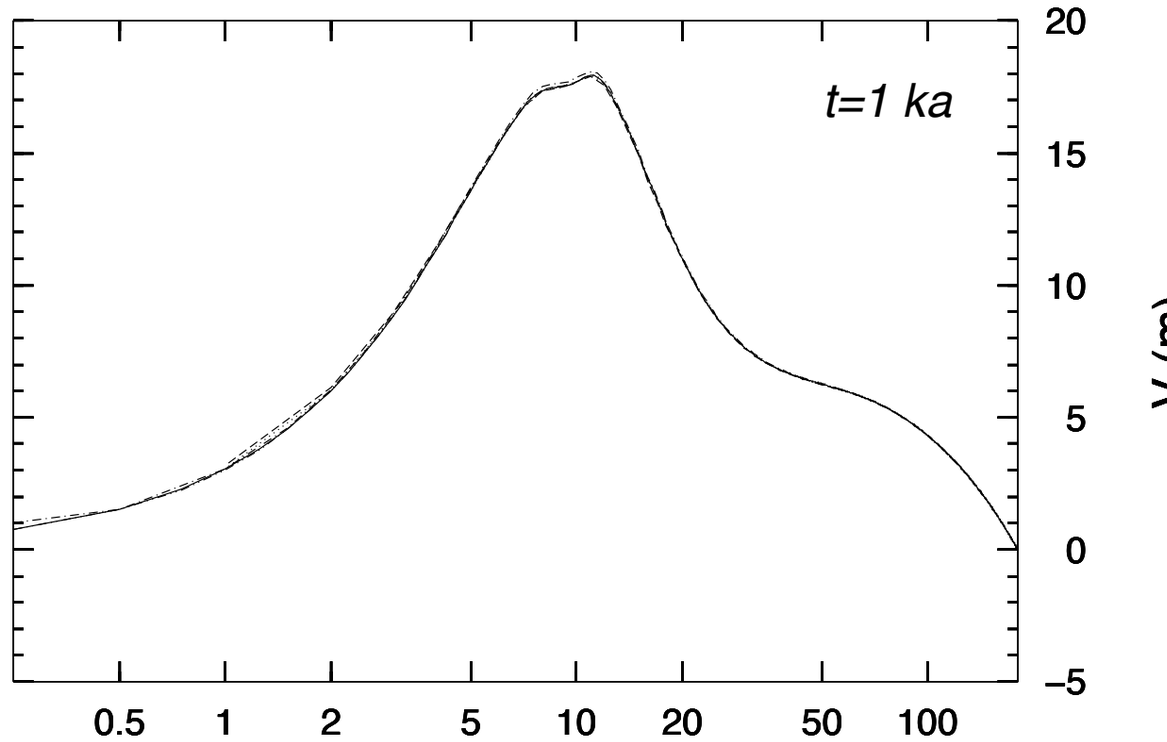
Sea surface variation (N)

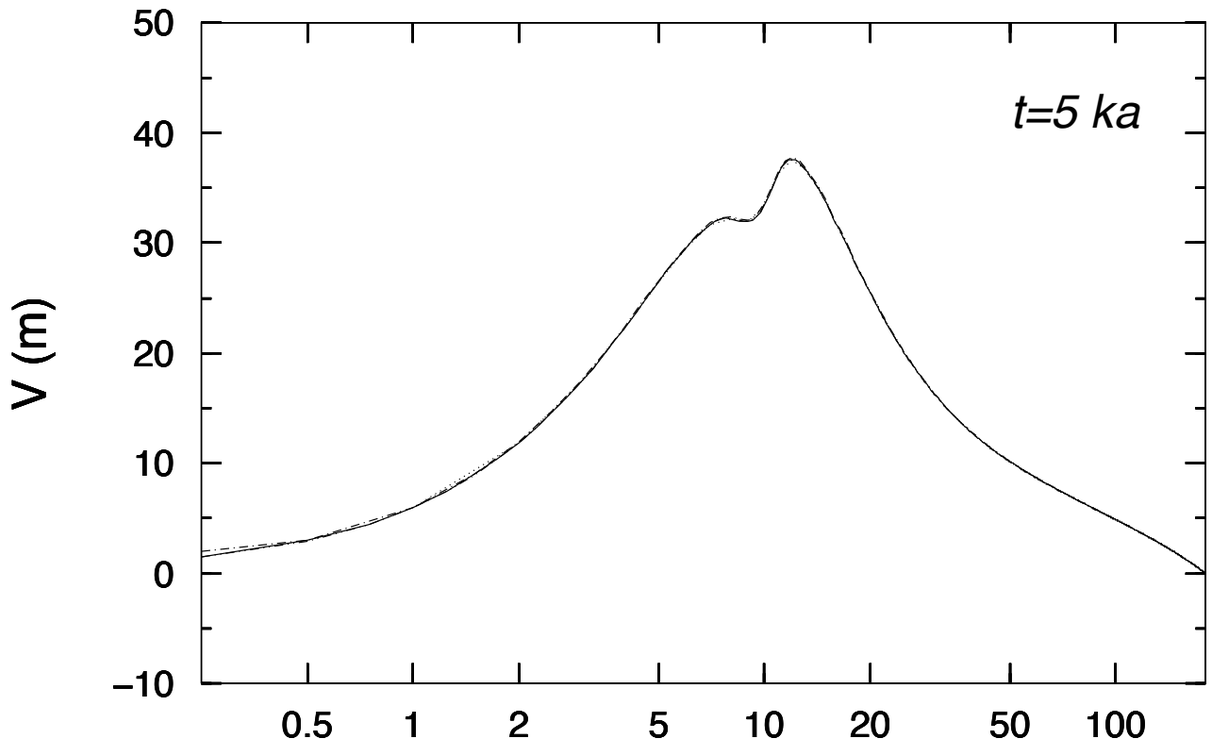


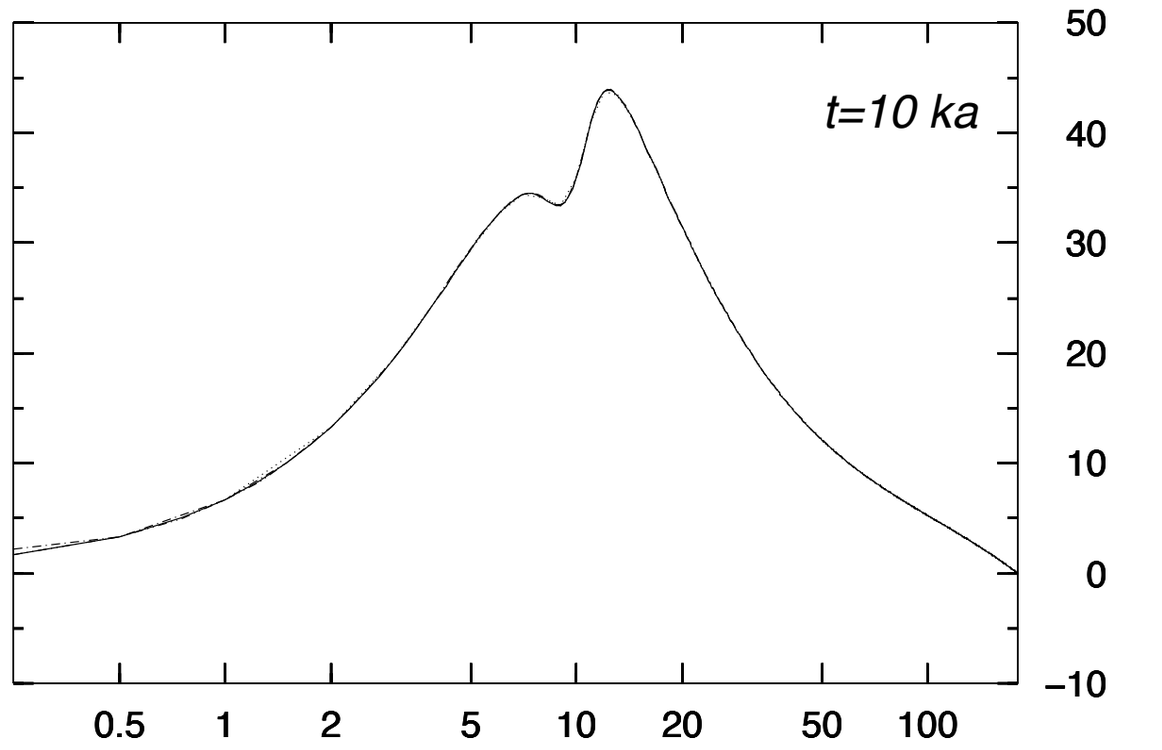
2011 Sep 20 19:25:22 **Compiled_by_GS_AUG2011**

Horizontal displacement (V)





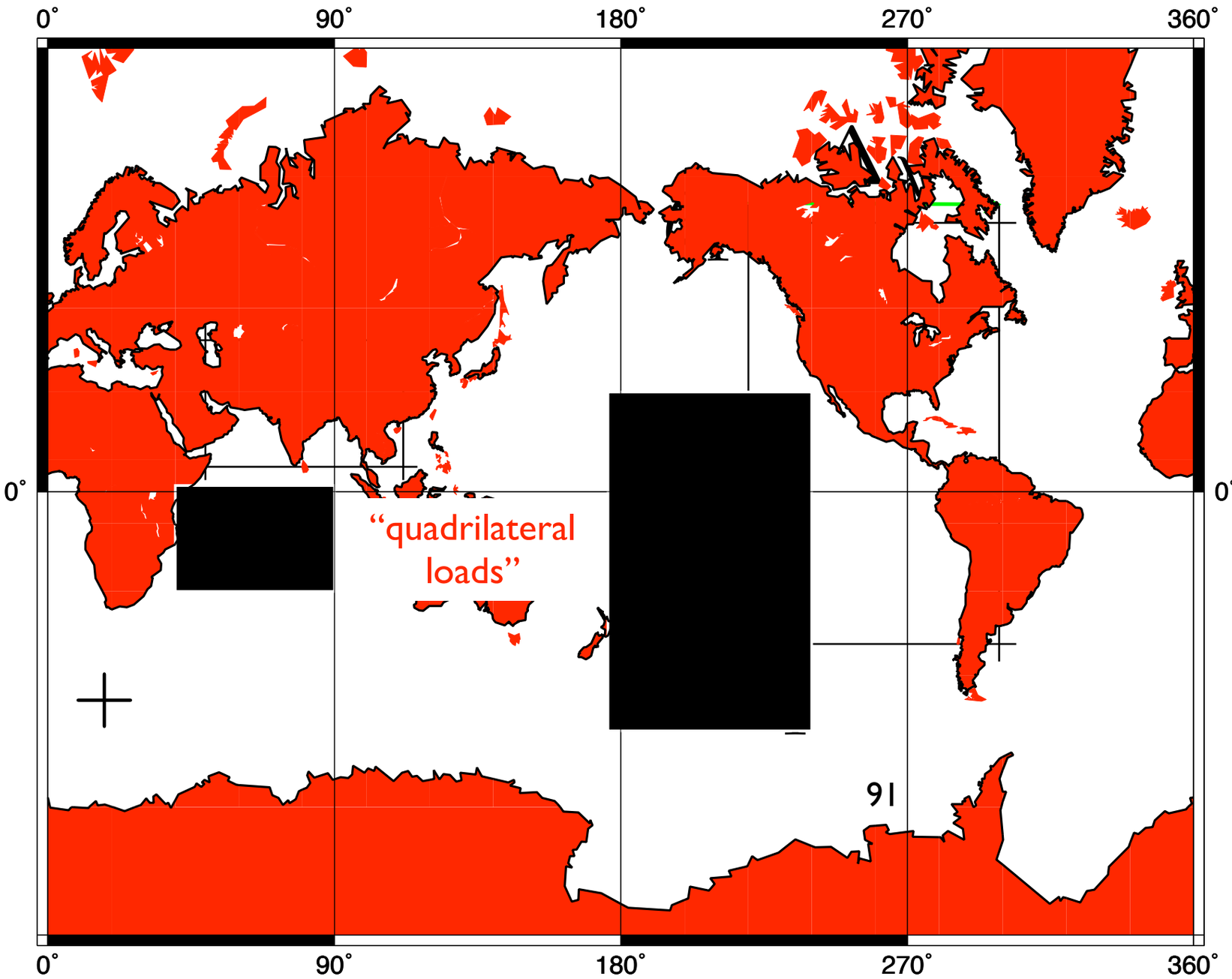


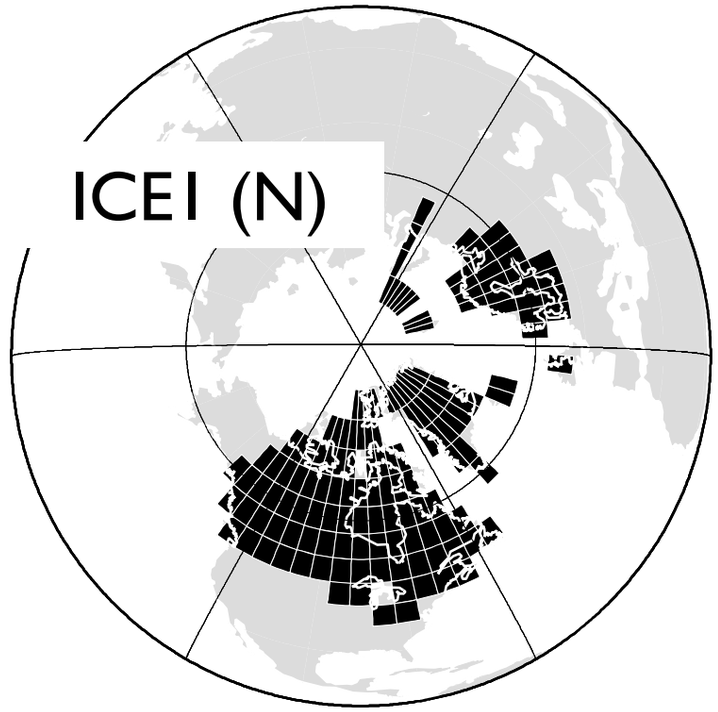


GMT 2011 Sep 20 19:21:20 **Compiled_by_GS_SEP2011**

work in progress

Test#1: non- axisymmetric loads





Lack of symmetry imposes numerical integration in latitude:

tricky integrals of (oscillating) Associated Legendre functions

Special topic (3/3):

present-day geodetic variations and the SLE

Is glacio-isostatic adjustment (GIA) affecting the Earth shape today?

Yes, in many respects:

✓present-day *sea level variations on a global scale and regional,*
✓*vertical movements and geoid height variations,*
✓*"local" sea level change at tide-gauges,*
✓*rate of change of harmonic components of geo-potential,*
✓*ground deformation (vertical + horizontal movements),*
✓*stress field variations in deglaciated areas...*
✓...

All these effects are referred to as *"Geodetic variations"*; they can be studied by the same fundamental equation used so far (the SLE).

We only consider some of these issues...

Solving the SLE we obtain the rate of present-day sea level change:

$$\dot{S}(\omega, t_p) = \left. \frac{dS(\omega, t)}{dt} \right|_{t=t_p}$$

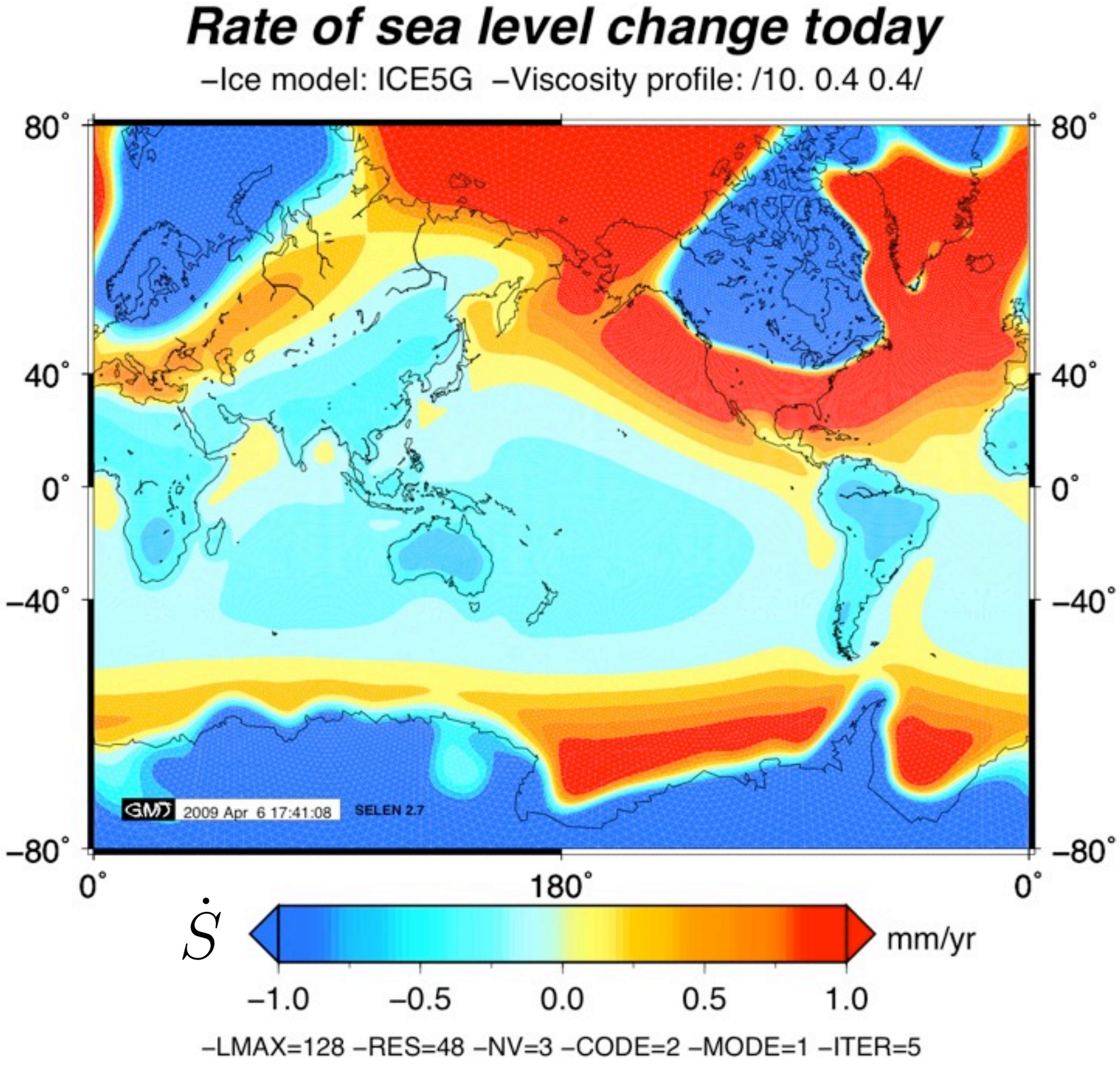
but also the rate of vertical displacement (*"vertical velocity"*)

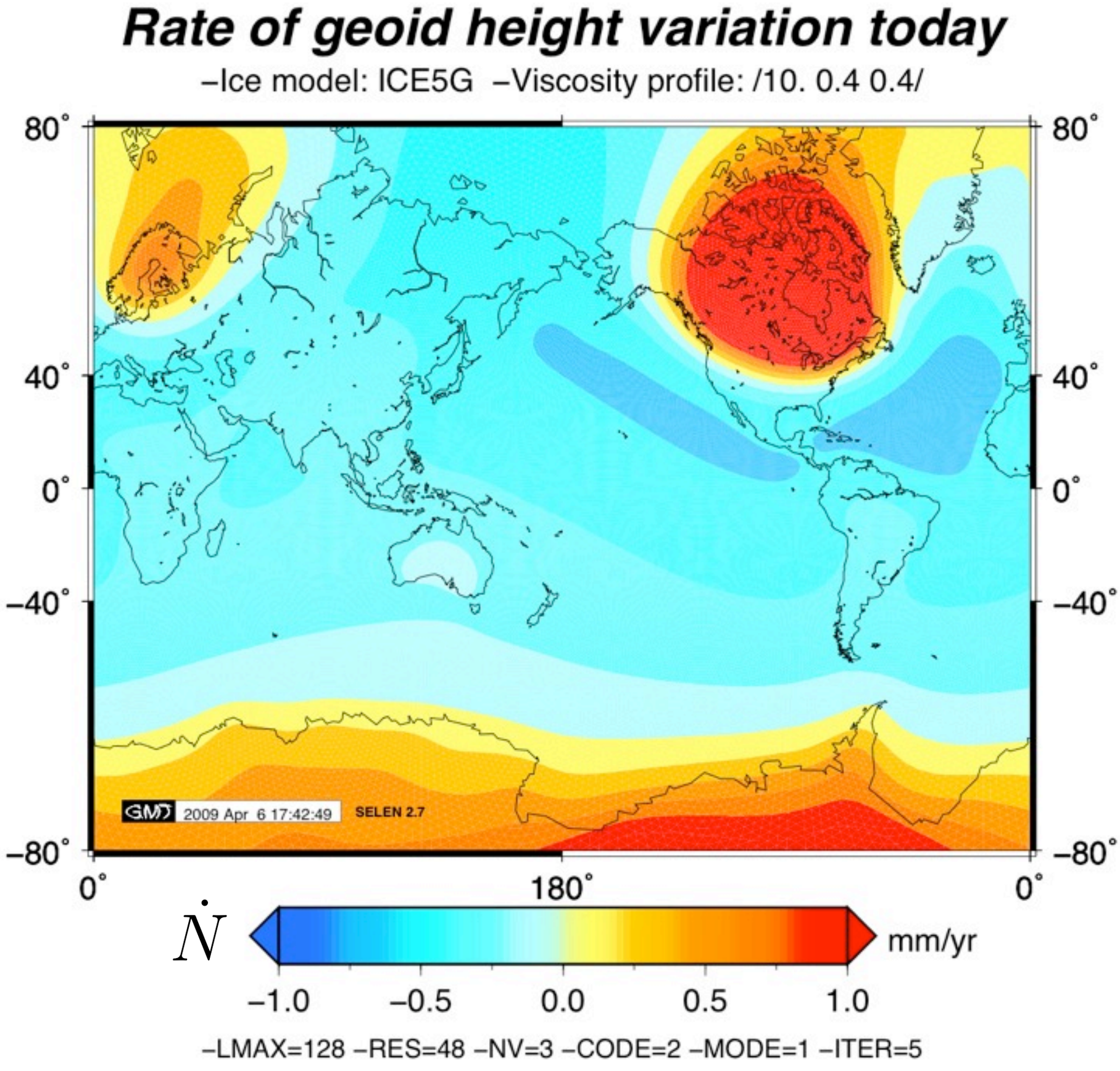
$$\dot{U}(\omega, t_p) = \left. \frac{dU(\omega, t)}{dt} \right|_{t=t_p}$$

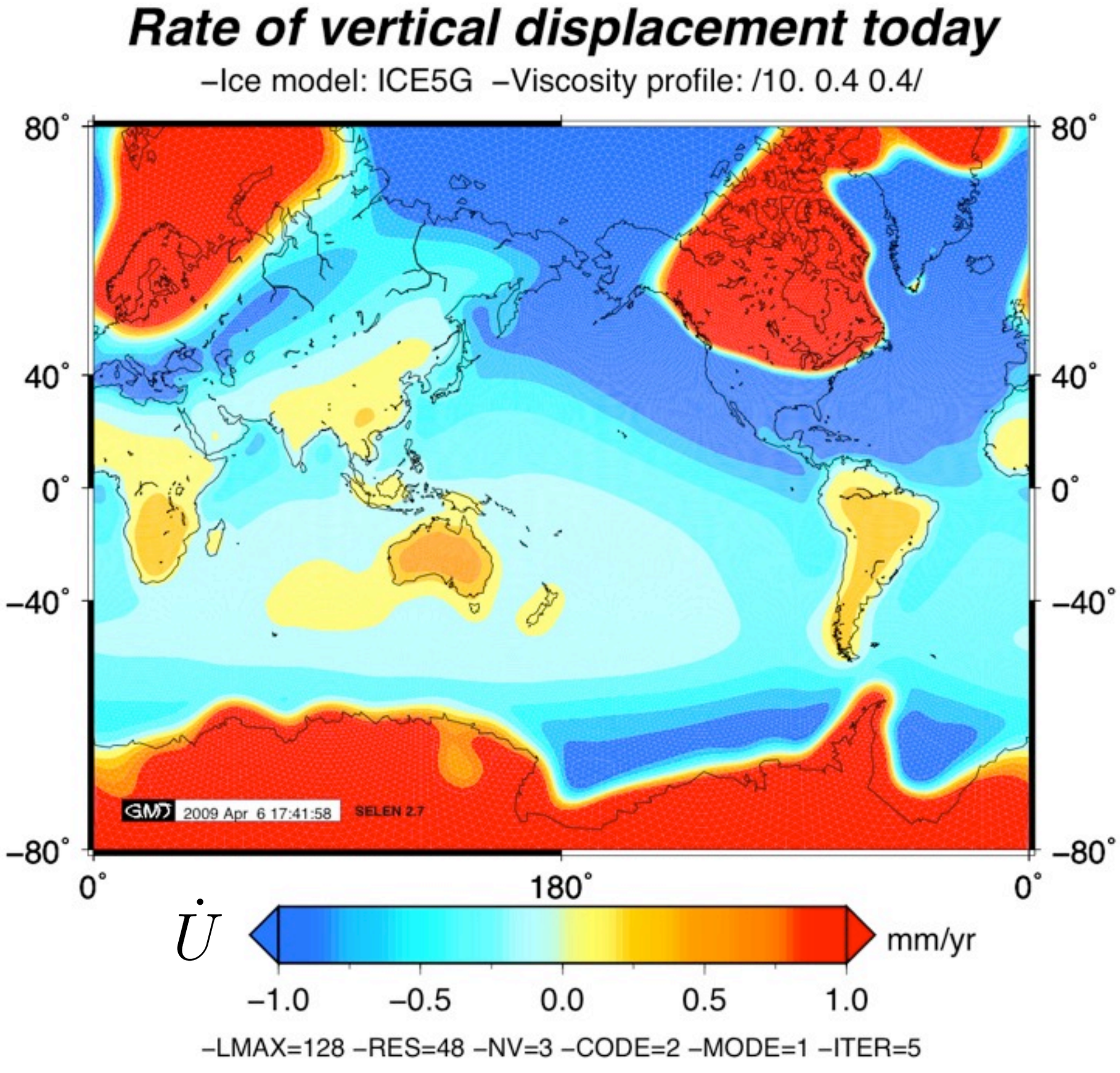
and the rate of sea surface height change:

$$\dot{N}(\omega, t_p) = \dot{U}(\omega, t_p) + \dot{S}(\omega, t_p)$$

These rates amount to several mm/yr, a *large* signal geodetically.

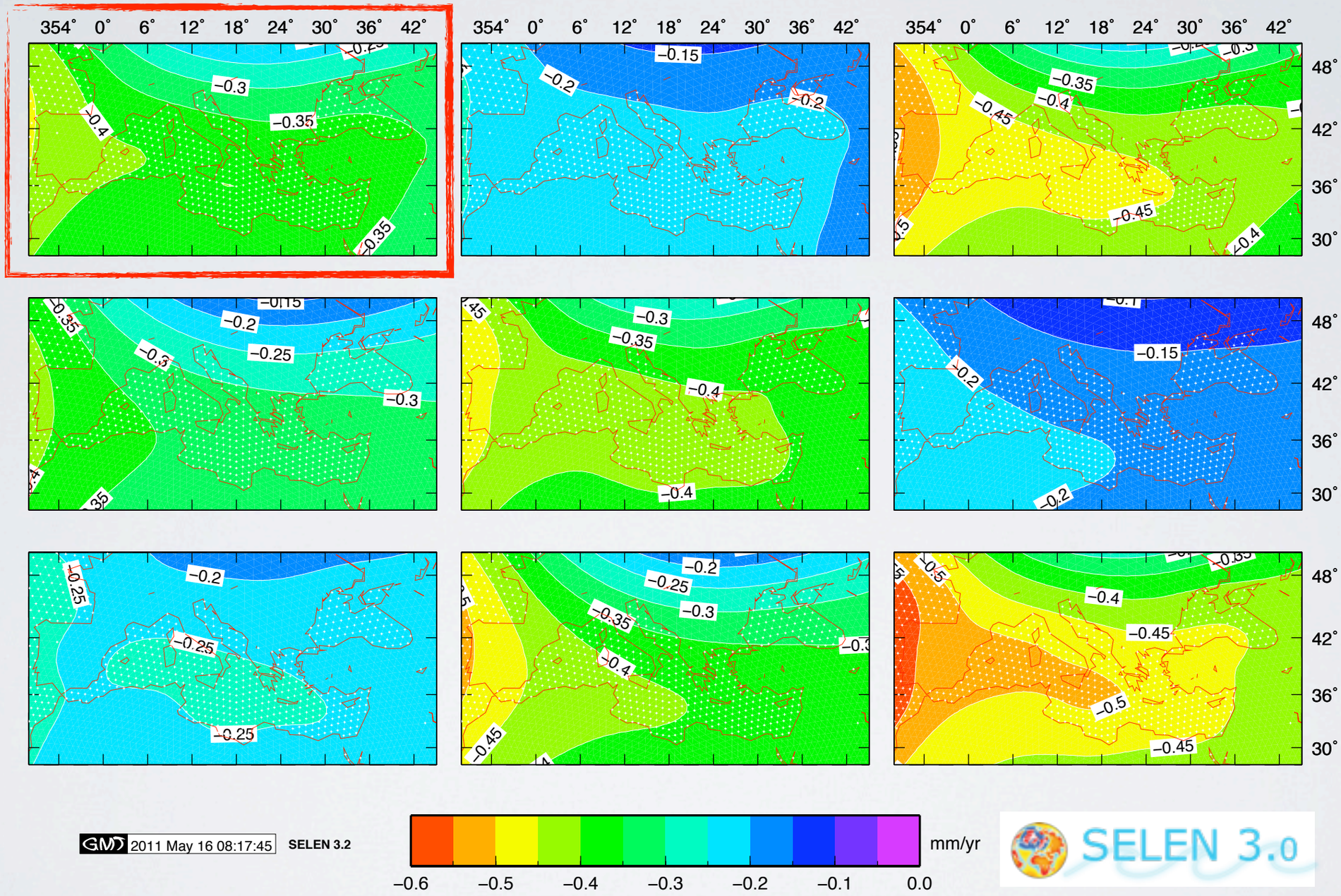

Rate of sea level change today
–Ice model: ICE5G –Viscosity profile: /10. 0.4 0.4/
80°
40°
0°
–40°
–80°
0°
180°
0°
GMT 2009 Apr 6 17:41:08
SELEN 2.7
$\dot{S}$
mm/yr
–1.0
–0.5
0.0
0.5
1.0
–LMAX=128 –RES=48 –NV=3 –CODE=2 –MODE=1 –ITER=5


Rate of geoid height variation today
–Ice model: ICE5G –Viscosity profile: /10. 0.4 0.4/
80°
40°
0°
–40°
–80°
0°
180°
0°
GMT 2009 Apr 6 17:42:49
SELEN 2.7
$\dot{N}$
mm/yr
–1.0
–0.5
0.0
0.5
1.0
–LMAX=128 –RES=48 –NV=3 –CODE=2 –MODE=1 –ITER=5


Rate of vertical displacement today
–Ice model: ICE5G –Viscosity profile: /10. 0.4 0.4/
80°
40°
0°
–40°
–80°
0°
180°
0°
GMT 2009 Apr 6 17:41:58
SELEN 2.7
$\dot{U}$
mm/yr
–1.0
–0.5
0.0
0.5
1.0
–LMAX=128 –RES=48 –NV=3 –CODE=2 –MODE=1 –ITER=5

Example #3: geoid subsidence in the MED

Rate of sea surface height variation today (MED)
GIA correction to altimetry data

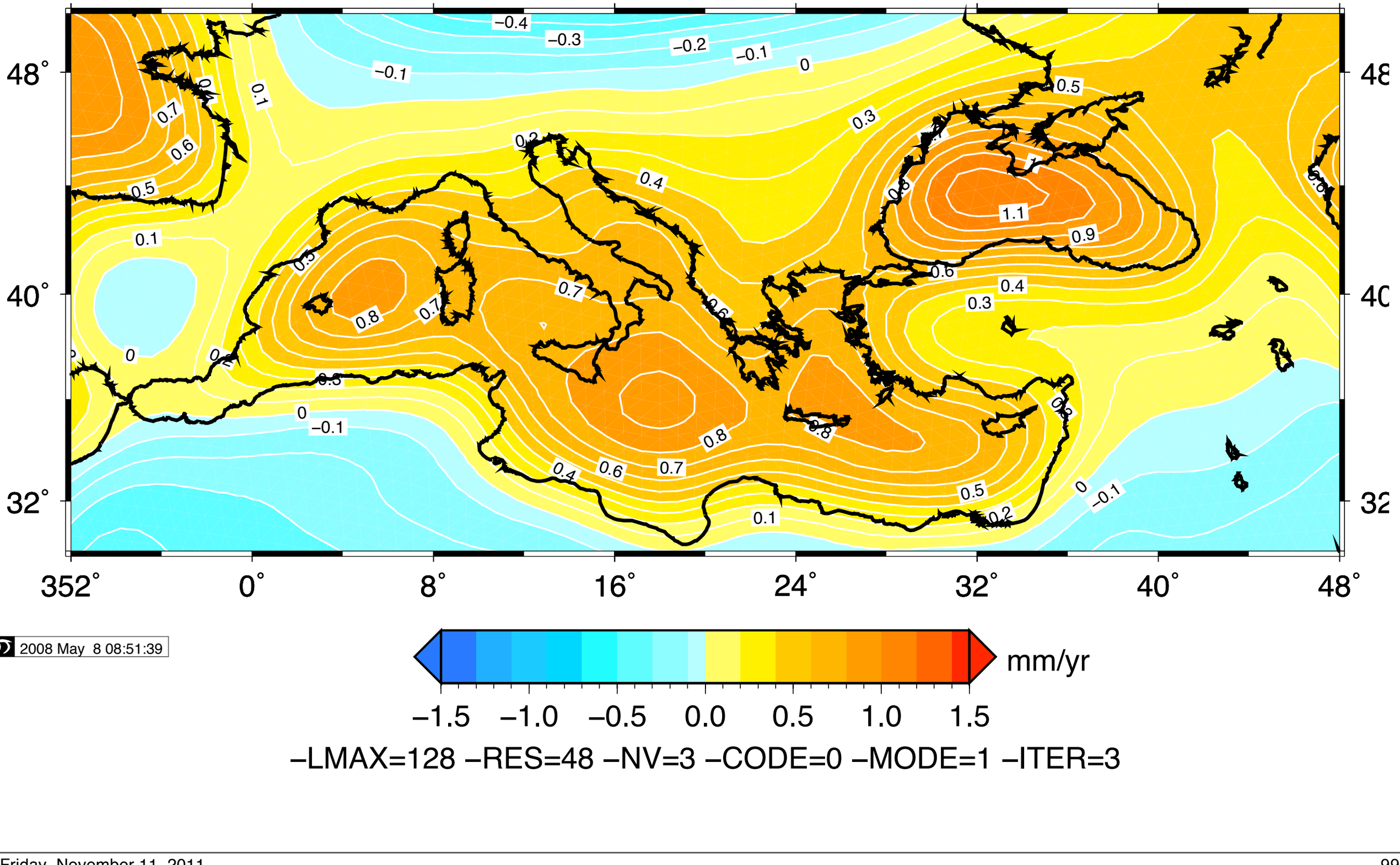


Nine “perturbed” viscosity profiles assuming ICE-5G (Peltier, 2004)

preferred rate: -0.30 +/- 0.10 mm/yr

Rate of sea level change today – MEDITERRANEAN

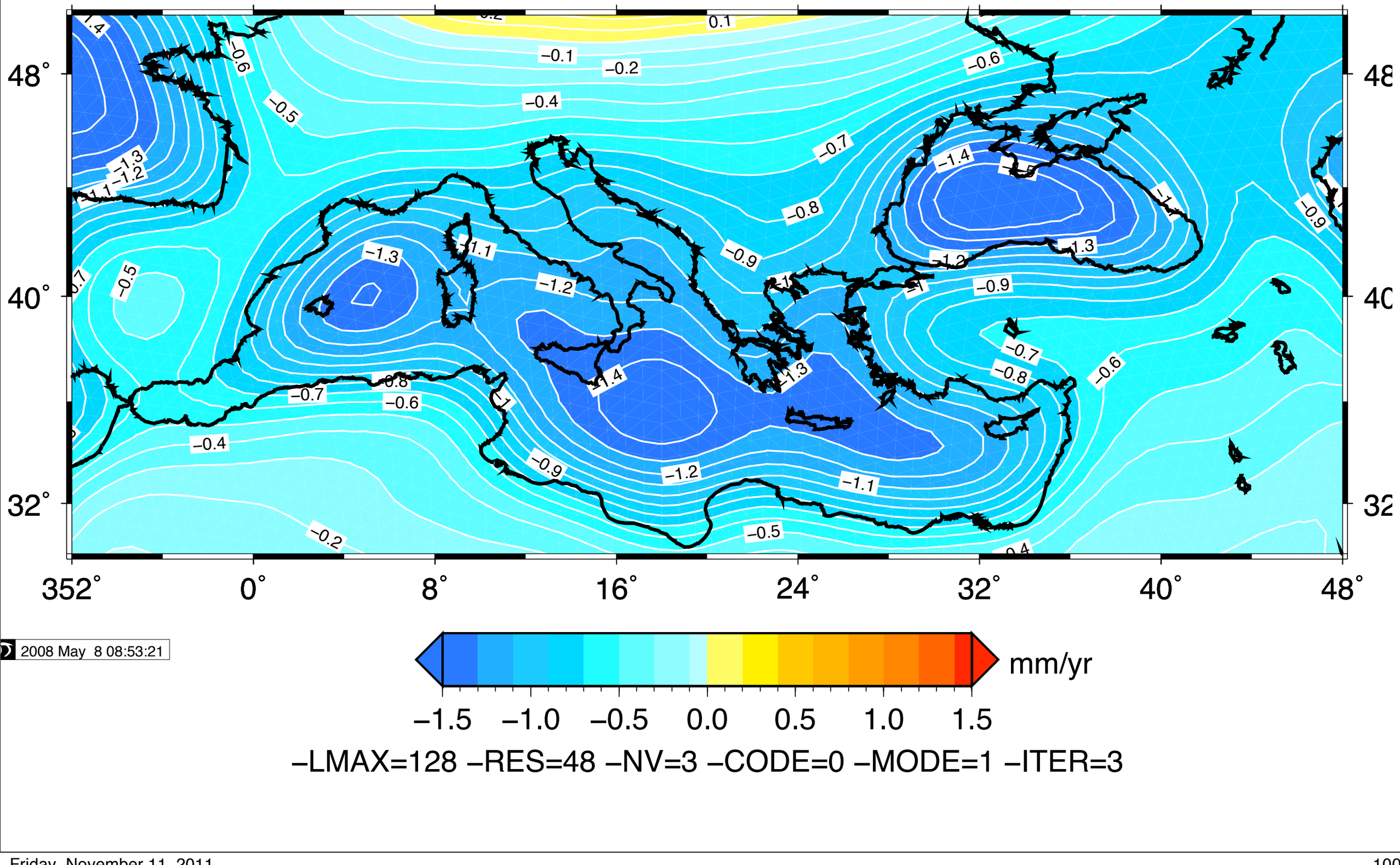
–Ice model: ICE5G –Viscosity profile: /10. 0.3 0.3/



–LMAX=128 –RES=48 –NV=3 –CODE=0 –MODE=1 –ITER=3

Rate of vertical uplift today – MEDITERRANEAN

–Ice model: ICE5G –Viscosity profile: /10. 0.3 0.3/



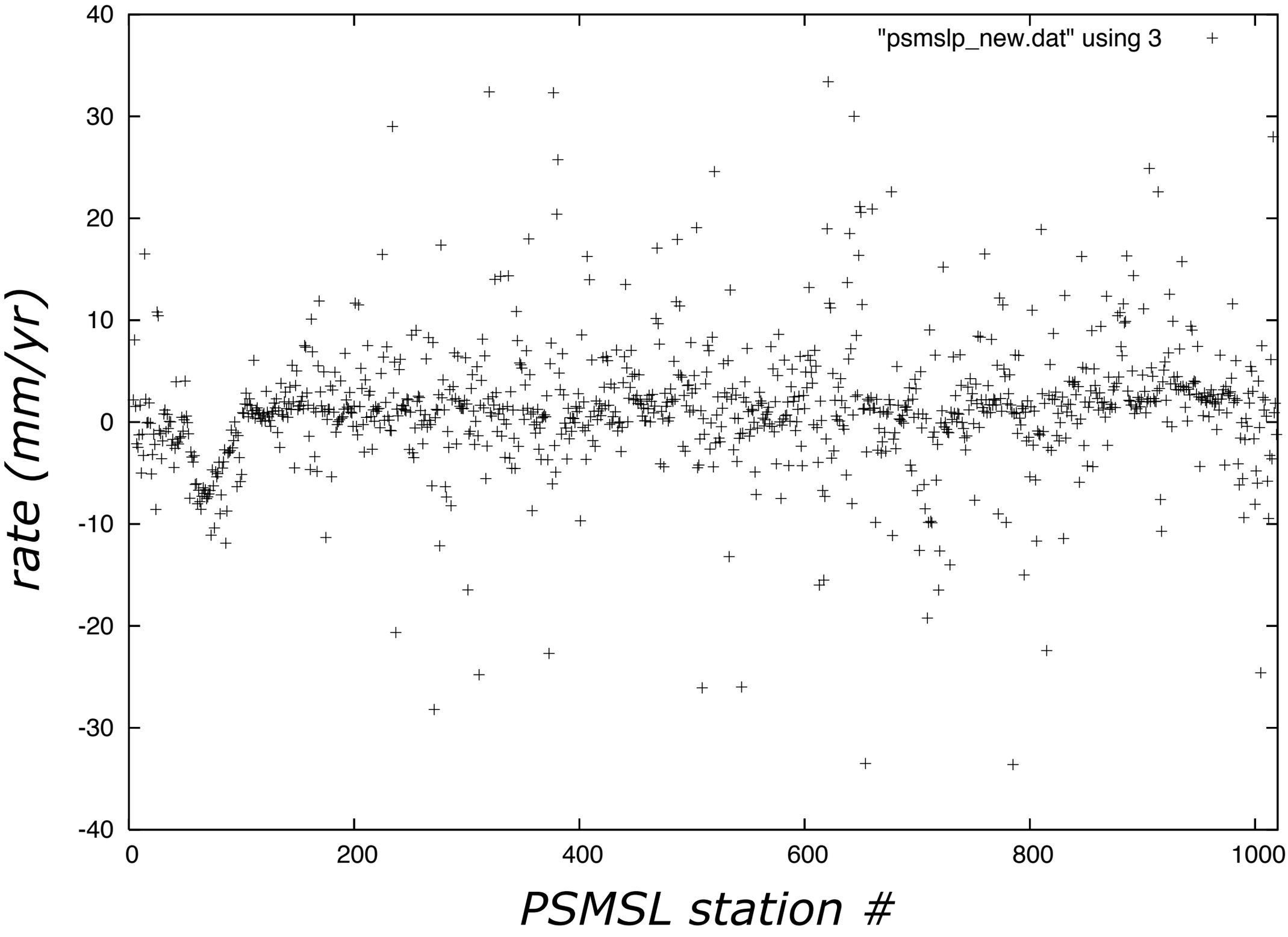
Role of GIA today

PSMSL tide-gauge observations

http://www.pol.ac.uk

PSMSL tide-gauge observations

is sea level "rising" or "falling" ?



Role of GIA today

PSMSL tide-gauge observations

```
2008.05.08 time=08.45.50
Ice model: ice5g.dat
Number of mantle layers:            3
Model code (see TABOO User guide):            0
Viscosity model: visco.dat
Thickness of the lithosphere (km): '65.'
Viscosity (bottom-to-top, Haskell units) '10.'
Viscosity (bottom-to-top, Haskell units) '0.3'
Viscosity (bottom-to-top, Haskell units) '0.3'
SLE iterations:            3
SLE mode of solution:            1
Maximum harmonic degree:          128
Tegmark resolution:           48
```

PSMSL code	yrs	range	trend mm/yr	error	lon deg	lat deg	s-dot mm/yr	n-dot mm/yr	u-dot mm/yr	PSMSL station name
010/001	40	1957 - 2003	2.23	0.46	338.067	64.150	0.697	-0.312	-1.008	REYKJAVIK
010/011	6	1958 - 1965	8.07	8.35	337.567	63.833	0.936	-0.325	-1.262	GRINDAVIK
015/011	22	1958 - 2001	1.50	0.34	353.233	62.014	0.761	-0.259	-1.020	TORSHAVN
025/001	44	1949 - 2003	-2.65	0.42	14.250	78.067	-2.260	0.133	2.393	BARENTSBURG
025/002	42	1949 - 1993	-1.36	0.46	14.250	78.067	-2.260	0.133	2.393	BARENTSBURG II (SPITSBERGEN)
025/021	20	1977 - 2004	-3.10	0.64	11.933	78.933	-1.197	0.051	1.248	NY-ALESUND
030/001	36	1953 - 1989	-1.26	0.62	62.583	76.200	-3.979	0.317	4.295	RUSSKAYA GAVAN
030/003	36	1953 - 1989	-1.28	0.62	62.583	76.183	-3.966	0.315	4.281	RUSSKAIA GAVAN II
030/007	19	1960 - 1978	2.43	2.52	60.217	69.600	-1.540	-0.021	1.519	BELYI NOS
030/014	28	1962 - 1990	-4.89	1.07	58.050	80.617	-3.273	0.223	3.496	KRENKELIA (HEISA OSTROV)
030/016	23	1950 - 1976	-3.10	1.91	52.700	72.367	-4.345	0.286	4.632	MALYE KARMAKULY
030/018	48	1952 - 2005	3.14	0.67	33.050	68.967	-5.597	0.463	6.060	MURMANSK
030/019	41	1952 - 1993	0.69	0.76	33.050	68.967	-5.597	0.463	6.060	MURMANSK II
030/025	8	1931 - 1938	-5.00	4.88	31.367	69.650	-5.489	0.481	5.970	LINAKHAMARI
030/305	42	1951 - 1992	2.72	0.64	59.083	70.450	-2.058	0.018	2.075	BOLVANSKII NOS (FEDOROVA)
030/308	40	1950 - 1989	0.59	0.57	60.750	69.817	-1.523	-0.026	1.497	UGORSKII SHAR (UGORSKII SHAR P
030/310	45	1950 - 2000	2.07	0.55	61.700	69.750	-1.348	-0.042	1.306	AMDERMA
030/315	42	1950 - 2000	1.16	0.48	64.517	69.250	-0.874	-0.092	0.782	UST KARA
030/318	37	1954 - 1990	2.48	1.02	67.583	71.417	-0.830	-0.104	0.726	MORZHOVAIA (HARASAVEI MYS)

• • • • • •

The rate of GLOBAL SEA LEVEL RISE is estimated through the analysis of "LONG" series of TG data!

Current estimates are collected in this table.

Table 3.1

Recent Determinations of Global Sea Level Rise from Tide Gauge Data

Author	Estimate (mm/yr)	Comments
Peltier and Tushingham (1989, 1991)	2.4 ± 0.9[a]	Global data
Barnett (1990)	1–2	Global data
Nakiboglu and Lambeck (1990)	1.15 ± 0.38	Global data
Trupin and Wahr (1990)	1.75 ± 0.13	Global data
Douglas (1991)	1.8 ± 0.1	Global data
Shennan and Woodworth (1992)	1.0 ± 0.15	U.K. and Europe
Mitrovica and Davis (1995)	1.1–1.6	Global data
Davis and Mitrovica (1996)	1.5 ± 0.3	U.S. east coast
Peltier (1996)	1.94 ± 0.6[a]	U.S. east coast
Peltier and Jiang (1997)	1.8 ± 0.6[a]	U.S. east coast
Douglas (1997)	1.8 ± 0.1	Global data

[a] Standard deviation of trends about their mean. The formal SE is a few tenths.

2: Stokes coefficients variations associated with GIA

By simple arguments (unpublished manuscript), we have obtained the Stokes coefficients variations (GRACE conventions)

$$\overline{c}^{G}_{lm} = +(-1)^m \sqrt{2-\delta_{0m}} \frac{Re(N_{lm})}{a}$$

$$\overline{s}^{G}_{lm} = -(-1)^m \sqrt{2-\delta_{0m}} \frac{Im(N_{lm})}{a},$$

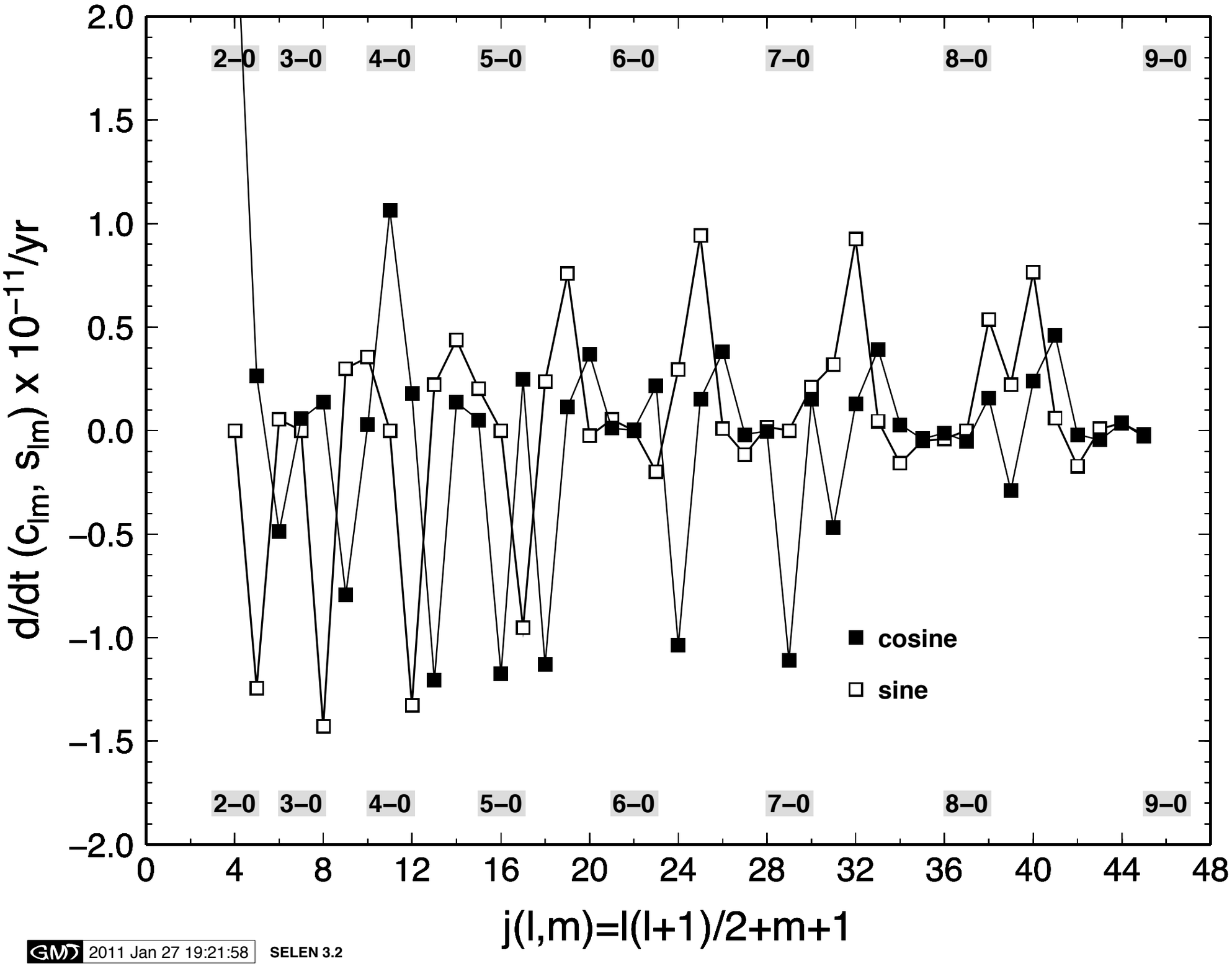
with:

N_{lm} Harmonics of the sea surface elevation change,

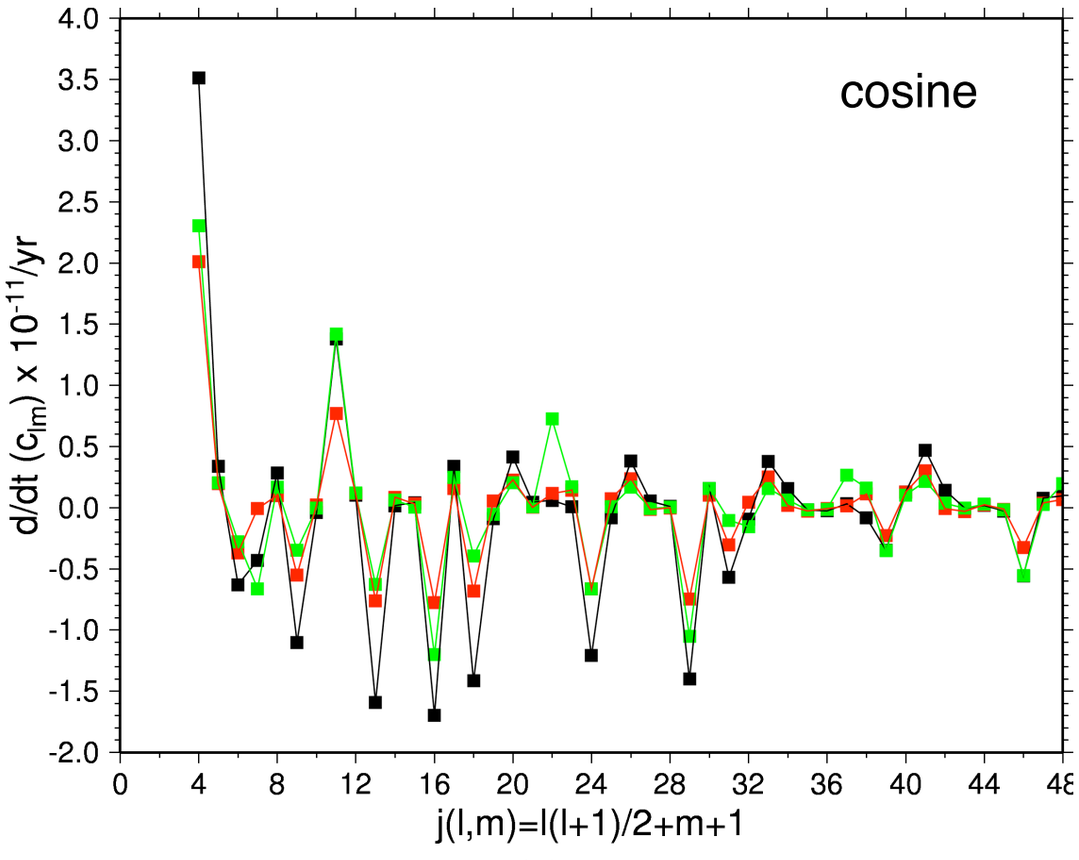
a Reference radius of the Earth.

Rate of change of the fully–normalized Stokes coefficients

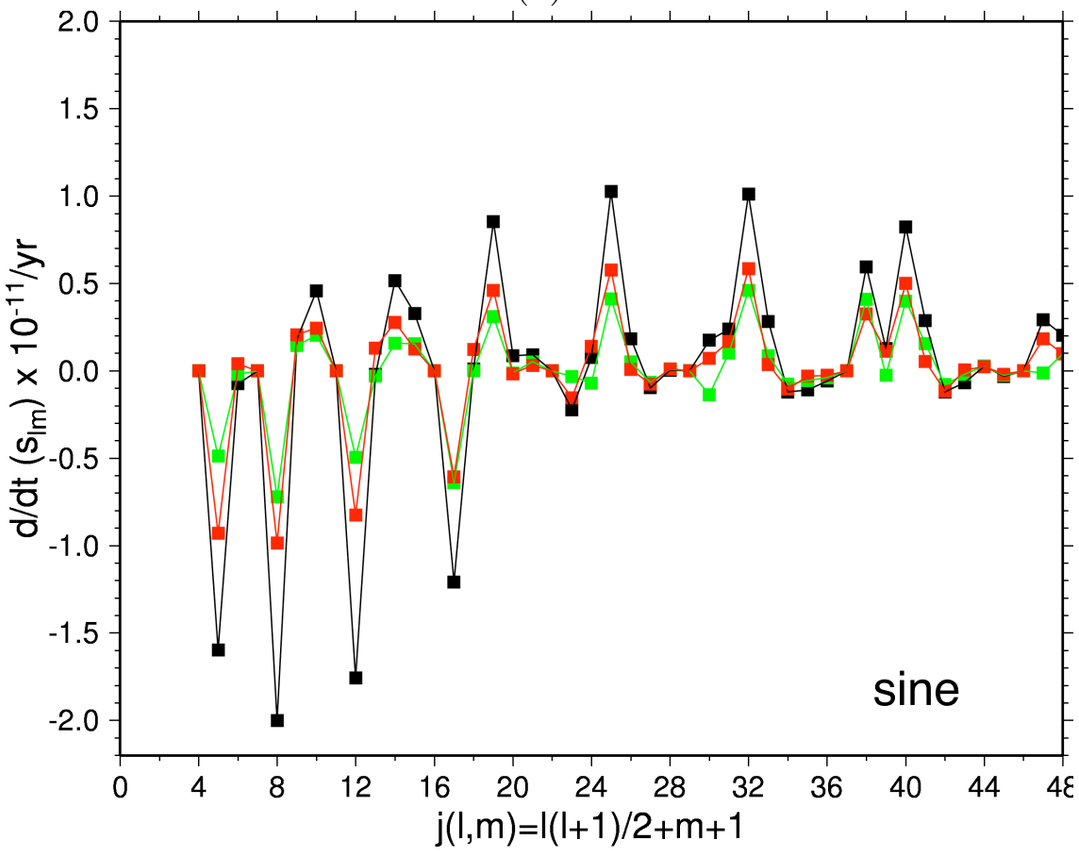
–Ice model: ICE5G –Viscosity profile: /4.0 0.4 0.4/ –LMAX=128 –RES=44 –NV=3 –CODE=2 –MODE=1 –ITER=3



GMT 2011 Jan 27 19:21:58 **SELEN 3.2**



(a)



(b)

COST ACTION ES0701
improved constraints on models of GIA

from the Ph.D. thesis of

Sofie Louise Sandberg Sorensen

"Changes of the Greenland ice sheet - derived from ICESat and GRACE data"

Figure 6.5:
Present-day rate of change of the fully normalized Stokes coefficients.

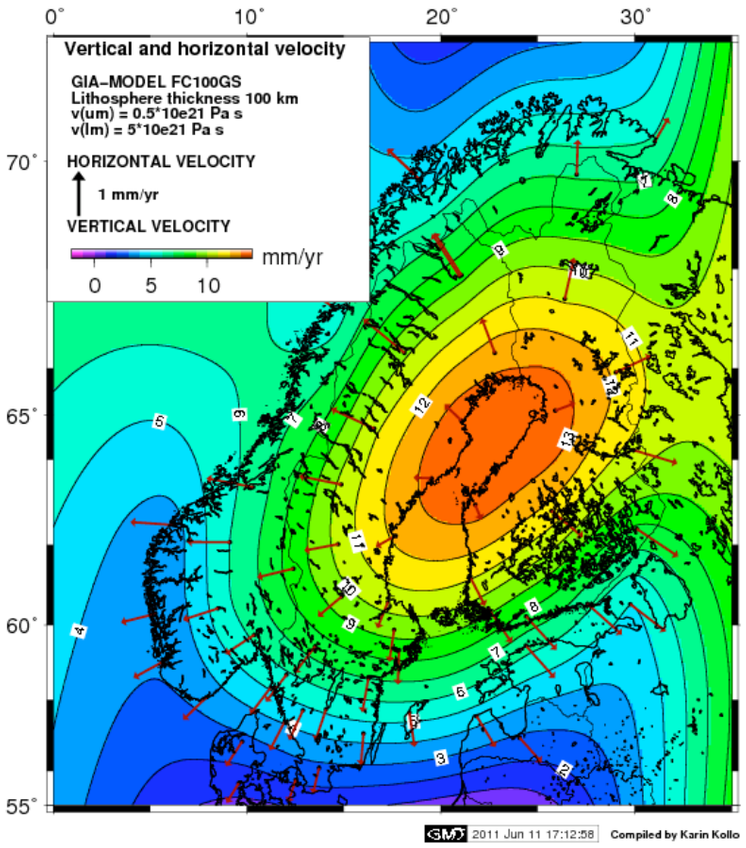
(a) Rate of change of the Clm coefficients,

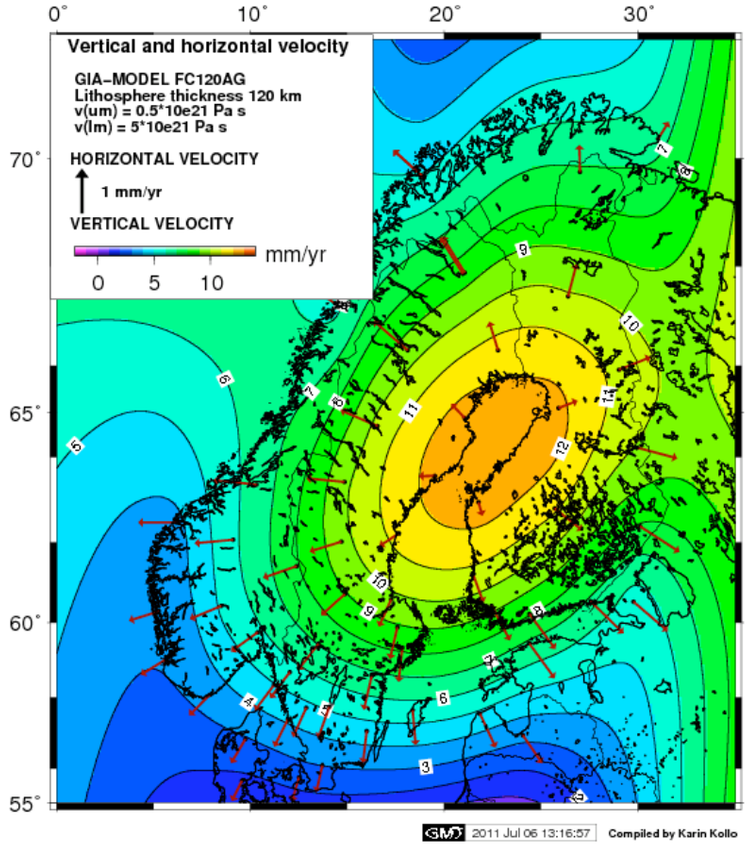
(b) Rate of change of the Slm coefficients.

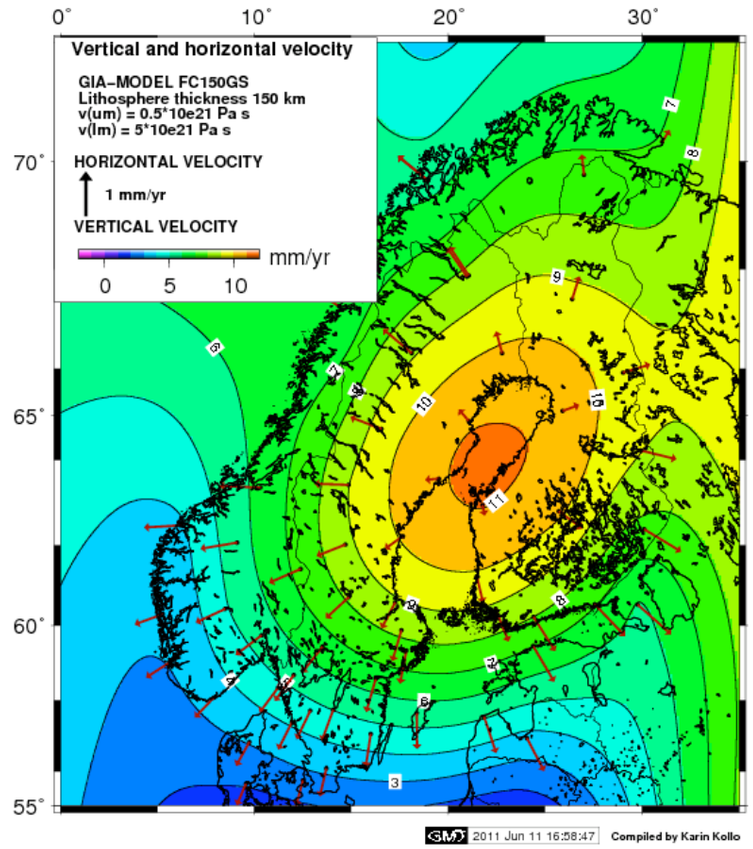
Red: ICE-5G(VM2),

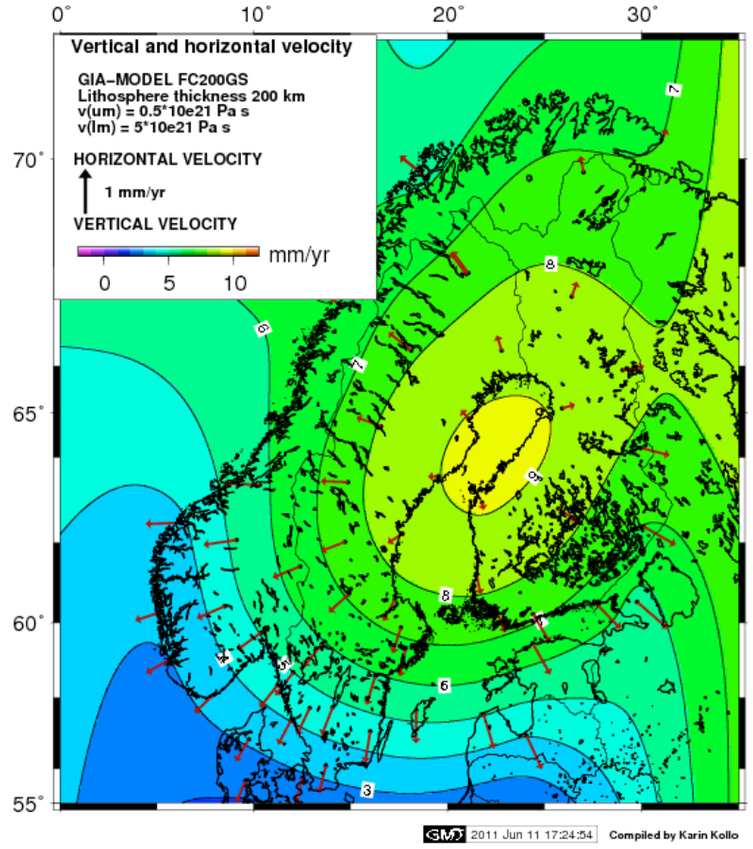
Green: ICE-3G(TP),

Black: ANU05(KL).


Vertical and horizontal velocity
GIA–MODEL FC100GS
Lithosphere thickness 100 km
v(um) = 0.5*10e21 Pa s
v(lm) = 5*10e21 Pa s
HORIZONTAL VELOCITY
1 mm/yr
VERTICAL VELOCITY
mm/yr
0 5 10
0° 10° 20° 30°
70° 65° 60° 55°
GMT 2011 Jun 11 17:12:58 Compiled by Karin Kollo


Vertical and horizontal velocity
GIA–MODEL FC120AG
Lithosphere thickness 120 km
v(um) = 0.5*10e21 Pa s
v(lm) = 5*10e21 Pa s
HORIZONTAL VELOCITY
1 mm/yr
VERTICAL VELOCITY
mm/yr
0 5 10
0° 10° 20° 30°
70° 65° 60° 55°
GMT 2011 Jul 06 13:16:57 Compiled by Karin Kollo


Vertical and horizontal velocity
GIA–MODEL FC150GS
Lithosphere thickness 150 km
v(um) = 0.5*10e21 Pa s
v(lm) = 5*10e21 Pa s
HORIZONTAL VELOCITY
1 mm/yr
VERTICAL VELOCITY
mm/yr
0 5 10
0° 10° 20° 30°
70° 65° 60° 55°
GMT 2011 Jun 11 16:58:47 Compiled by Karin Kollo


Vertical and horizontal velocity
GIA–MODEL FC200GS
Lithosphere thickness 200 km
v(um) = 0.5*10e21 Pa s
v(lm) = 5*10e21 Pa s
HORIZONTAL VELOCITY
1 mm/yr
VERTICAL VELOCITY
mm/yr
0 5 10
0° 10° 20° 30°
70° 65° 60° 55°
GMT 2011 Jun 11 17:24:54 Compiled by Karin Kollo