

Electromagnetic Induction in the Earth's Oceans II

Going big!

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Outline

- Introduction
- Parallel modelling in the time domain
- Seafloor cable voltages
- Conclusions
- Outlook

Introduction

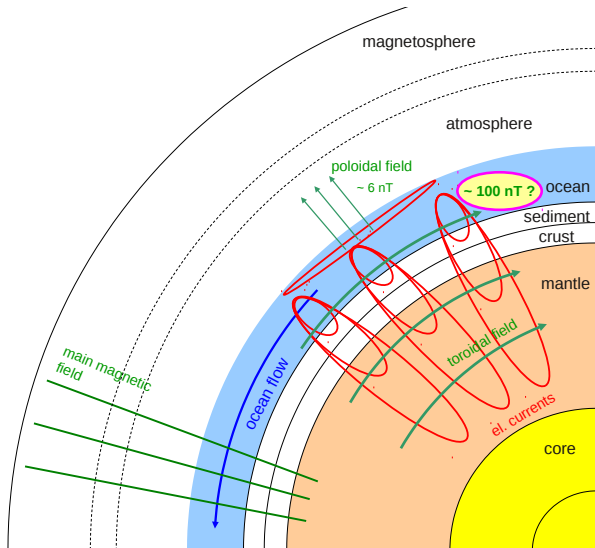
Motivation

- ▶ Recovery of ocean circulations and its temporal variations from satellite observations is a possibility, but should be further studied (Vennerstrom et al., The Impact of Combined Magnetic and Electric Field Analysis and of Ocean Circulation Effects on Swarm Mission Performance, Swarm Science Study 2005)
- ▶ Applications:
 - ▶ source term for mantle induction studies
 - ▶ long-term monitoring of ocean currents
 - ▶ numerical prediction of the signal as correction for other data analyses

Introduction

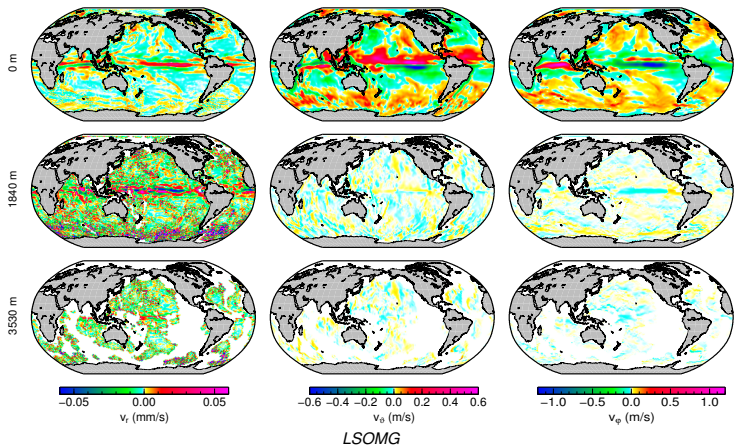
Basic physics

- ▶ oceanic currents moving in Earth's main magnetic field induce secondary magnetic fields; both poloidal and toroidal
- ▶ electric currents are coupled to the conductive crust and mantle
- ▶ high lateral conductivity contrast between continents and oceans also plays an important role
- ▶ magnetic signatures are observable at sea-bottom, ocean-island, and coastal geomagnetic observatories, and newly also by low-orbit satellites



Introduction

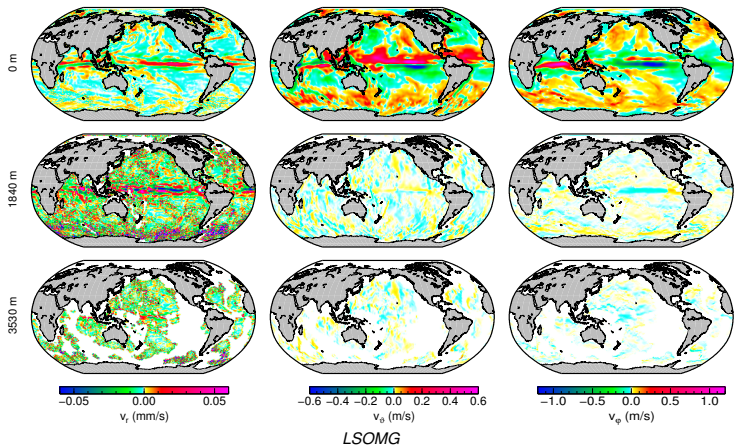
Global ocean circulation



- ▶ driven by wind stresses and heat fluxes on the ocean surface
- ▶ variations on seasonal and longer time scales

Introduction

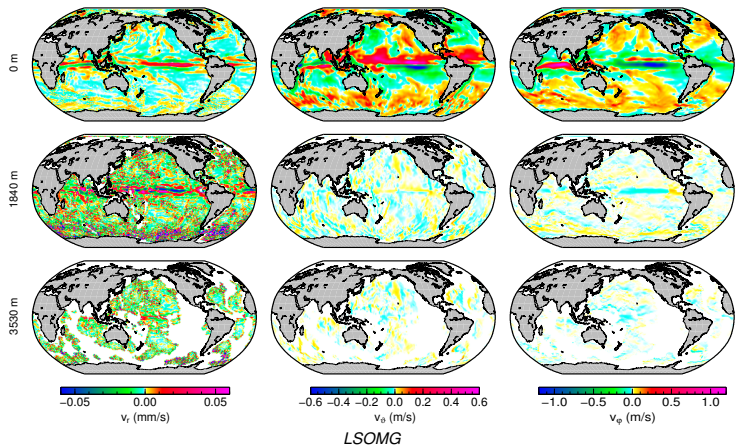
Global ocean circulation



- ▶ vertical flows non-negligible
- ▶ vertical stratification of the horizontal flows

Introduction

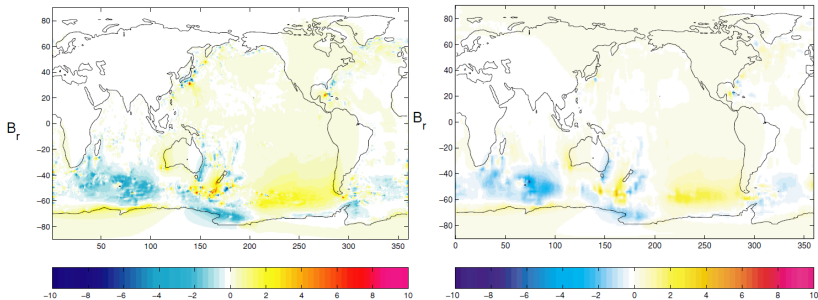
Global ocean circulation



- ▶ 3-D modelling
- ▶ baroclinic approach, hydrostatic and Boussinesq approx.

Introduction

Magnetic signatures of ocean circulation

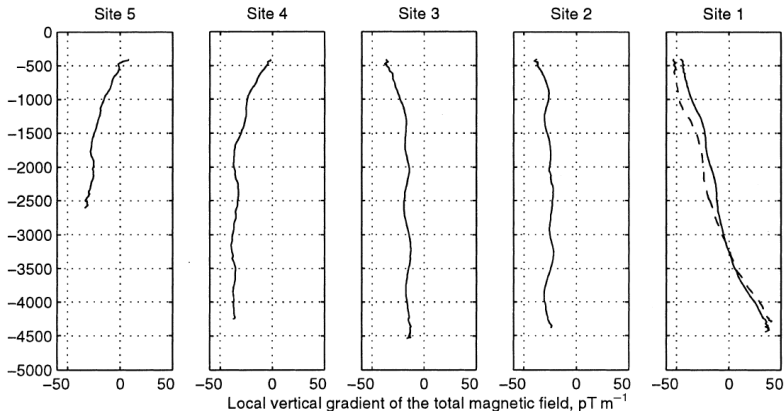


OCCAM (left) and ECCO (right) magnetic signatures (*Manoj et al. 2006*)

- predictions strongly dependent on ocean flow model

Introduction

Magnetic signatures of ocean circulation

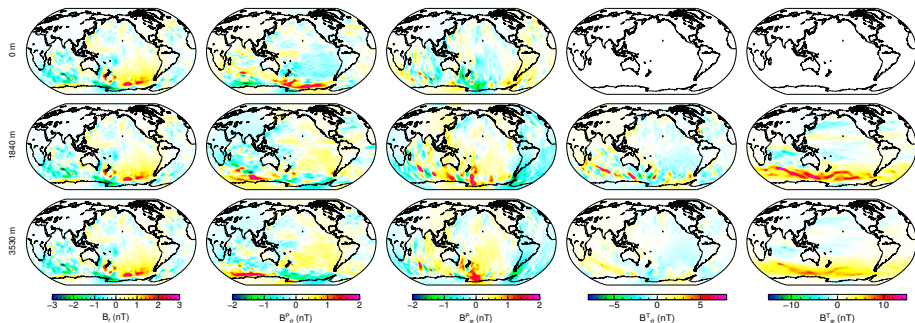


Gradient profiles in the Tasman sea (*Lilley et al. 2001*)

- significant amplitudes in the deep oceans

Introduction

Magnetic signatures of ocean circulation



Velínský *et al.*, 2019

- ▶ maximum amplitude of toroidal field ≈ 15 nT at depth 1800 m (eastward ACC signature)
- ▶ radial field independent on depth
- ▶ overturn of horizontal poloidal field

Parallel modelling in the time domain

EM induction equation with Lorentz forcing

$$\begin{aligned}\operatorname{curl}(\rho \operatorname{curl} \mathbf{B}) + \mu_0 \frac{\partial \mathbf{B}}{\partial t} &= \mu_0 \operatorname{curl}(\mathbf{v} \times \mathbf{B}_0) \quad \text{in } G \\ \operatorname{div} \mathbf{B} &= \mathbf{0} \quad \text{in } G \\ \mathbf{B}^{(\text{ext})} &= \mathbf{0} \quad \text{on } \partial G\end{aligned}$$

in general $\rho = \rho(T, s) = \rho(r, \vartheta, \varphi; t)$

Parallel modelling in the time domain

Spherical harmonic finite-element parameterization

- ▶ expansion of magnetic field in vector spherical harmonics and 1-D finite elements
- ▶ Galerkin method
- ▶ enforcing solenoidality by Lagrange multipliers method
- ▶ spatial discretization leads to a system of linear ODEs:

$$\mathbf{A}(t) \cdot \mathbf{x}(t) + \mathbf{M} \cdot \frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{b}(t)$$

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- ▶ matrix $\mathbf{M}$ has block-tridiagonal structure with diagonal blocks

$$\mathbf{M} = \begin{pmatrix} \boxed{} & \boxed{} & & & & & \\ \boxed{} & \boxed{} & \boxed{} & & & & \\ & \boxed{} & \boxed{} & \boxed{} & & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & \boxed{} & \boxed{} & \boxed{} & \\ & & & & \boxed{} & \boxed{} & \\ & & & & & \boxed{} & \boxed{} \end{pmatrix}$$

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- ▶ matrix $\mathbf{A}$ has block-tridiagonal structure with sparse or dense blocks

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Parallel modelling in the time domain

Crank-Nicolson time discretization

$$\frac{\mathbf{A}_{i+1} \cdot \mathbf{x}_{i+1} + \mathbf{A}_i \cdot \mathbf{x}_i}{2} + \mathbf{M} \cdot \frac{\mathbf{x}_{i+1} - \mathbf{x}_i}{\Delta t} = \frac{\mathbf{b}_{i+1} + \mathbf{b}_i}{2}$$

Parallel modelling in the time domain

Sequential approach

$$(2\mathbf{M} + \Delta t \mathbf{A}_{i+1}) \cdot \mathbf{x}_{i+1} = (2\mathbf{M} - \Delta t \mathbf{A}_i) \cdot \mathbf{x}_i + \Delta t (\mathbf{b}_{i+1} + \mathbf{b}_i)$$

► system properties

- fast inversion of 1-D preconditioner $(2\mathbf{M} + \Delta t \mathbf{A}_{1D})$ (system decoupled in j , m , and in toroidal-poloidal decomposition)
- fast matrix-free multiplication by $\mathbf{M}$ and $\mathbf{A}_i$ using FFT; saves memory, but requires shared memory across j , m
- allows parallelization over layers, but 1-D layers are much faster than 3-D: easy with shared-memory, difficult with distributed-memory systems
- slow and memory-intensive factorization of full 3-D system $(2\mathbf{M} + \Delta t \mathbf{A}_{i+1})$

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- ▶ direct solver

- ▶ practical for moderate resolutions $J \leq 80$ and time-independent resistivity ($\mathbf{A}_i = \mathbf{A}_{3D} \forall i$)

Parallel modelling in the time domain

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- ▶ iterative solver, e.g. BiCG, GMRES, BiCGStab, BiCGStab(l)

- ▶ usable for high resolution
- ▶ slow; effective parallelization limited by number of 3-D layers

Parallel modelling in the time domain

Parallel approach in time

$$\begin{pmatrix} \mathbf{D}_1 & & & & \\ \mathbf{L}_1 & \mathbf{D}_2 & & & \\ & \mathbf{L}_2 & \mathbf{D}_3 & & \\ & & \ddots & \ddots & \\ & & & \mathbf{L}_{I-1} & \mathbf{D}_I \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \vdots \\ \mathbf{x}_I \end{pmatrix} = \begin{pmatrix} \mathbf{b}_{1/2} \\ \mathbf{b}_{3/2} \\ \mathbf{b}_{5/2} \\ \vdots \\ \mathbf{b}_{(2I-1)/2} \end{pmatrix}$$

$$\mathbf{D}_i = \mathbf{A}_i/2 + \mathbf{M}/\Delta t$$

$$\mathbf{L}_i = \mathbf{A}_i/2 - \mathbf{M}/\Delta t$$

$$\mathbf{b}_{i+1/2} = (\mathbf{b}_{i+1} + \mathbf{b}_i) / 2$$

Parallel modelling in the time domain

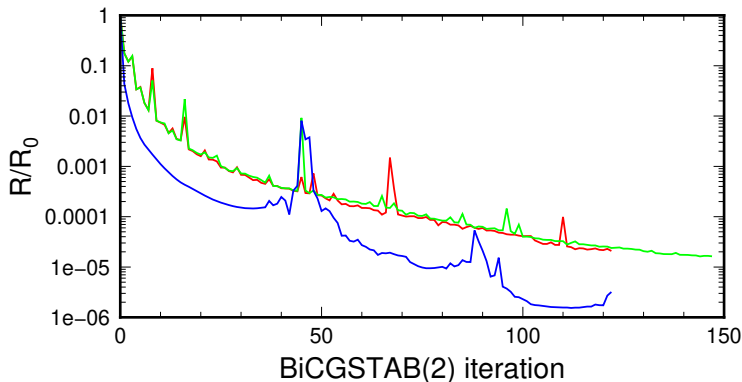
Parallel approach in time

- ▶ apply iterative solver to the entire system
- ▶ I time levels distributed among N processors
- ▶ ideally I divisible by N
- ▶ linear (chain) topology
- ▶ unidirectional communication between nodes
- ▶ full solution at one time level sent/received in each 3-D multiplication
- ▶ preconditioner: 1-D sequential solution staggered in j, m
- ▶ internal shared-memory parallelization in 3-D multiplication (matrix-free, FFT, layers)

Parallel modelling in the time domain

Parallel approach in time

- ▶ BiCGSTAB(l) (Sleijpen & Fokkema, 1993): hybrid Krylov-space method
- ▶ alternates l biconjugate-gradient steps with a GMRES step
- ▶ suitable for non-symmetric systems
- ▶ does not require the implementation of a transpose-matrix product



Parallel modelling in the time domain

Embarassingly parallel scaling

Case 1

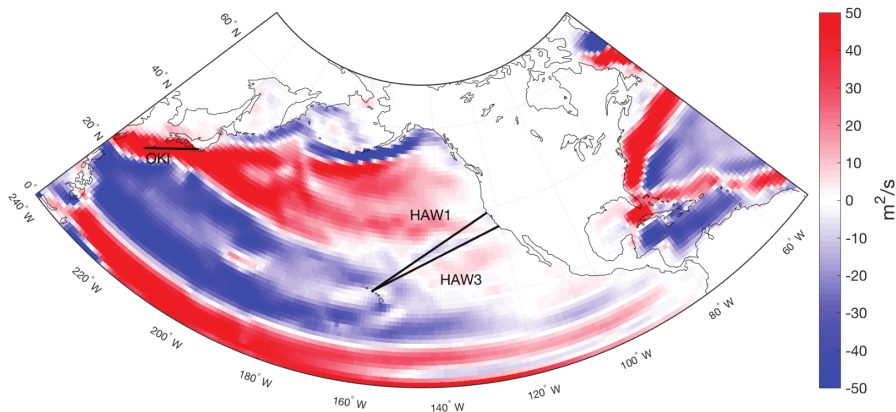
- ▶ cluster parameters: 86 nodes, 24 cores and 128 GB RAM per node
- ▶ $N = 1376 = 86 \times 4 \times 4$ time levels total
- ▶ at given resolution 3 GB RAM per time level needed
- ▶ 4 MPI/node, 4 time levels, 6 OpenMP threads each
- ▶ wall-clock runtime: 10 hrs

Case 2

- ▶ single high-end PC: 12 cores, 192 GB RAM
- ▶ $N = 1392 = 48 \times 29$ time levels total
- ▶ 4 MPI processes, 12 time levels, 3 OpenMP threads each
- ▶ repeat 29 times
- ▶ wall-clock runtime: 70 days

Seafloor cable voltages

- ▶ Goal: to compare the predicted and observed voltages across long-distant submarine cables in the Pacific



Seafloor cable voltages

Modelling setup

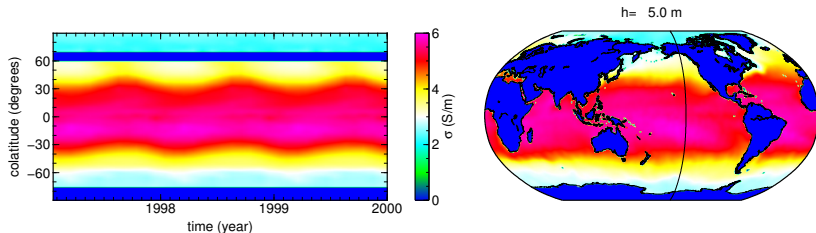
- ▶ 4-D ocean conductivity ($1^\circ \times 1^\circ$, WOA, Tyler *et al.* 2017)
- ▶ 1-D mantle conductivity (Grayver *et al.* 2017)
- ▶ ECCOV4r3 horizontal flows ($0.5^\circ \times 0.5^\circ$, Forget *et al.* 2015)
- ▶ main field from IGRF12
- ▶ $J = 240$
- ▶ $\Delta t = 1$ day
- ▶ time span 1997.0–2002.0
- ▶ 50 ocean layers
- ▶ 120 mantle layers
- ▶ E-field obtained by postprocessing

$$\mathbf{E} = \frac{\rho}{\mu_0} (\text{curl } \mathbf{B}) - \mathbf{v} \times \mathbf{B}_0$$

Seafloor cable voltages

World Ocean Atlas

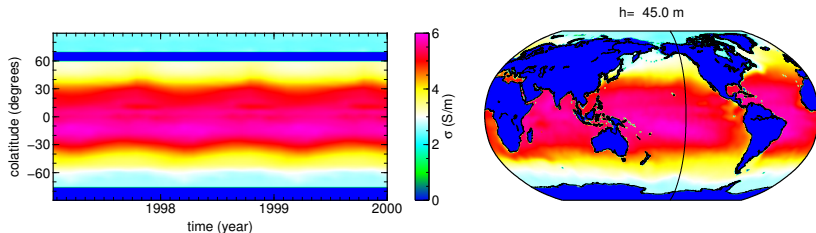
- ▶ 4-D global map of ocean electrical conductivity reprocessed from collocated measurements of salinity and temperature (WOA, Tyler *et al.* 2017)
- ? effect of conductivity time variations: alternative run with time-averaged conductivity



Seafloor cable voltages

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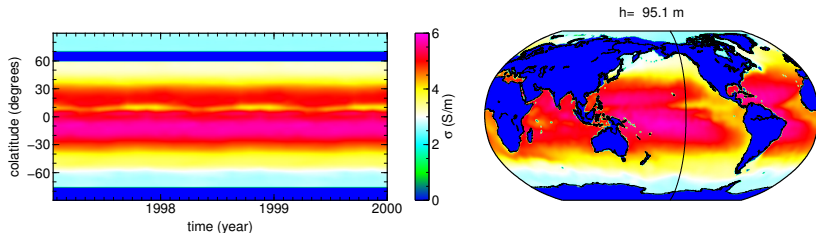
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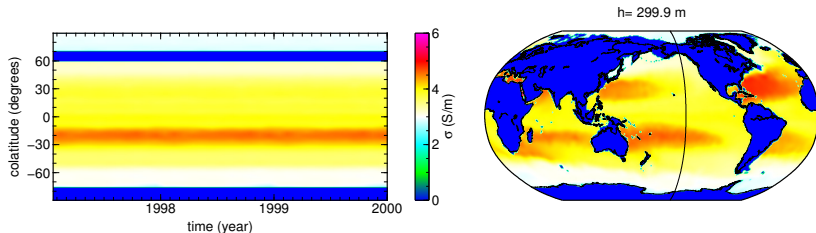
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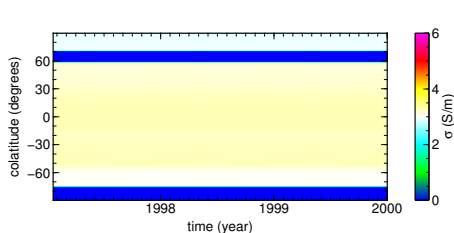
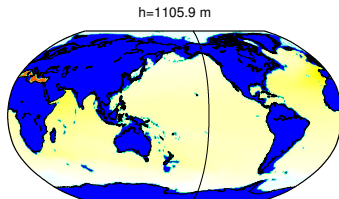
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Seafloor cable voltages

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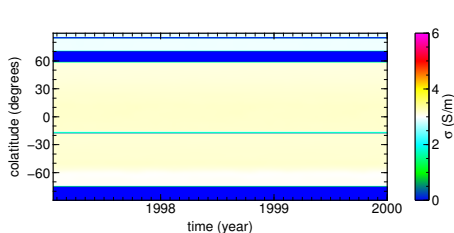
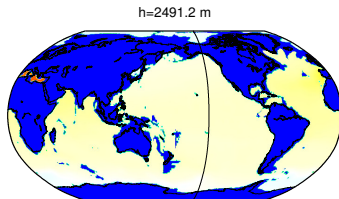
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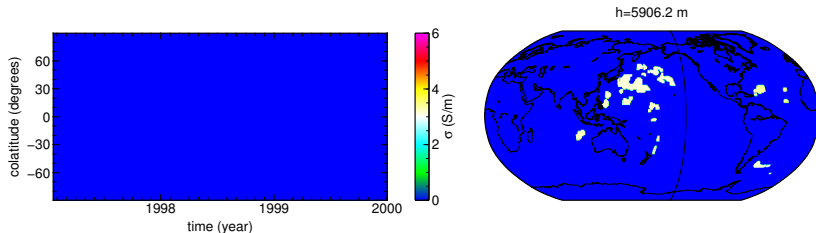
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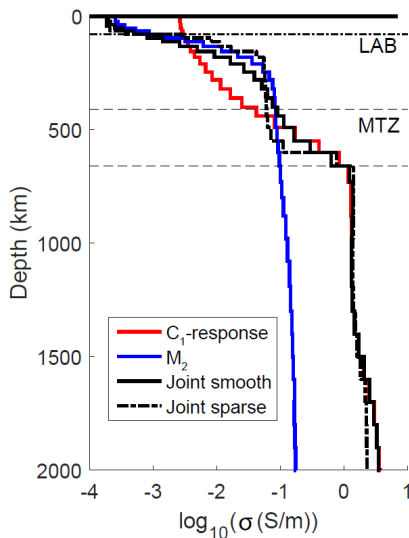
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Seafloor cable voltages

1-D mantle

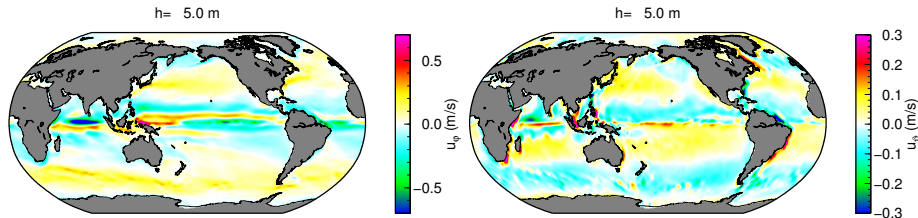
- ▶ 1-D mantle conductivity model based on satellite data using M_2 and magnetospheric sources (Grayver *et al.* 2017)



Seafloor cable voltages

ECCO

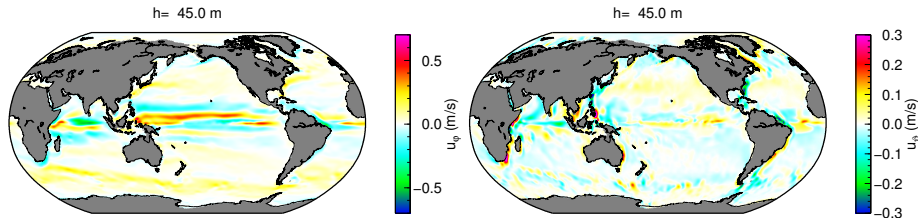
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Seafloor cable voltages

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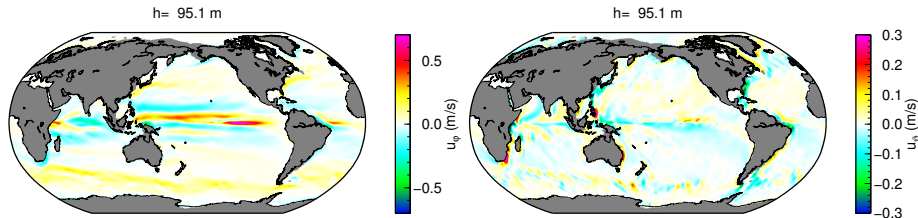
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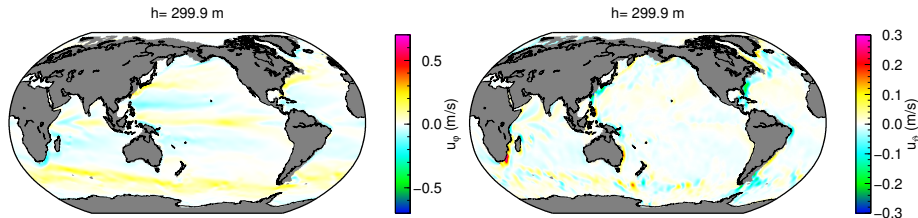
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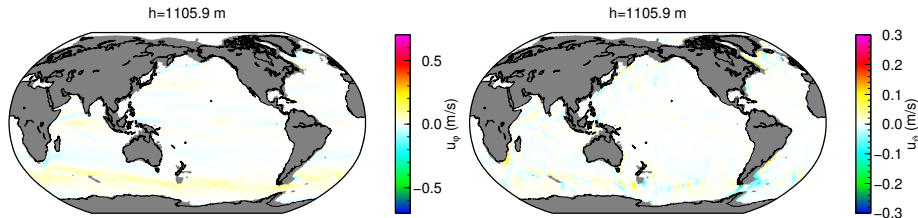
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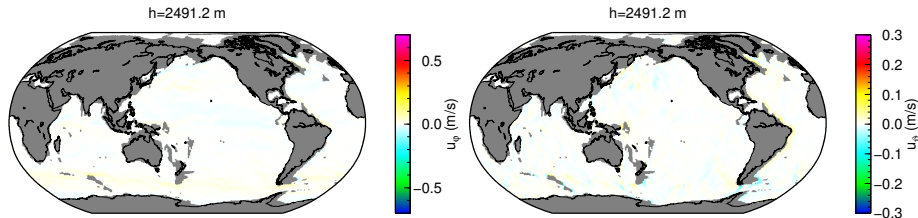
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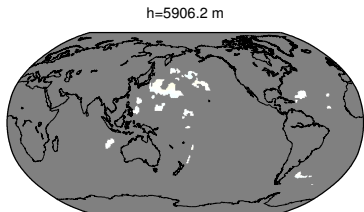
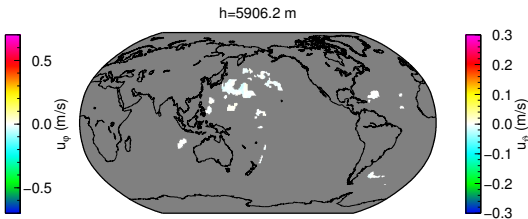
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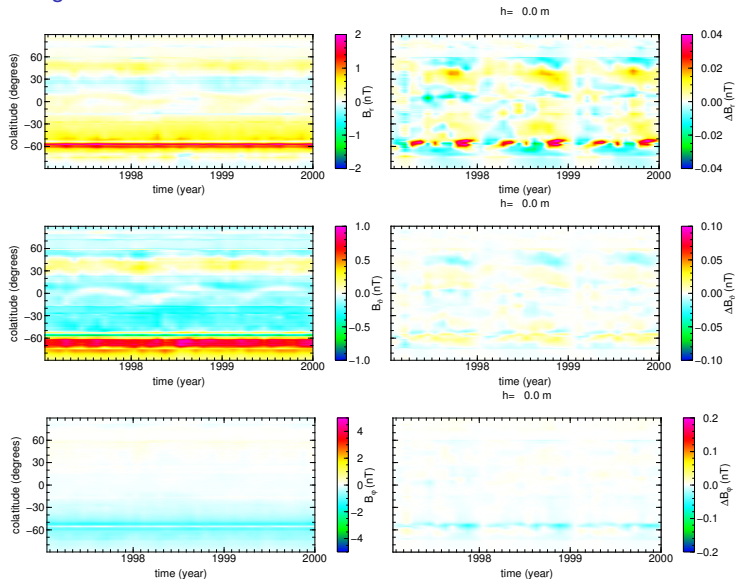
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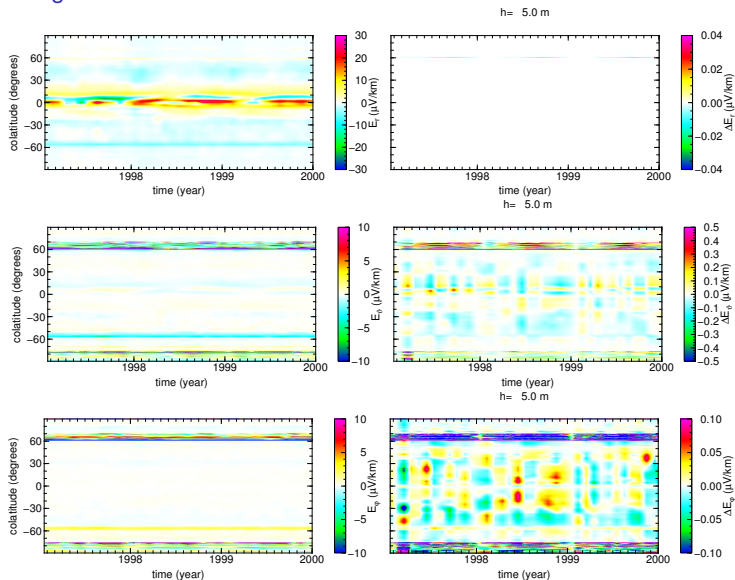
Seafloor cable voltages

Predicted magnetic and electric fields



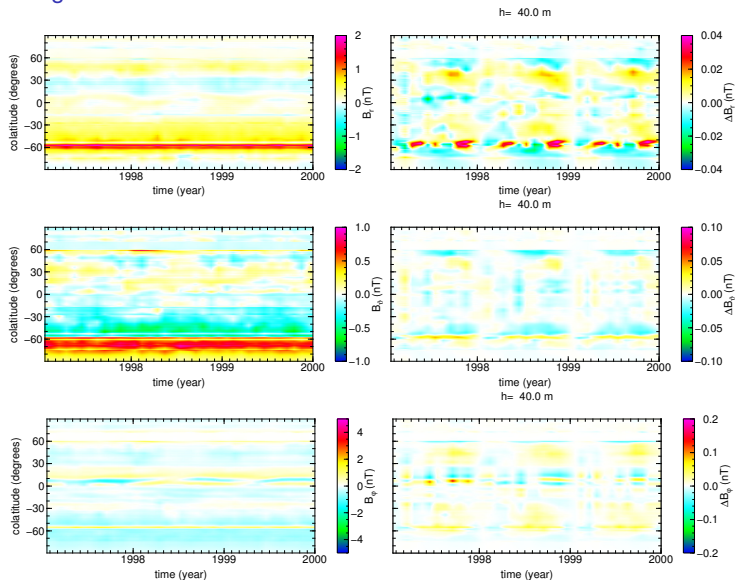
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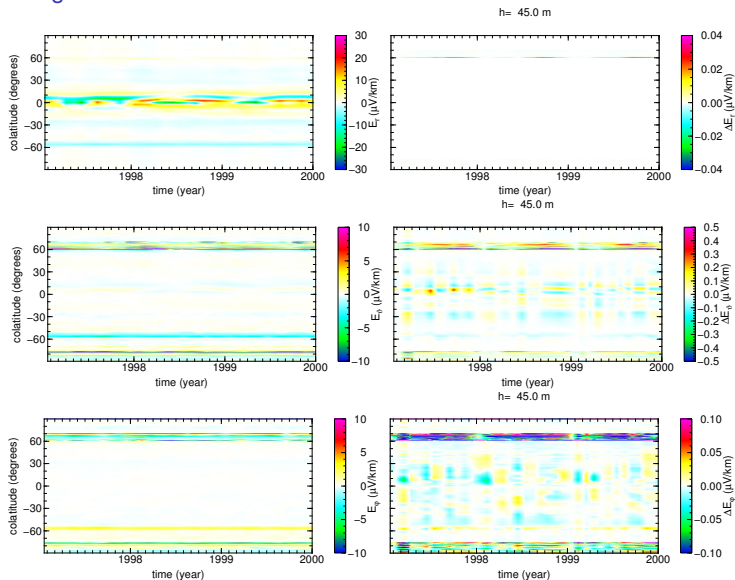
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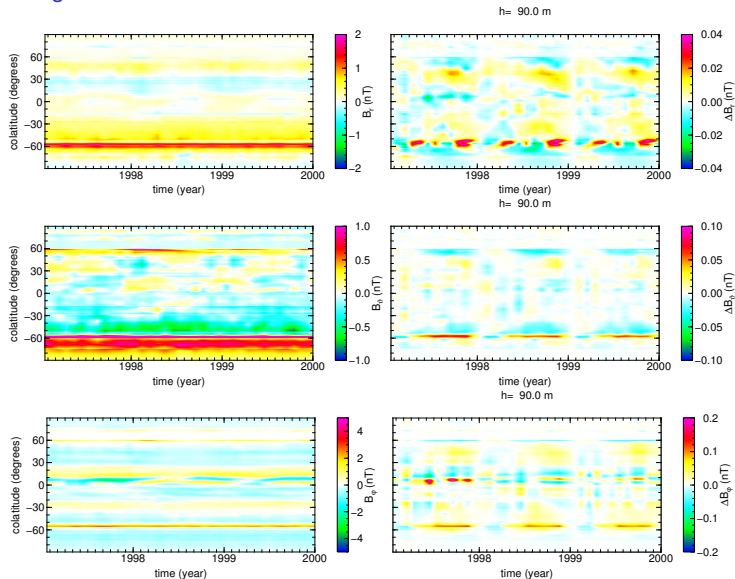
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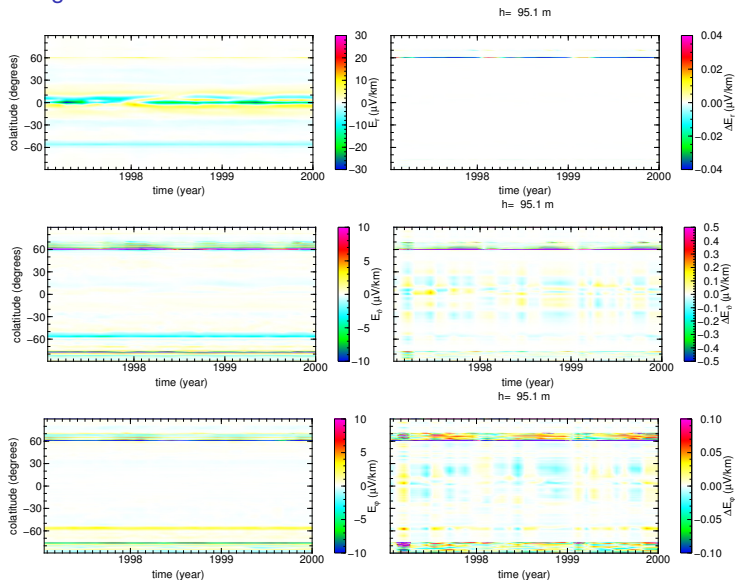
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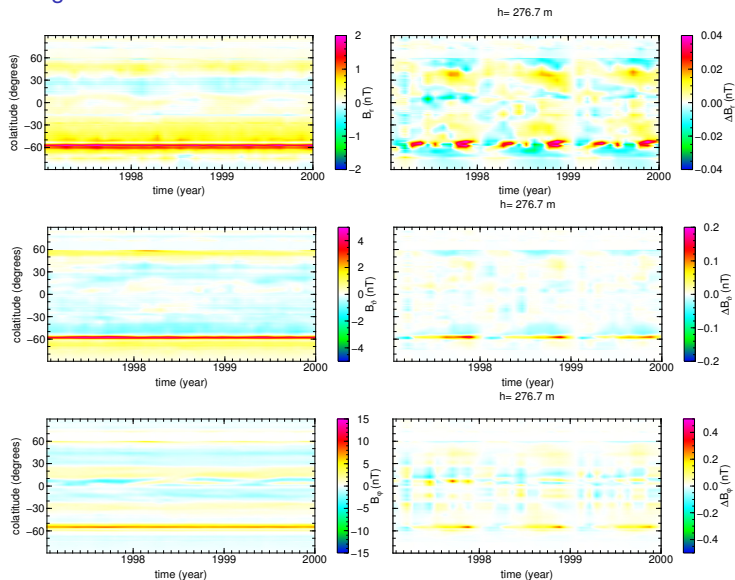
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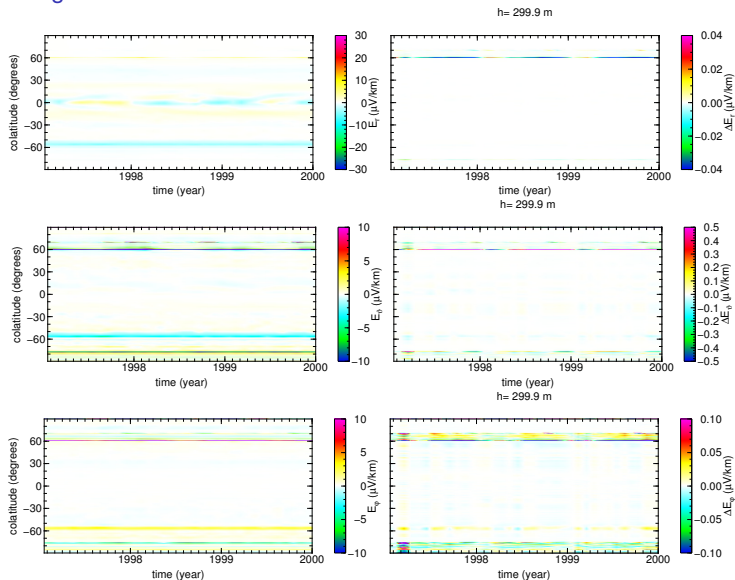
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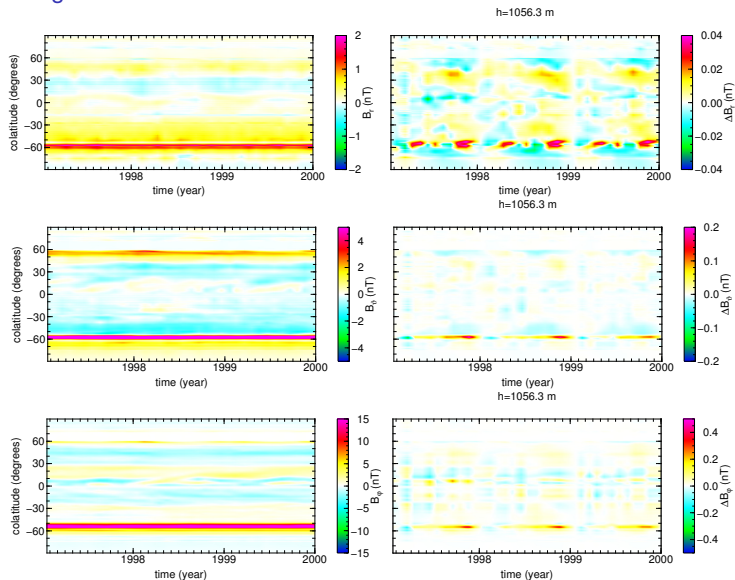
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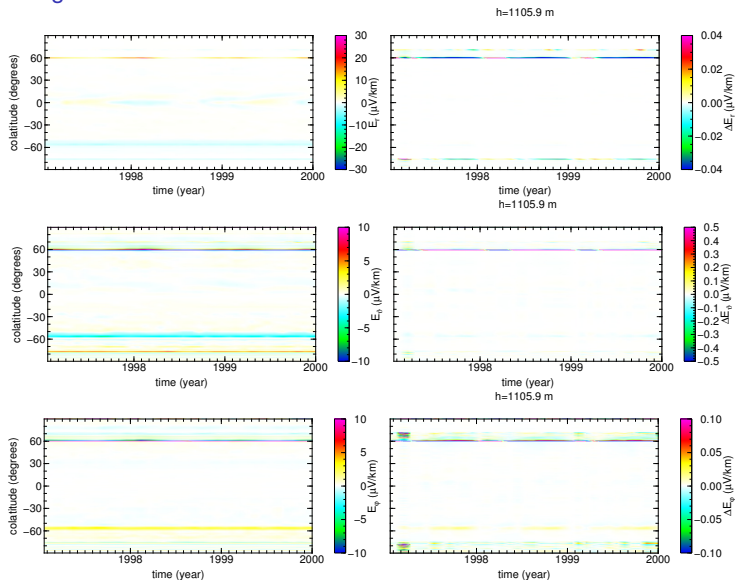
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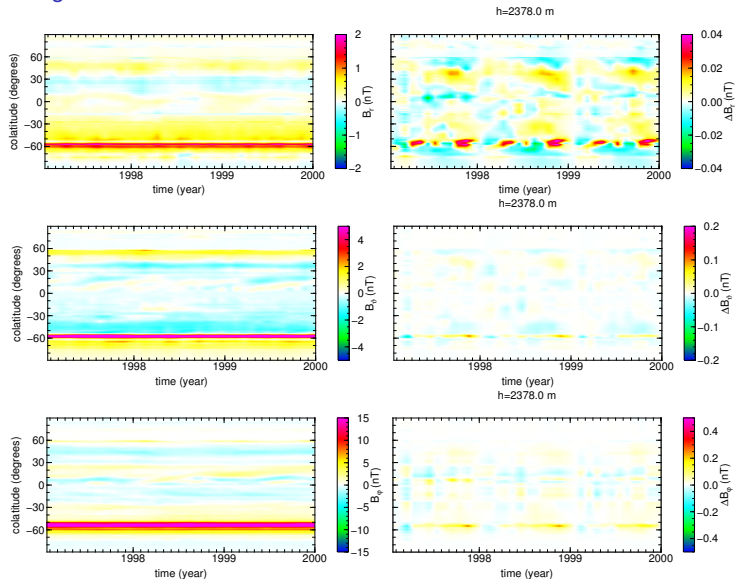
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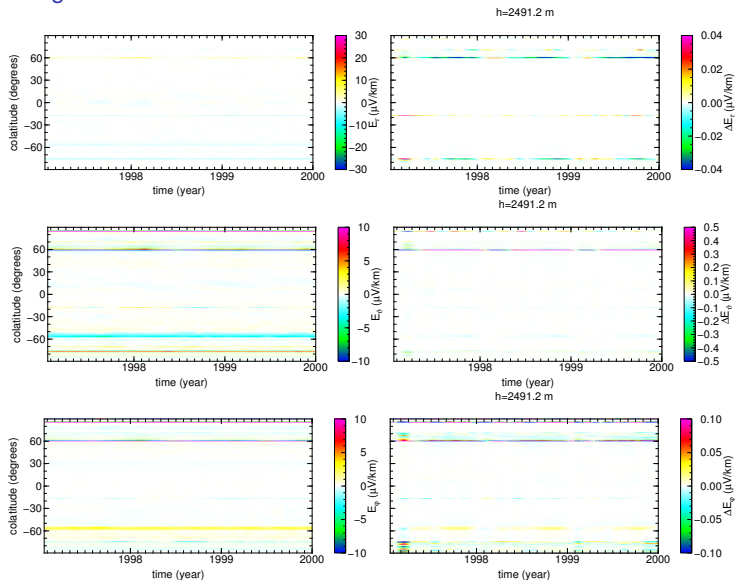
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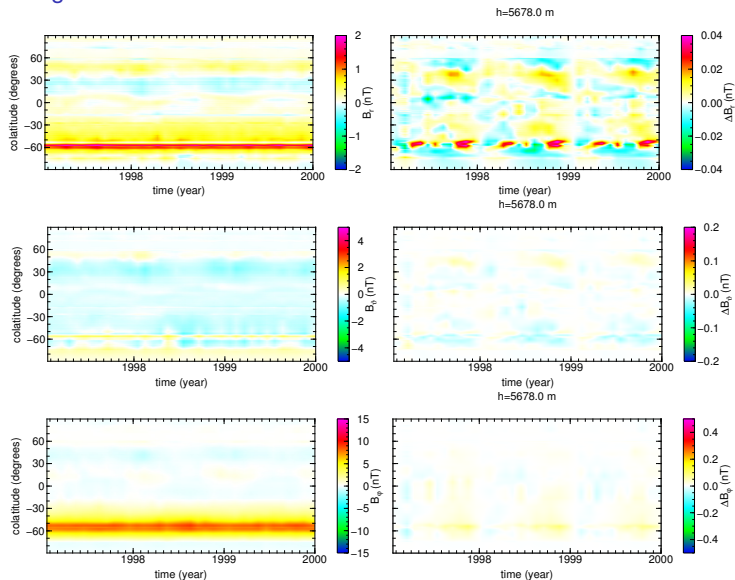
Seafloor cable voltages

Predicted magnetic and electric fields



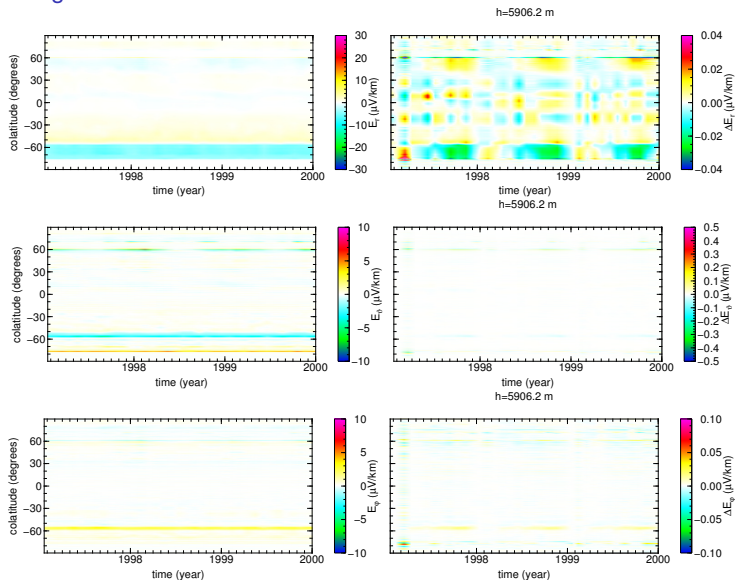
Seafloor cable voltages

Predicted magnetic and electric fields



Seafloor cable voltages

Predicted magnetic and electric fields



Seafloor cable voltages

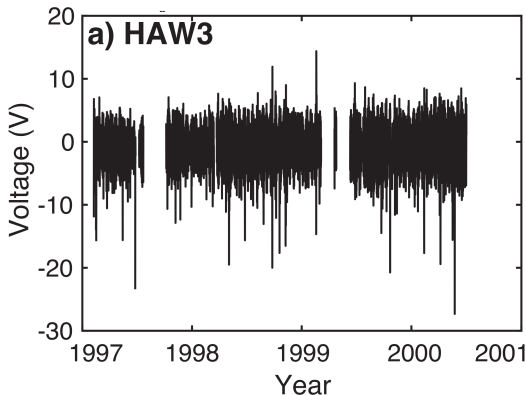
Observations from forward modelling

- ▶ the effect of seasonal conductivity variations in the oceans is less than 5 %
- ▶ this is not as important as other factors (e.g., use of a different ocean flow model)
- ▶ mostly zonal magnetic and electric field are affected

Seafloor cable voltages

Submarine cable data processing

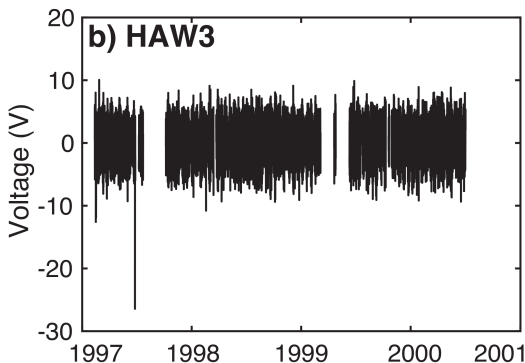
- ▶ hourly data
- ▶ removal of noisy days ($A_p > 20$)
- ▶ least-square fit of 12 dominant tidal frequencies and their removal
- ▶ smoothing by cubic splines (90 days or 30.5 days knot distance)



Seafloor cable voltages

Submarine cable data processing

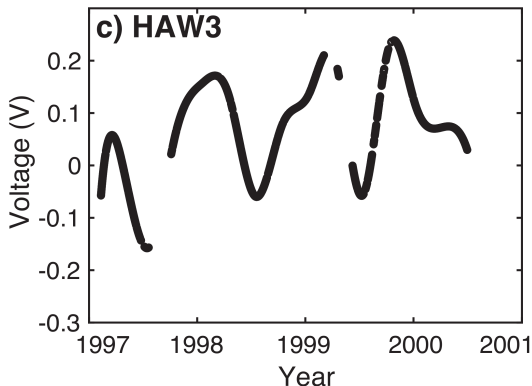
- ▶ hourly data
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- ▶ least-square fit of 12 dominant tidal frequencies and their removal
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Seafloor cable voltages

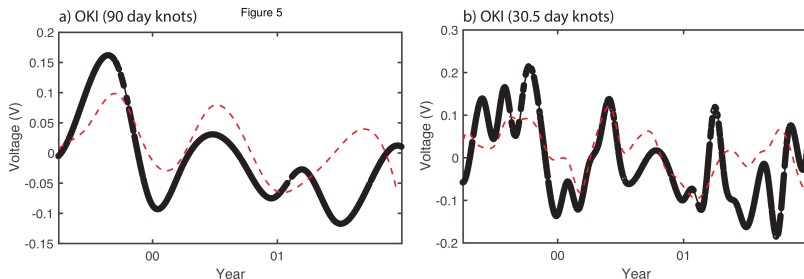
Submarine cable data processing

- ▶ hourly data
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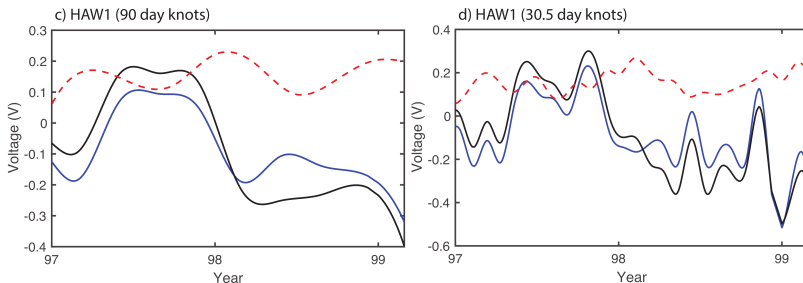
Seafloor cable voltages

Comparison of predicted and observed voltages



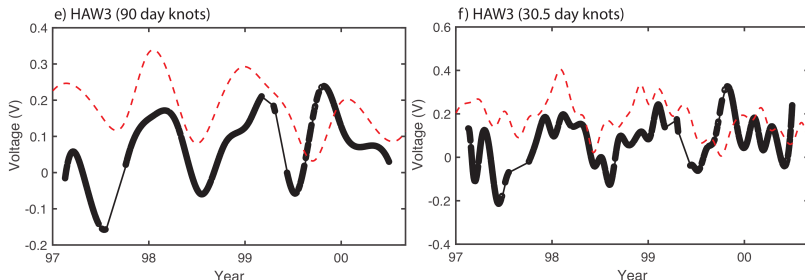
Seafloor cable voltages

Comparison of predicted and observed voltages



Seafloor cable voltages

Comparison of predicted and observed voltages



Seafloor cable voltages

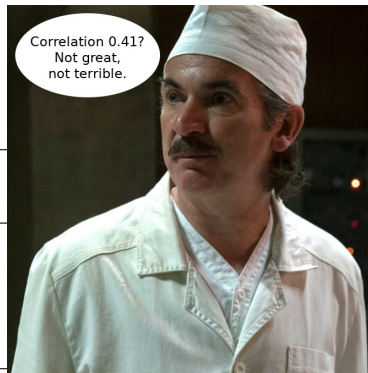
Comparison of predicted and observed voltages

Cable	30.5 day spline fit		90 day spline fit	
	R	R^2	R	R^2
HAW1N	-0.2675	0.0716	-0.3591	0.1290
HAW1S	-0.1540	0.0237	-0.1779	0.0316
HAW3	0.3129	0.0979	0.4111	0.1690
OKI	0.5920	0.3505	0.6526	0.4259

Seafloor cable voltages

Comparison of predicted and observed voltages

Cable	30.5 day spline fit	
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OKI	0.5920	0.3505



Conclusions

- ▶ NOAA's installation in Florida routinely measures ocean transport by cable voltage (200 km)
- ▶ first attempt to correlate long-term observations of long-distance submarine voltage cables with numerical predictions
- ▶ possible sources of noise:
 - ▶ electric field induced by secular variations
 - ▶ imperfect removal of external-field induced signals (e.g. seasonal ionosphere variations)
 - ▶ local effects where cables are grounded
 - ▶ coherence of ocean transports across the cable

Outlook

- ▶ back to subsurface magnetic field measurements (suspended string of magnetometers in ACC)
- ▶ regional modelling (e.g. for the Florida or Okinawa cables)