

Viscoelasticity in mantle convection

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Content

Motivation

Method

Testing: Elastic slab

Results: Regime of Convection

(Numerical?) instabilities

Results: Stress, Topography

Future

"Almost all solid materials, including essentially all rocks at relatively low temperatures and pressures, behave elastically when the applied forces are not too large."

- @ Lithospheric flexure
- @ Lithospheric buckling
- @ Chandler wobble period
- @ Post-Glacial Rebound
- @ Topography/geoid of planets

Mariana trench - lithospheric flexure

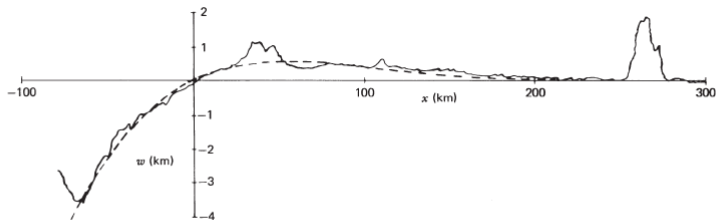
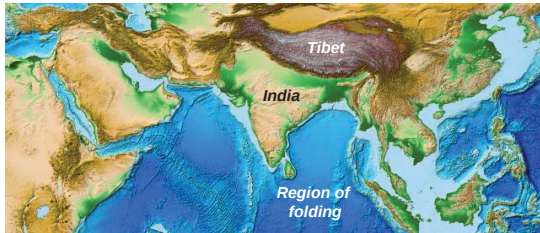


Figure 3.35 Comparison of a bathymetric profile across the Mariana trench (solid line) with the universal lithospheric deflection profile given by Equation (3.159) (dashed line); $x_b = 55$ km and $w_b = 0.5$ km.

Picture taken from G. Schubert, D.L. Turcotte - Geodynamics

S. Schmalholz - Analytical solutions for deformation structures

Uplift of Tibetan plateau and folding of the Indo-Australian plate



Olive et al. [in review]

4. ELASTICITY AND MODES OF EXTENSIONAL FAULTING

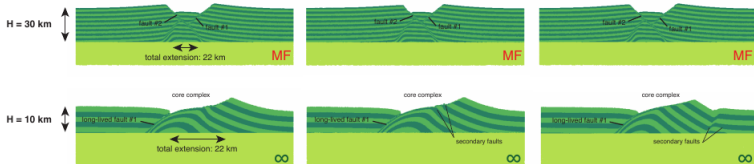
Snapshots of model deformation after 22 km of extension:

ELASTO-PLASTIC

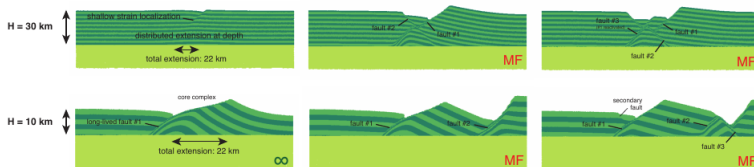
$U = 0.2 \text{ cm/yr}$

$U = 2 \text{ cm/yr}$

$U = 20 \text{ cm/yr}$



VISCO-PLASTIC, $\eta_L = 10^{24} \text{ Pa.s}$



Anelastic Boussinesq approximation

$$-\nabla p + \nabla \cdot \boldsymbol{\tau} = \text{Ra } \bar{\alpha} \bar{\rho} T \vec{e}_r$$

$$\nabla \cdot (\bar{\rho} \vec{v}) = 0$$

$$\bar{\rho} \bar{C}_p \frac{DT}{Dt} = \nabla \cdot (\bar{k} \nabla T) - Di_s \bar{\alpha} \bar{\rho} T v_r + \bar{\rho} \bar{H} + \frac{Di_s}{Ra} \boldsymbol{\tau} : \mathbb{D}$$

$$\tau_{ij} = \eta (v_{i,j} + v_{j,i} - \frac{2}{3} v_{k,k} \delta_{ij})$$

Constitutive equation

$$\boldsymbol{\tau} + \mathcal{D} \overset{\nabla}{\boldsymbol{\tau}} = 2\eta \mathbb{D} \quad \mathcal{D}(p, T) := \tau_{\text{rel}} \frac{\kappa}{h^2} = \frac{\eta \kappa}{G h^2}$$

$$\overset{\nabla}{\boldsymbol{\tau}} := \frac{\partial \boldsymbol{\tau}}{\partial t} + \color{red}{\vec{v}} \cdot \nabla \boldsymbol{\tau} + (\color{blue}{\boldsymbol{\tau}} \mathbb{W} - \mathbb{W} \boldsymbol{\tau}) - \color{yellow}{a}(\mathbb{D} \boldsymbol{\tau} + \boldsymbol{\tau} \mathbb{D})$$

advection co-rotation co-deformation

Mixed Euler (order of accuracy 1)

$$\tilde{\boldsymbol{\tau}}^{n-1} := \boldsymbol{\tau}^{n-1} + dt (\mathbb{W} \boldsymbol{\tau} - \boldsymbol{\tau} \mathbb{W} + a (\mathbb{D} \boldsymbol{\tau} + \boldsymbol{\tau} \mathbb{D}) - \vec{v} \cdot \nabla \boldsymbol{\tau})^{n-1}$$

$$\boldsymbol{\tau}^n = 2\eta_{\text{num}}^n \mathbb{D}^n + \frac{\mathcal{D}^n}{dt + \mathcal{D}^n} \tilde{\boldsymbol{\tau}}^{n-1}; \quad \eta_{\text{num}} := \frac{dt}{dt + \mathcal{D}}$$

Viscoelasticity parameter Z

$$Z := \frac{dt}{dt + \mathcal{D}}; \quad \boldsymbol{\tau}^n = 2Z \eta^n \mathbb{D}^n + (1 - Z) \tilde{\boldsymbol{\tau}}^{n-1}$$

Numerical viscosity

$$\eta_{\text{num}} := \frac{dt}{dt + \mathcal{D}} \eta$$

Right hand side

$$\nabla \cdot [(1 - Z) \tilde{\boldsymbol{\tau}}^{n-1}]$$

Objectivity

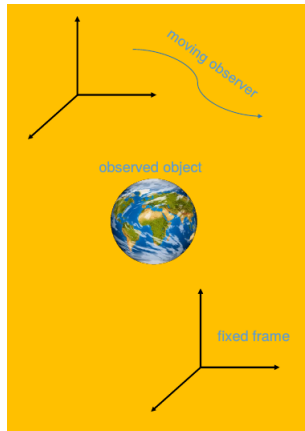
Simple case: Conservation of a quantity, solved by observer attached to a rigid body (continuum with no deformation) and by observer flying around:

$$\text{Rate}(A) = 0$$

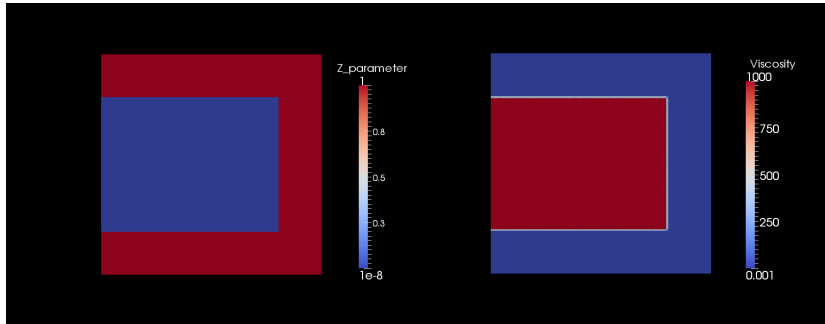
For scalars: $\frac{D}{Dt}A = 0$ does the trick.

Tensors: $\frac{D}{Dt}\mathbb{A} = 0$, what would it mean?

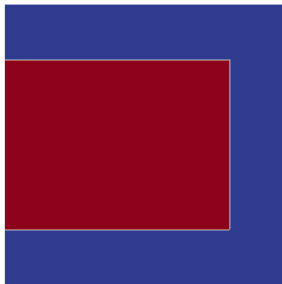
Simplest way to solve it is with Jaumann's co-rotational rate (accounting for $\mathbb{A}' = \mathbb{R}\mathbb{A}\mathbb{R}^T$).



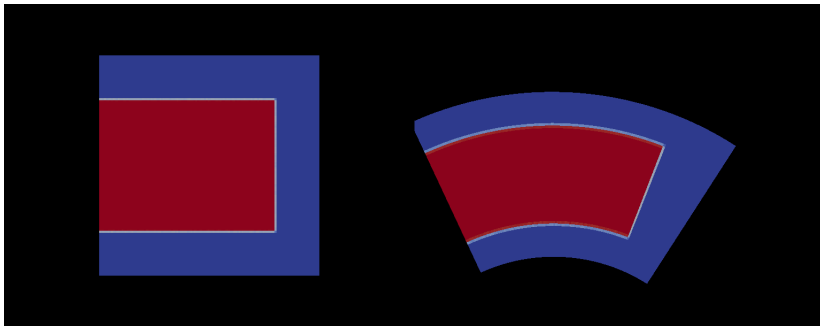
Testing: Elastic slab/recovery experiment



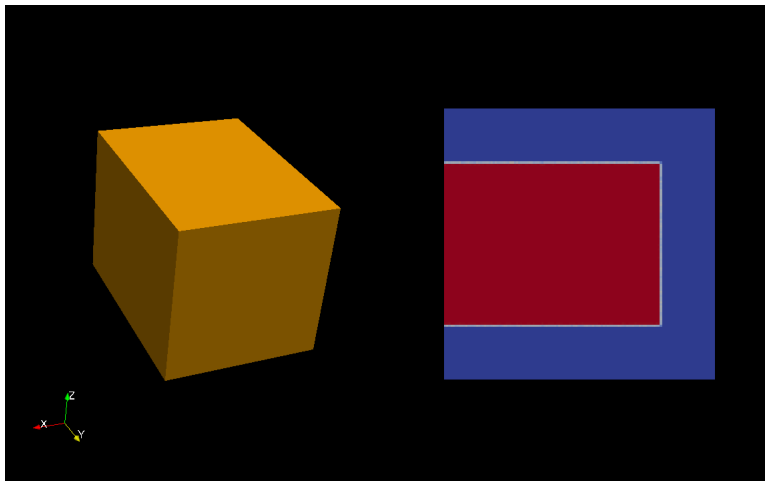
advection of stress via donor-cell



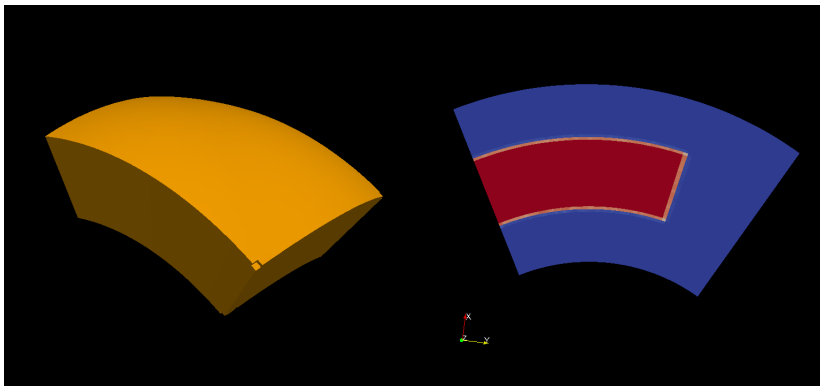
Stress on tracers



3D cartesian



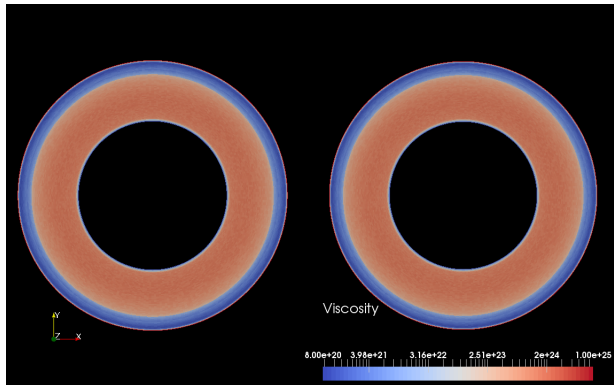
3D spherical



Surface yield stress 20 MPa

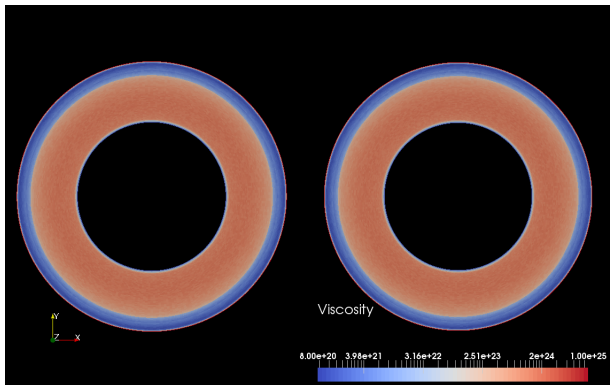
Plasticity: $\eta_p = \tau_{\text{yield}} / \mathbb{D}_{\text{eff}}$

Composition, phase transitions, Core cooling

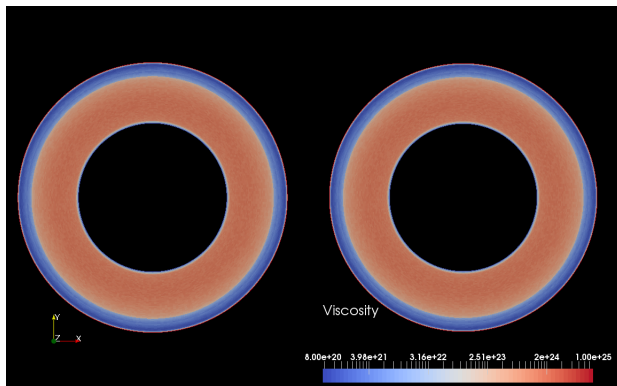


Surface yield stress 40 MPa

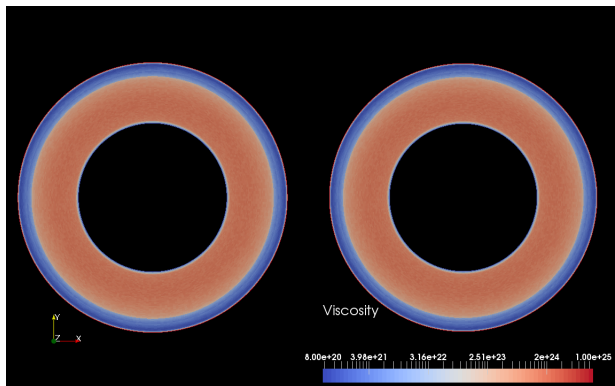
Regime of convection different with elasticity



Surface yield stress 80 MPa



Surface yield stress 100 MPa



Harder 1991

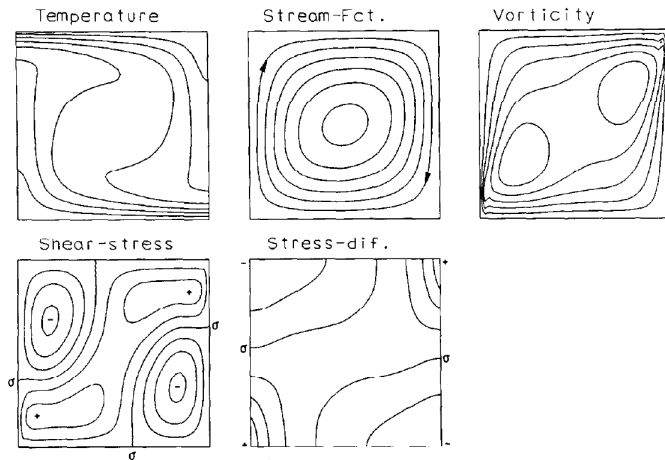
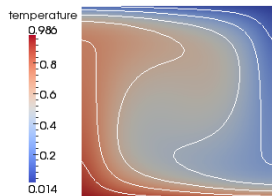
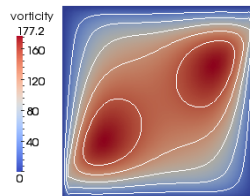
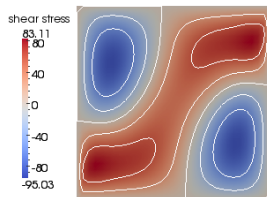
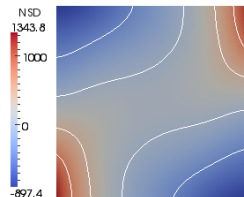
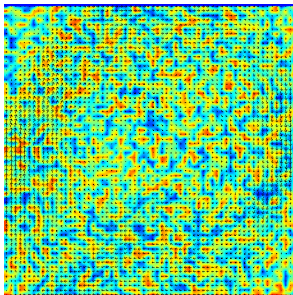


Fig. 6. As Fig. 4, but with co-rotational Maxwell rheology ($a = 0$, $\lambda = 0.0015$, $De = 0.52$).

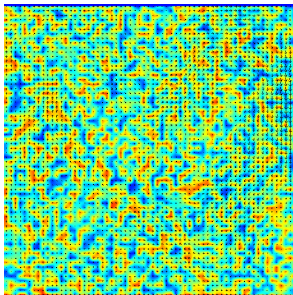
Code - comparison with Harder 1991



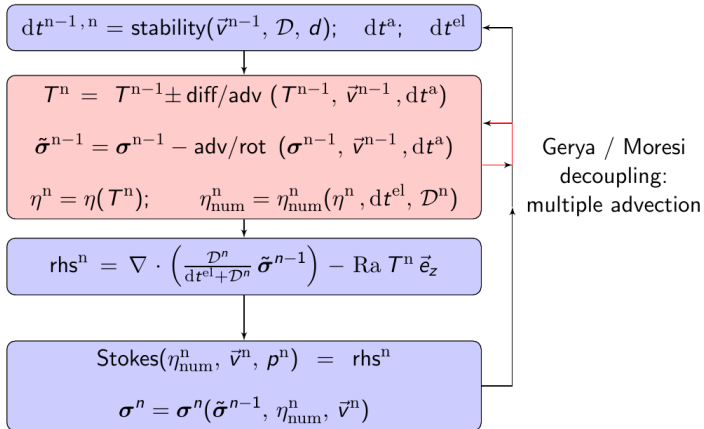
Stability



Stability - art

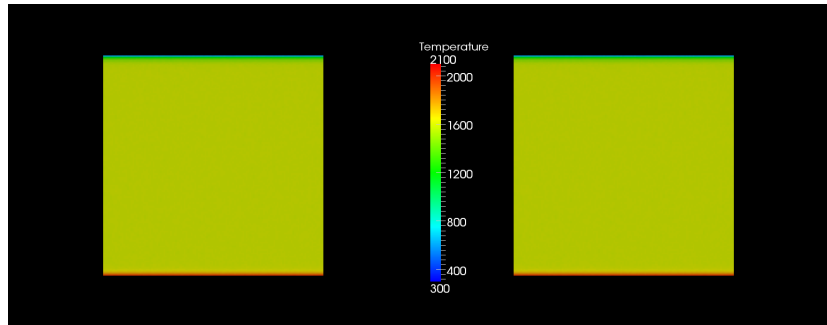


Time step decoupling



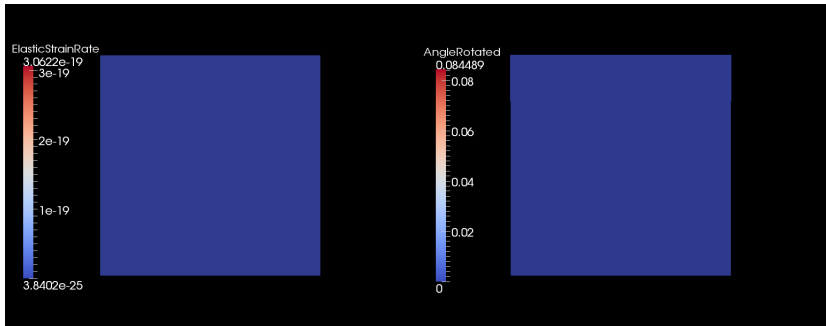
Results: Purely thermal convection

Dimensional setup: $D=3000\text{km}$, $\eta_0 = 1\text{e}22$ (1600 K),
 $k = 3\text{W/mK}$, $\Delta T_{\text{sad}} = 1811\text{K}$, $\alpha = 3.2\text{e-}5$ 1/K, $E=240\text{e}3$ J/mol,
 $V=0.2\text{e-}6$



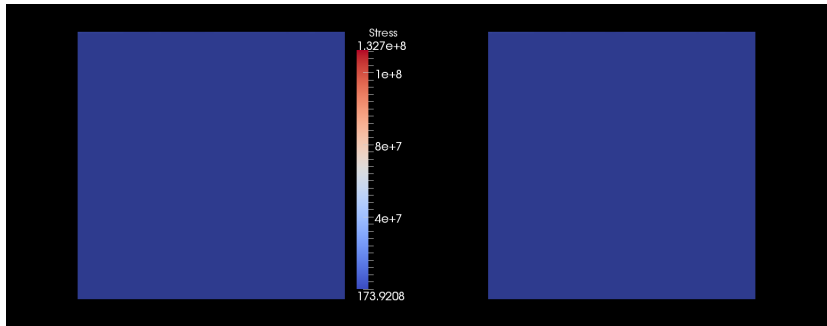
Purely thermal convection

Are these results trustworthy? Rotated angle, acceleration

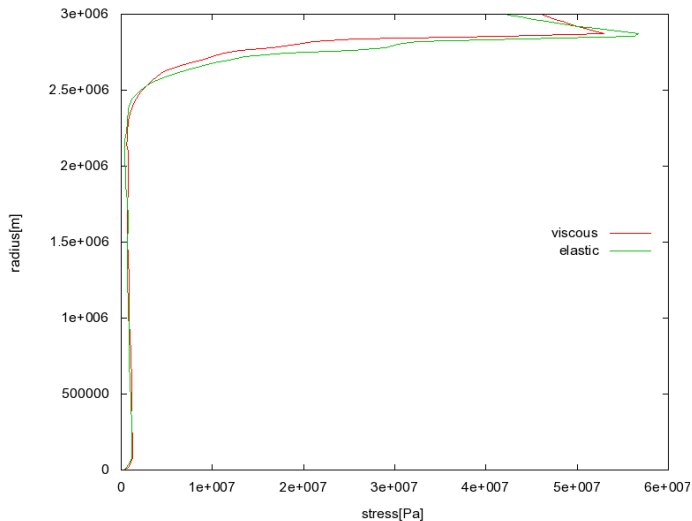


Purely thermal convection - stress

Stresses are significantly different in the lithosphere

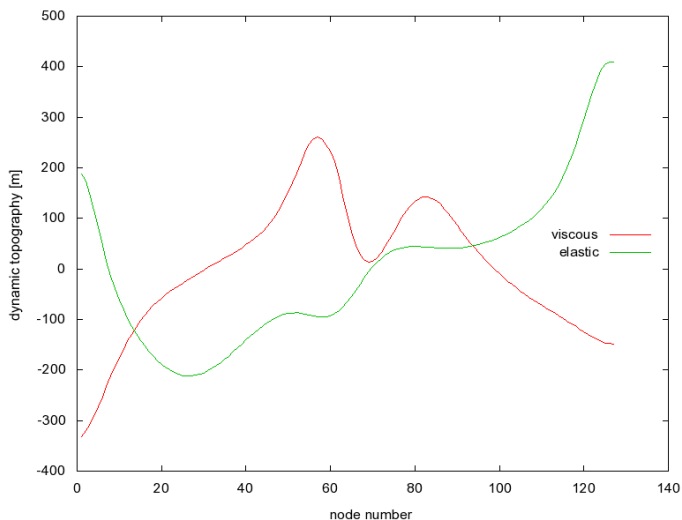


Stress profile



Topography

Dynamic topography. Now running: sticky air



Future possibilities

Regional models - elasticity important (and they often have it):

- Boundary conditions often questioned
- No coupling between BCS and what's happening in the model
- Method of embedded regional models

We are now on the edge of being able to compute low resolution regional models as sections of a global model!

YinYang - not Venus: melting

