

Inversion of Moment Tensors in Anisotropic Media

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Motivation

- Inverted moment tensor may show volumetric components e.g. Island, W-Bohemia, Long Valley – volcanic areas!
- Volumetric components are often interpreted as tensile faulting.
- Inversion algorithms assume isotropic properties of rocks along the ray path and in the source region.
- Many rocks are anisotropic, fault zone may be regions of increased anisotropy due to local long-term deformations.
- Anisotropy influences moment tensors
for tensile and shear sources, moment tensors in anisotropic media may comprise apparent (non-real) volumetric components.
- Anisotropy effects wave amplitudes.

Motivation

- What happens if we neglect anisotropy in inversion?
- Inversion for moment tensors considering anisotropy?
- Do volumetric components of events in W-Bohemia change if we consider anisotropy?
- Can we determine elastic properties in the source region from inverted moment tensors or simultaneous inversion for moment tensors and elastic properties?

Outline

1. Dependence of moment tensor components on orientation of the elastic tensor
2. Influence of anisotropy on radiated amplitudes
3. Inversion neglecting anisotropy
4. Inversion accounting for anisotropy
5. Example: W-Bohemia
6. Conclusion
7. Appendix: more examples, inversion for anisotropy in the source area, simultaneous inversion for the moment tensor and anisotropy in the source area, formulas

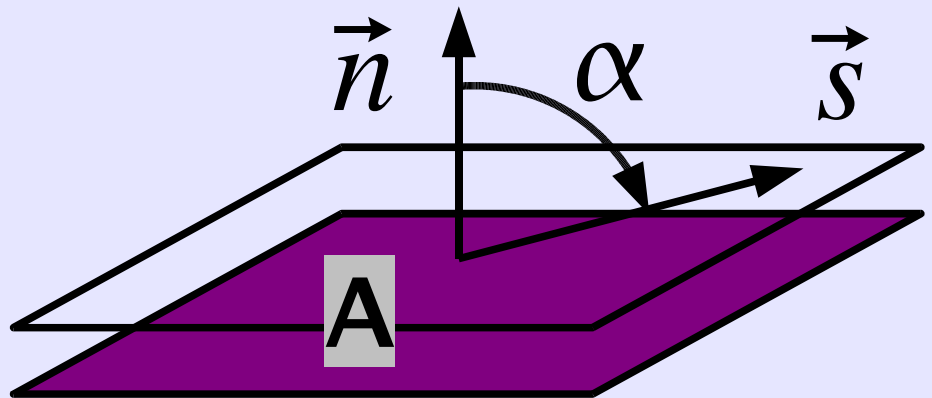
Theory – Seismic Moment Tensor

isotropic materials:

$$M_{jk} = \lambda \mathbf{s}_j \mathbf{n}_j \delta_{jk} A + \mu (\mathbf{s}_j \mathbf{n}_k + \mathbf{n}_j \mathbf{s}_k) A$$

shear source is purely DC
horizontal shear source:

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{13} & 0 & 0 \end{bmatrix}$$



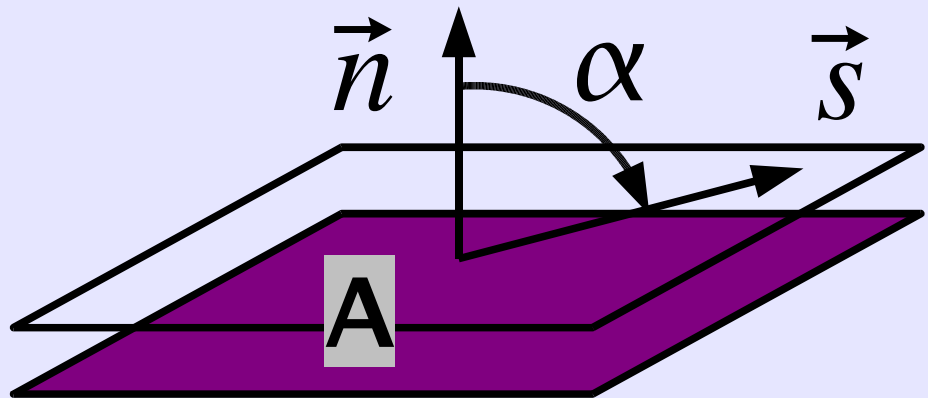
Theory – Seismic Moment Tensor

anisotropic materials:

$$M_{kl} = s_i n_j c_{ijkl} A$$

shear sources may create DC+ISO+CLVD
horizontal shear source:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}$$

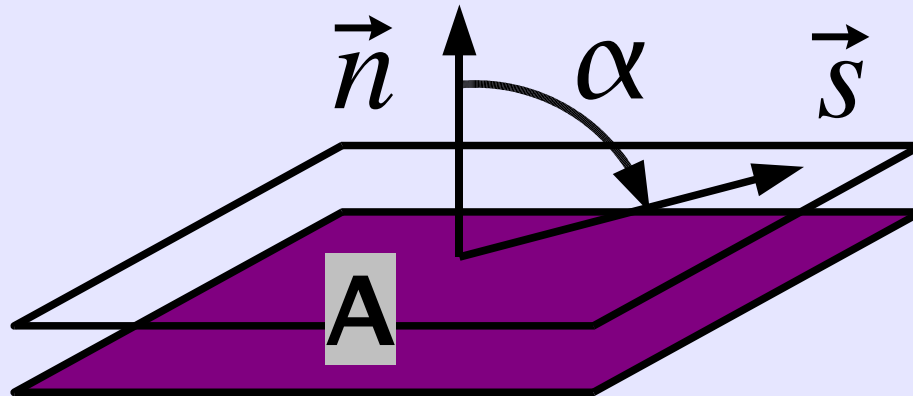


Theory – Seismic Moment Tensor

characteristic features of M:

- seismic moment: $M_0 = \sqrt{M_{ij} * M_{ij} / 2}$
- **DC**: double couple – shear component
- **ISO**: volumetric component
- **CLVD**: compensated linear vector dipole
- **slip inclination**: angle between fault normal

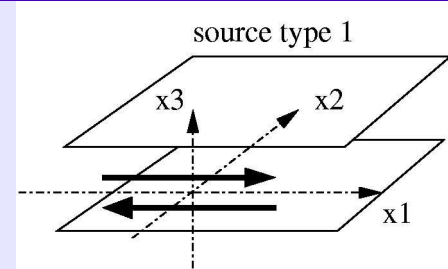
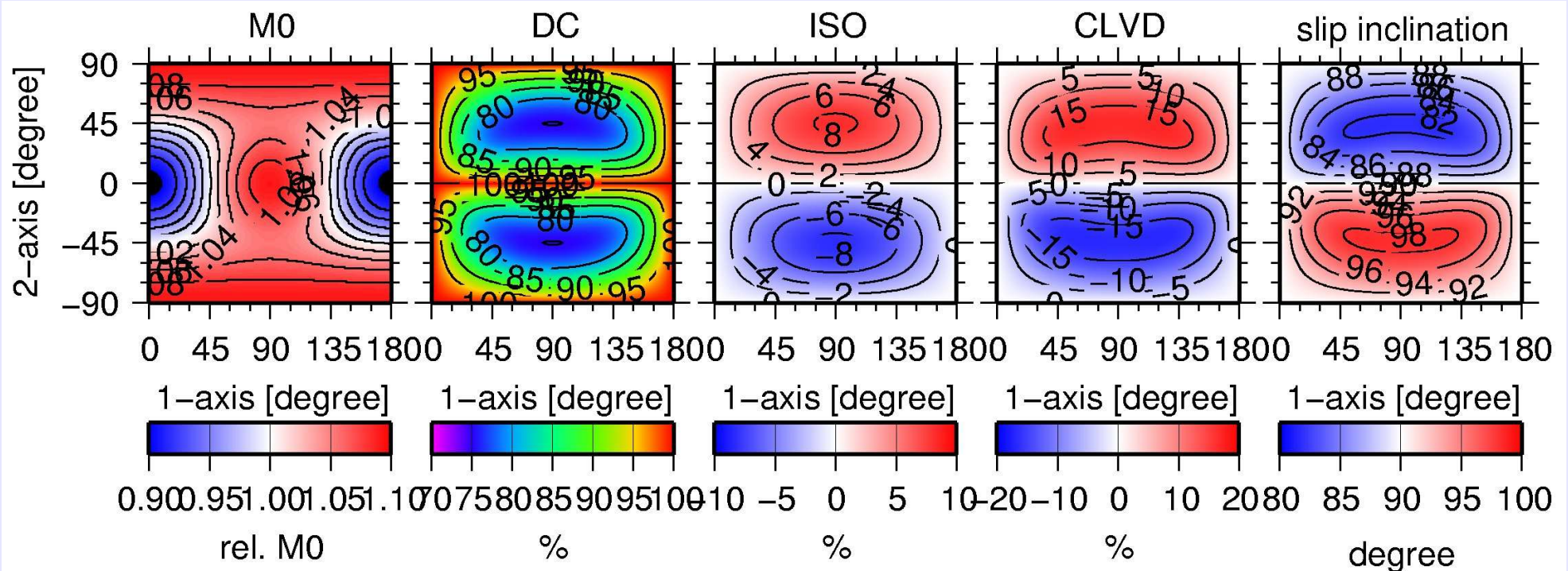
and slip direction



Modelling

shear source

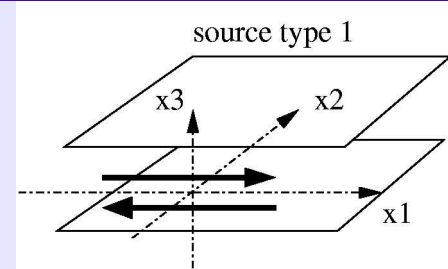
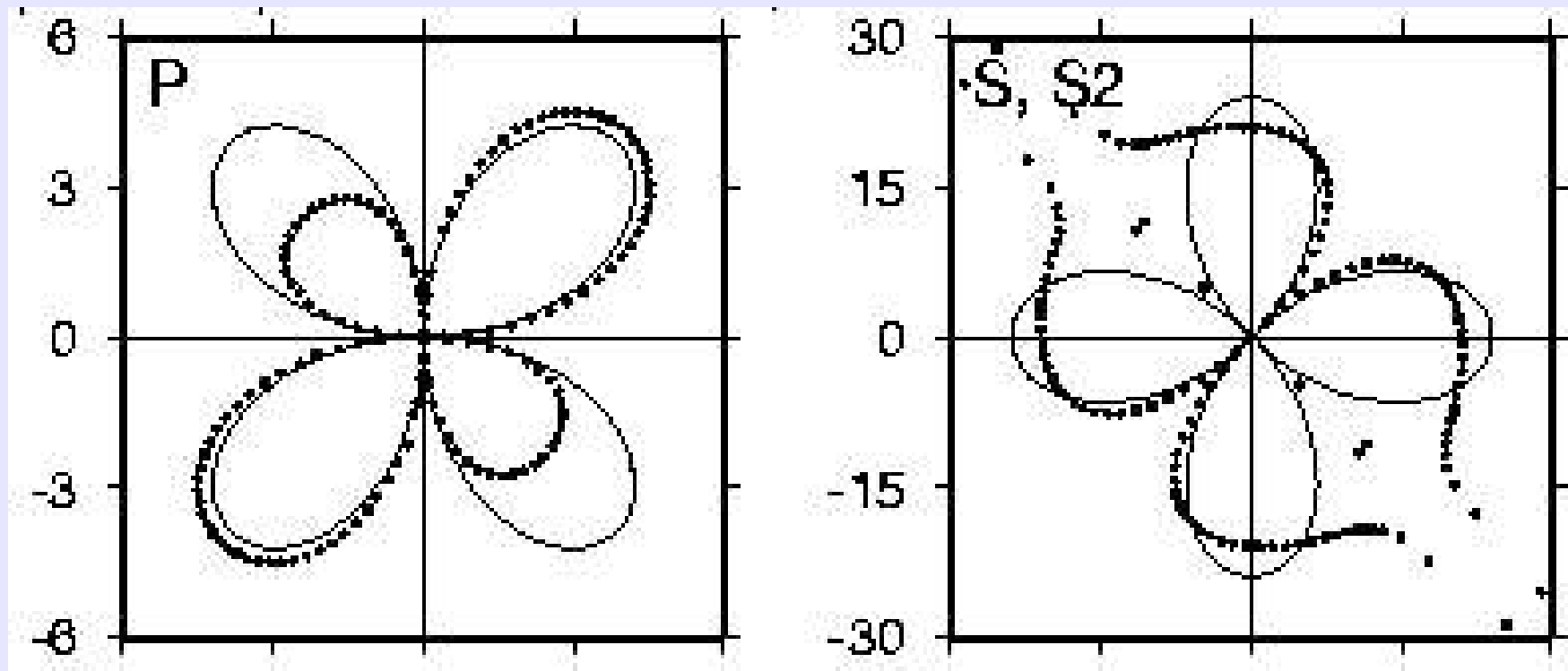
- transversely isotropic medium (10% anisotropy)
- rotate elastic tensor about 2 axes
- decompose **M** after rotation



Radiation Pattern

shear source: P+S waves

- dots: homogeneous, transversely isotropic (10%)
- line: homogeneous, isotropic

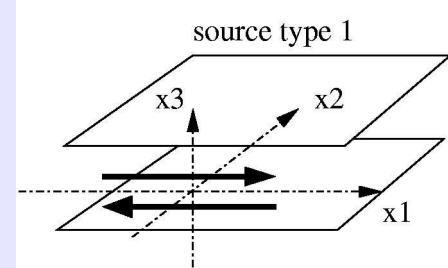


(Rößler et al., 2004)

Inversion – Green's Functions for isotropic Media

fault orientation

- synthetic event: pure shear fault
- medium in the source region: anisotropic
- synthetic waveforms: isotropic or anisotropic
- invert amplitude spectra assuming isotropic media
- use P/S waves (vertical/transverse components)
- grey: final solution, lines: solutions with similar misfit



isotropic

anisotropic waveforms

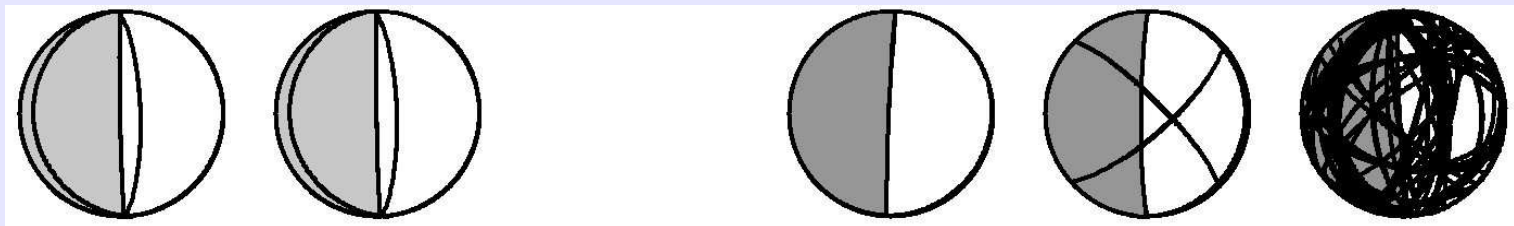
P

P+S

qP(Tr(M)=0)

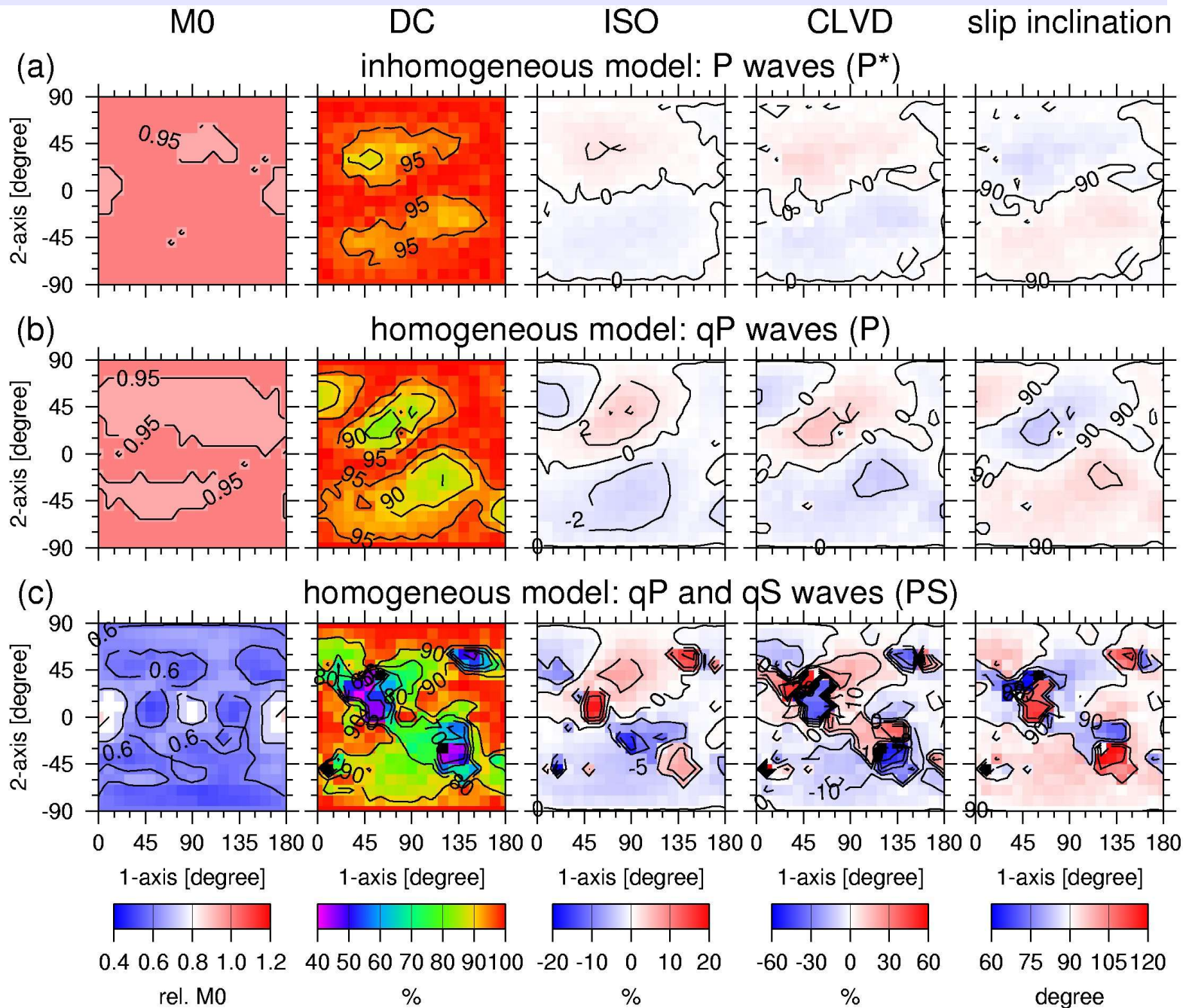
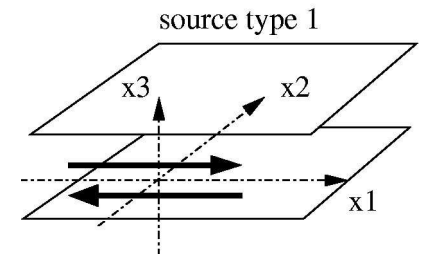
qP

qP+qS



The orientation of the fault plane can be derived even if anisotropy is neglected.

Inversion – Green's Functions for isotropic Media



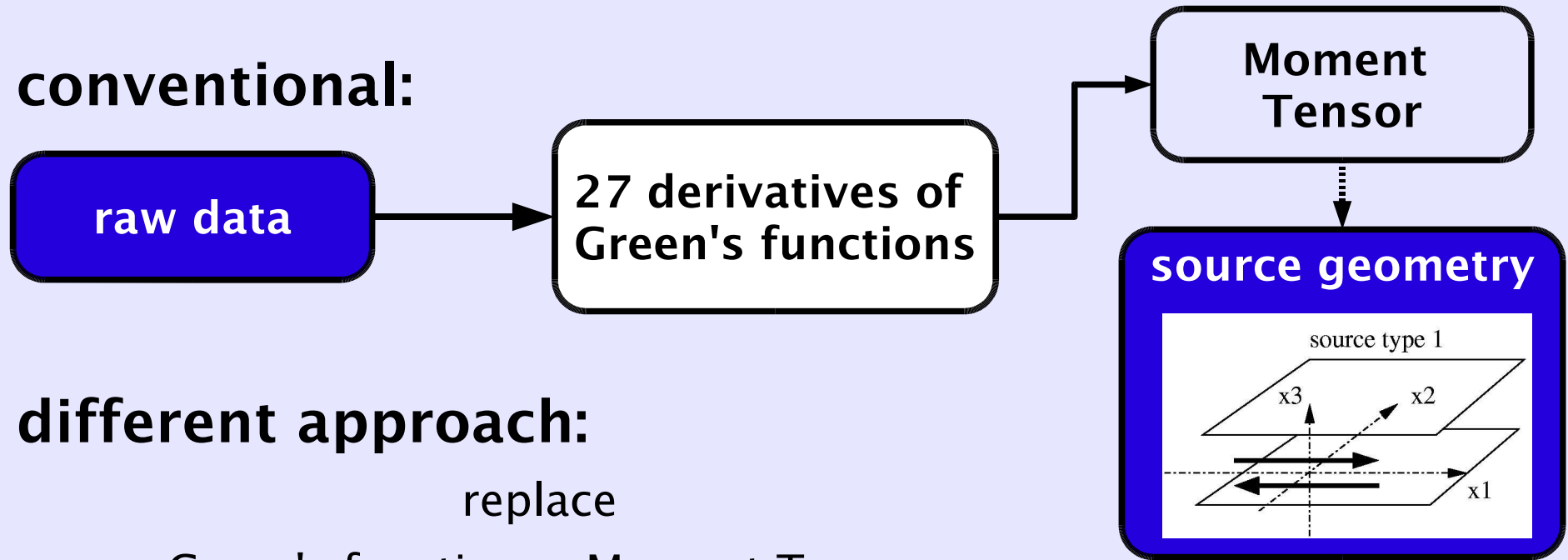
source region:
anisotropic
along ray path:
isotropic

source region:
anisotropic
along ray path:
anisotropic

source region:
anisotropic
along ray path:
anisotropic

Inversion – single Event

conventional:

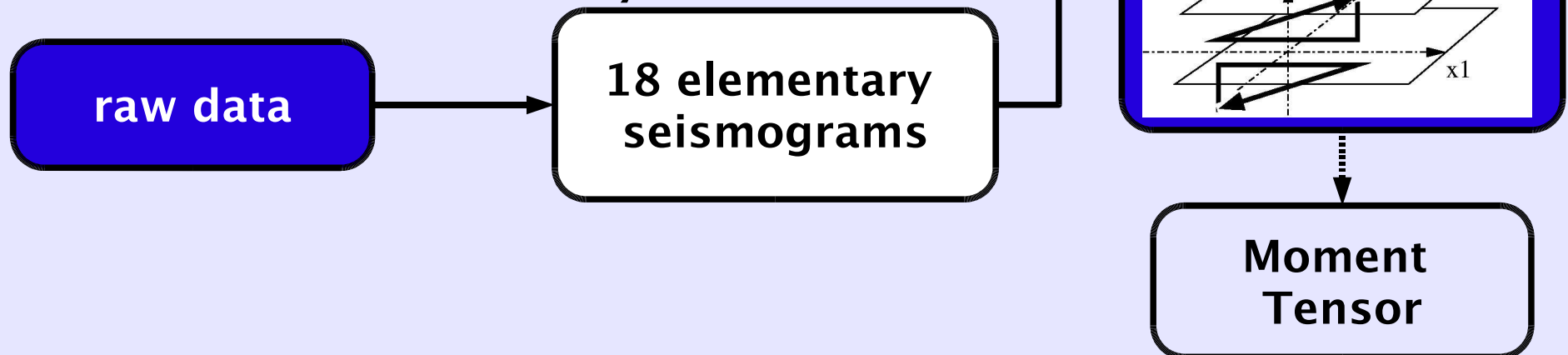


different approach:

replace

Green's function + Moment Tensor
by

18 elementary seismograms for anisotropic media
and 6 elementary sources



Inversion – single Event

Event – W-Bohemia:

16.10.2000, 15:51:24

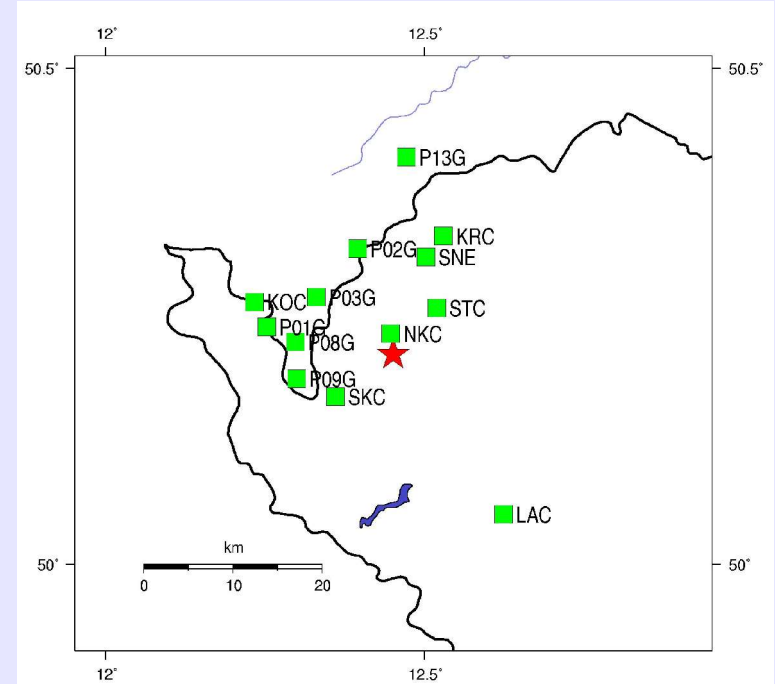
N: 50.21° E: 12.45° Z: 8.4 km

(courtesy to T. Fischer)

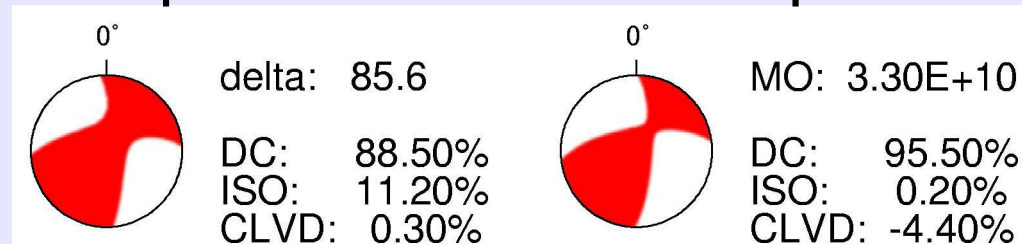
Inversion: 13 stations
anisotropic 1D model
(Vavrycuk, 1993)

Result (preliminary):

- $M_w \sim 1.1$
- N-S oriented near-vertical tensile event
- $\sim 86^\circ$ slip inclination (crack opening)
- volumetric effect is **NOT directly visible** in the moment tensor



P-radiation modelled from inverted source geometry for
isotropic anisotropic medium



Conclusion

- 1) anisotropy (source region, ray path):
 - influence on the moment tensor
 - may distort volumetric components (+/-)
 - influence on radiated amplitudes
- 2) neglect anisotropy during inversion:
 - error dependent on the strength of anisotropy
 - erroneous volumetric components
 - problems in interpretation
 - fault plane may be determined from qP ($\text{Tr}(\mathbf{M})=0$)
- 3) isotropic ray path:
 - similar patterns for synthetic and inverted MT
 - invert for elastic constants in the source region from $\mathbf{M}$

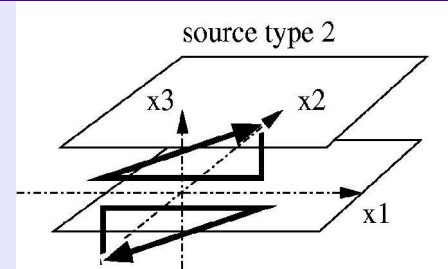
Conclusion

- 4) new inversion algorithm for **M**
 - invert directly for the source geometry
 - use elementary seismograms f. anisotropic media

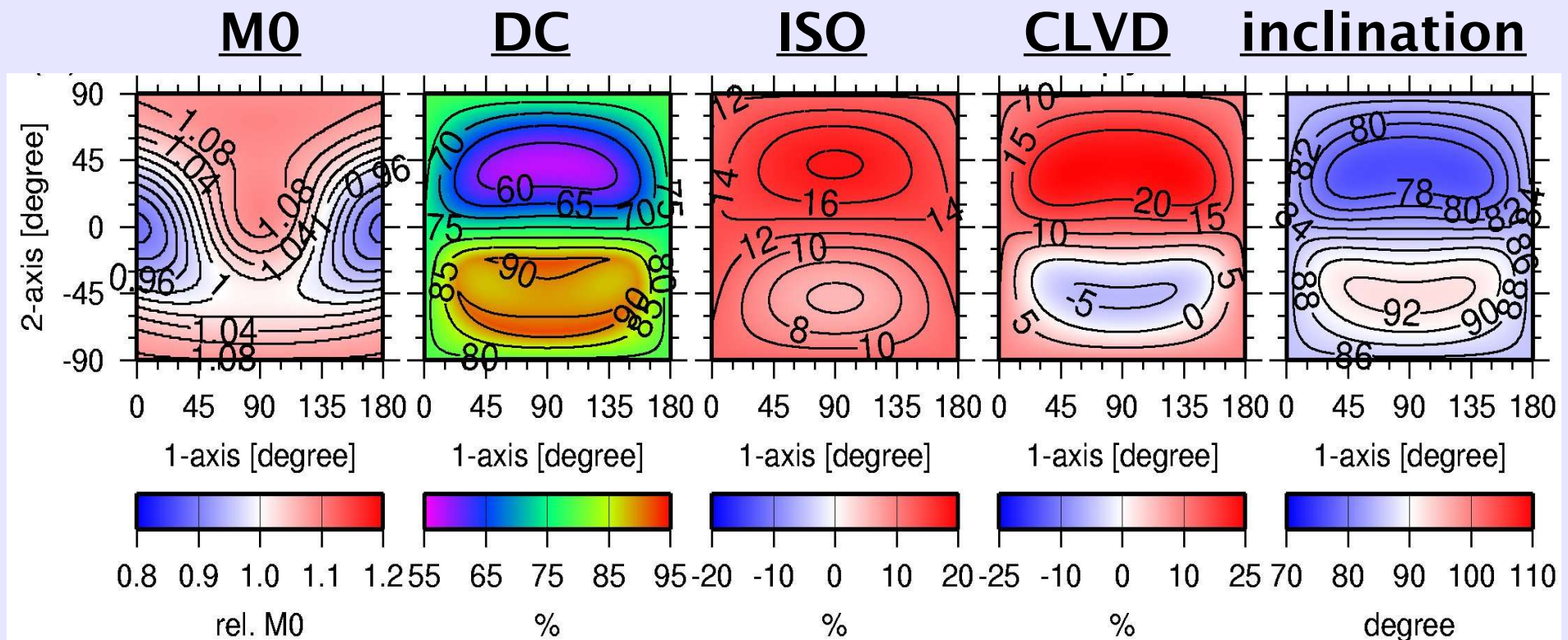
- 5) inverted event from W-Bohemia
 - show tensile faulting (4° opening)
 - vertical N-S trending fault
 - tensile character is not visible in **M!!!**

Appendix – Modelling

tensile source



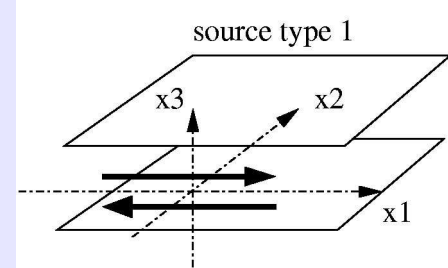
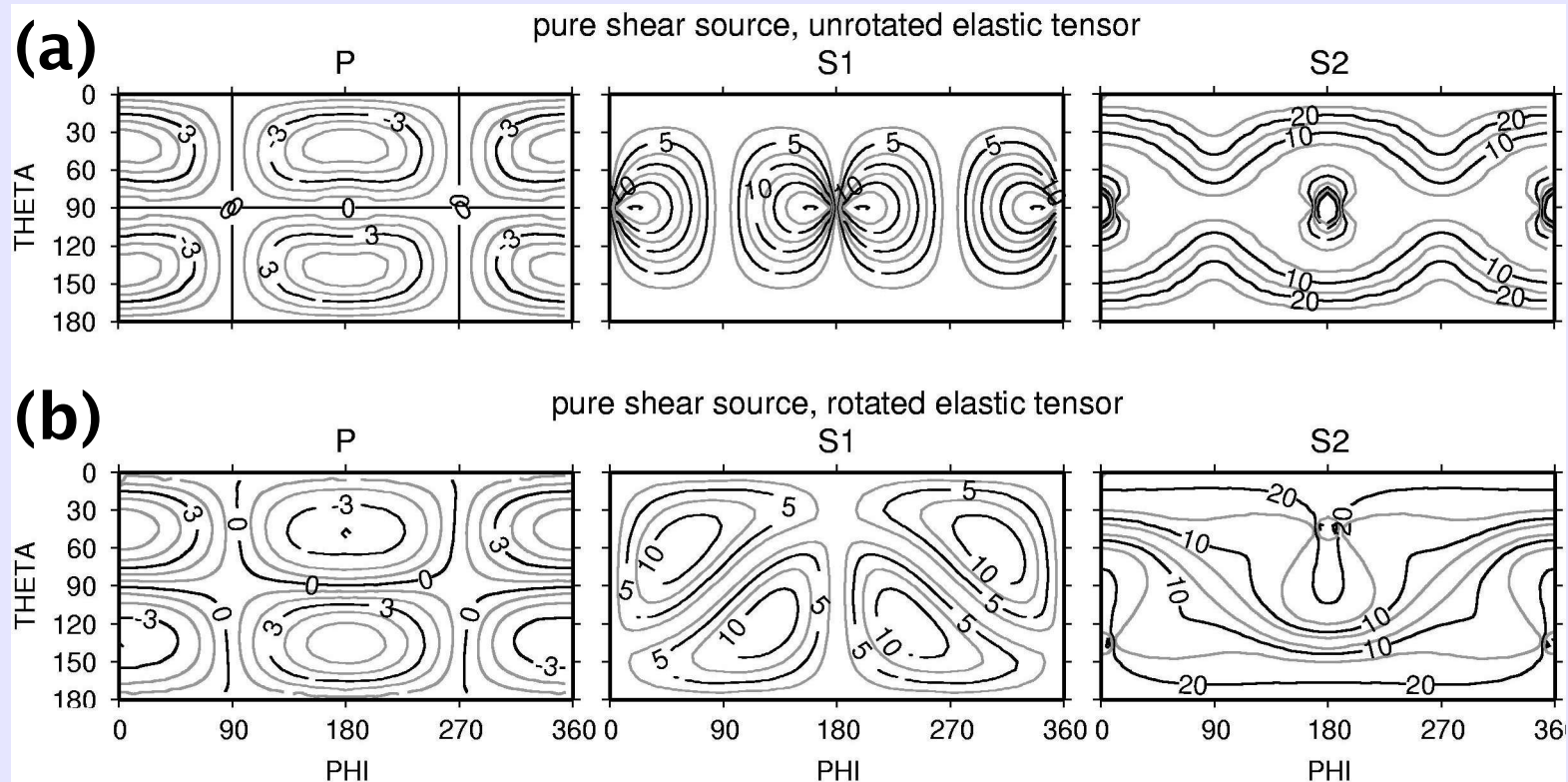
- transverse isotropic medium (10% anisotropy)
- rotate elastic tensor about 2 Euler angles
- decomposition for every set of Euler angles



Appendix – Radiation Pattern

shear source: P+S waves

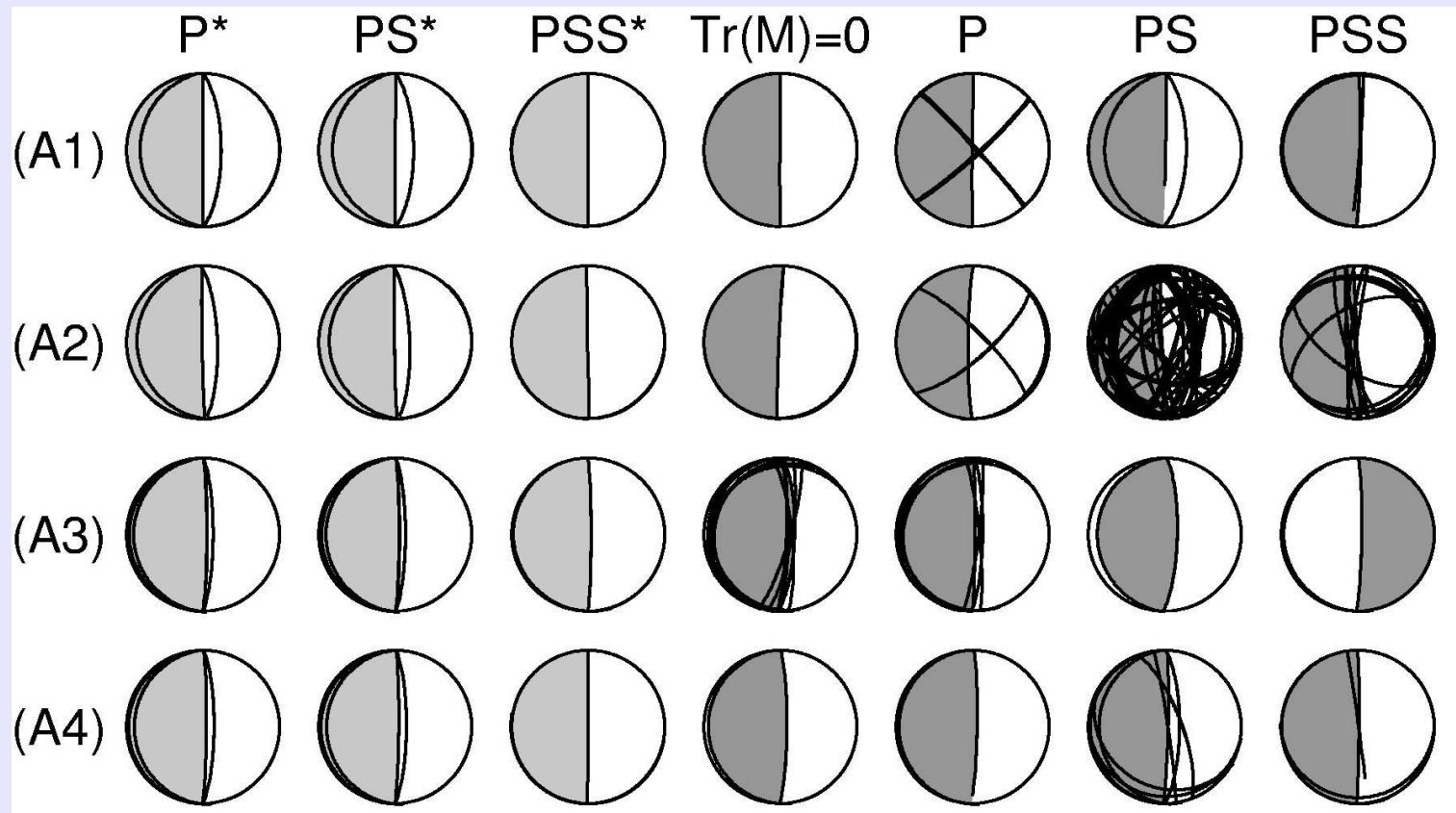
- homogeneous, transverse isotropic (10%)
- elastic tensor: unrotated (a)
rotated by 45° about x_2 (b)
- theta, phi: inclination, azimuth of the ray



Appendix – Green's Functions for isotropic Media

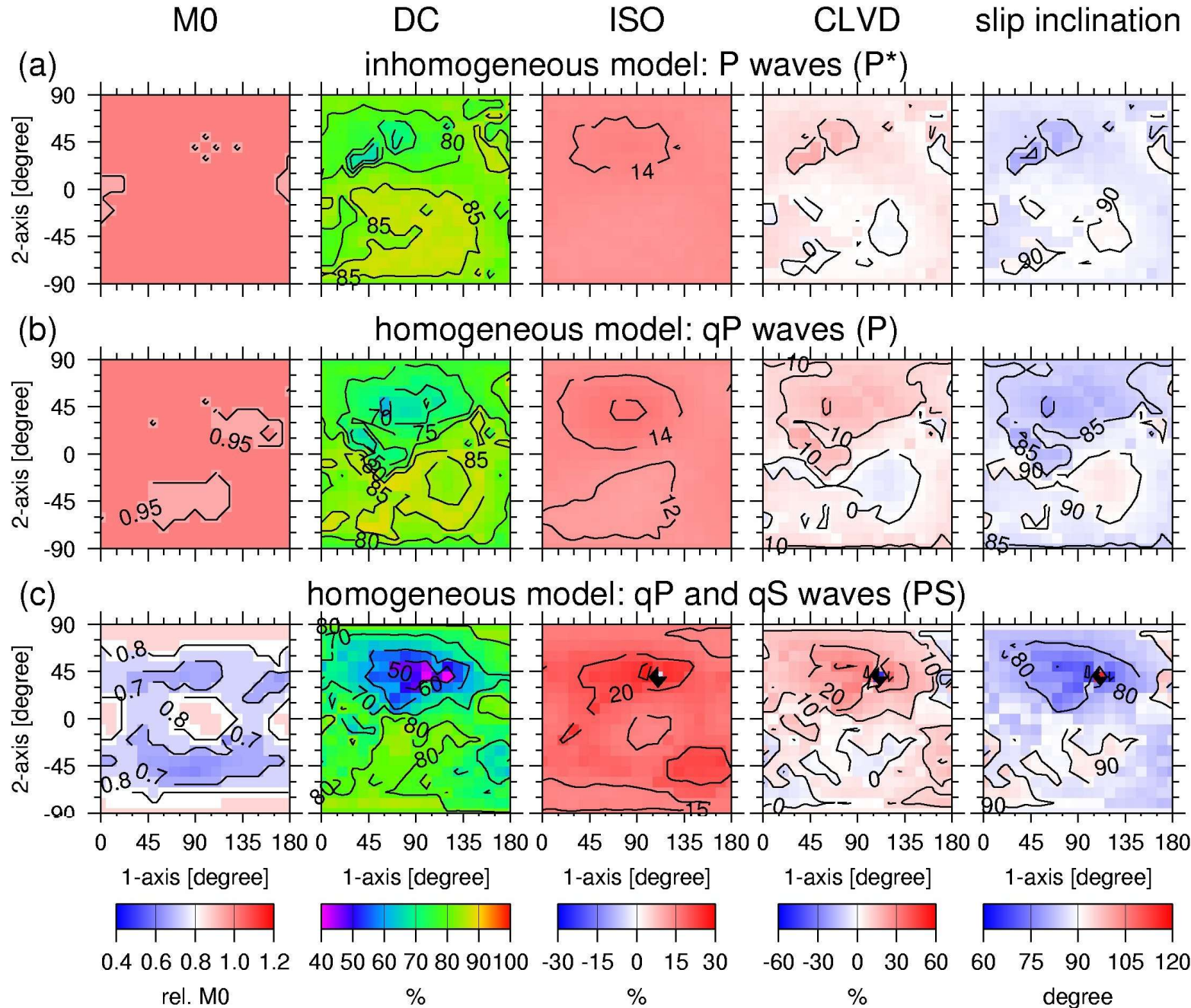
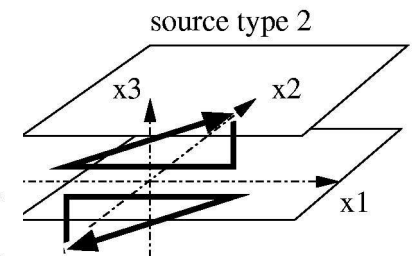
inverted focal mechanisms

- inverted from wave forms in anisotropic media
- shear source: A1,A2, tensile source: A3,A4
- fault normal || axis of symmetry: (A1), (A3)
- rotated axis of symmetry: (A2), (A4)



Appendix – Green's Functions for isotropic Media

Inversion, waveforms in rotated elastic media



source region:
anisotropic
along ray path:
isotropic

source region:
anisotropic
along ray path:
anisotropic

source region:
anisotropic
along ray path:
anisotropic

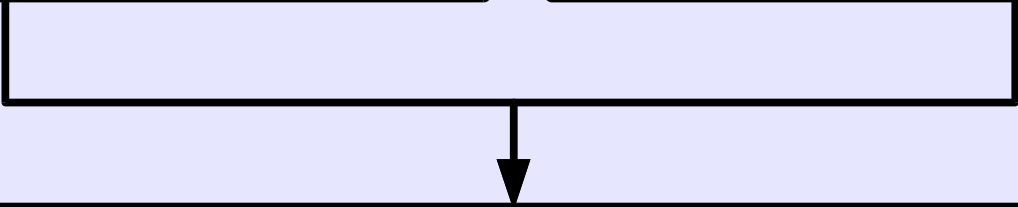
Appendix – Elastic Constants

Data:

- synthetic waveforms of events with different fault orientations
- noise added to data
- isotropic components of inverted Moment Tensors

Fit:

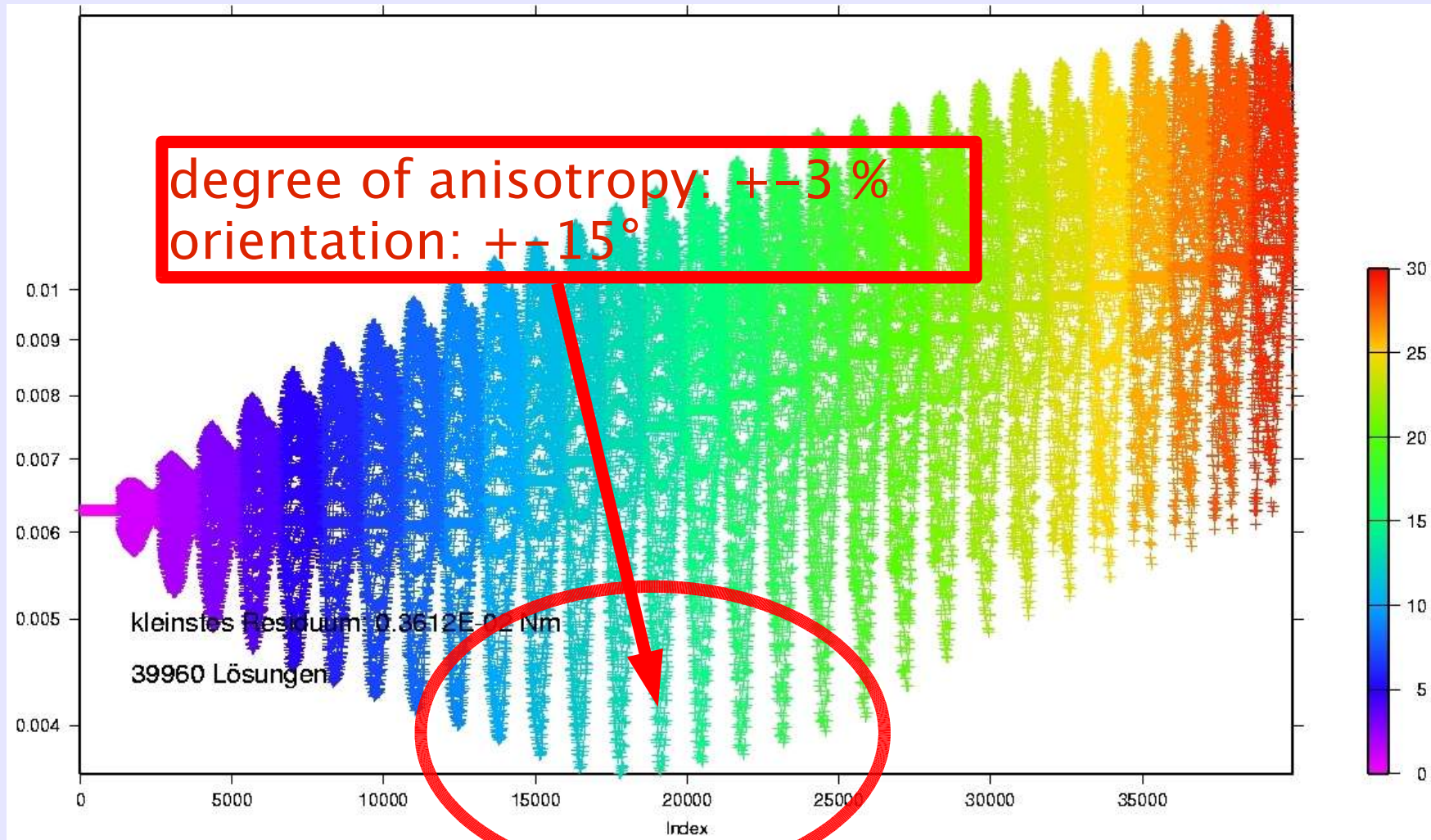
- assume pure shear fault
- correct fault orientation
- grid search for elastic constants
- fit isotropic Moment Tensor



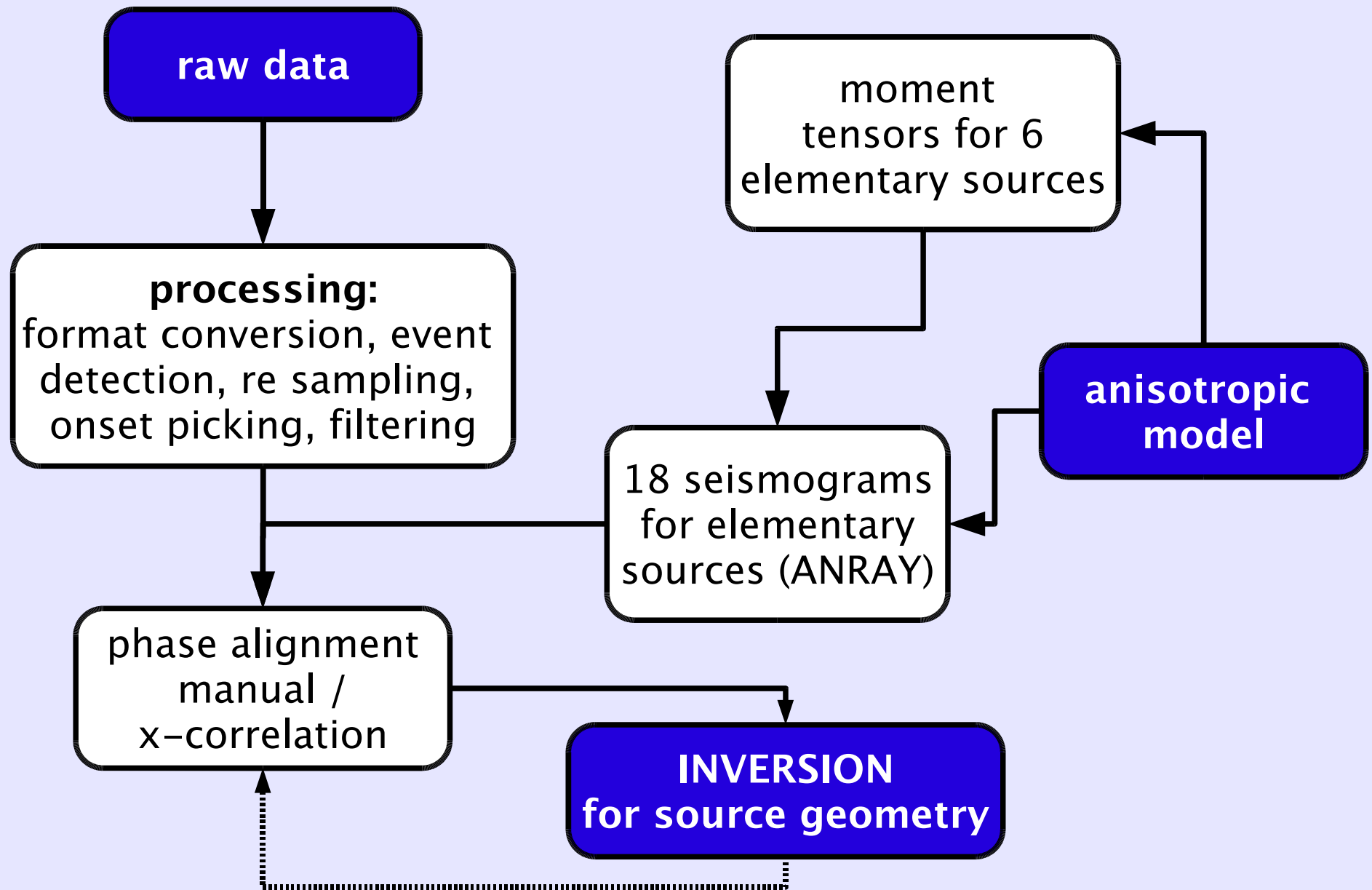
```
graph TD; Data[Data] --- Join[ ]; Fit[Fit] --- Join; Join --> Equation[Equation]
```

$$\mathbf{M}_{kl}^0 - \mathbf{s}_i \mathbf{n}_j \mathbf{c}_{ijkl} = \textit{Minimum}$$

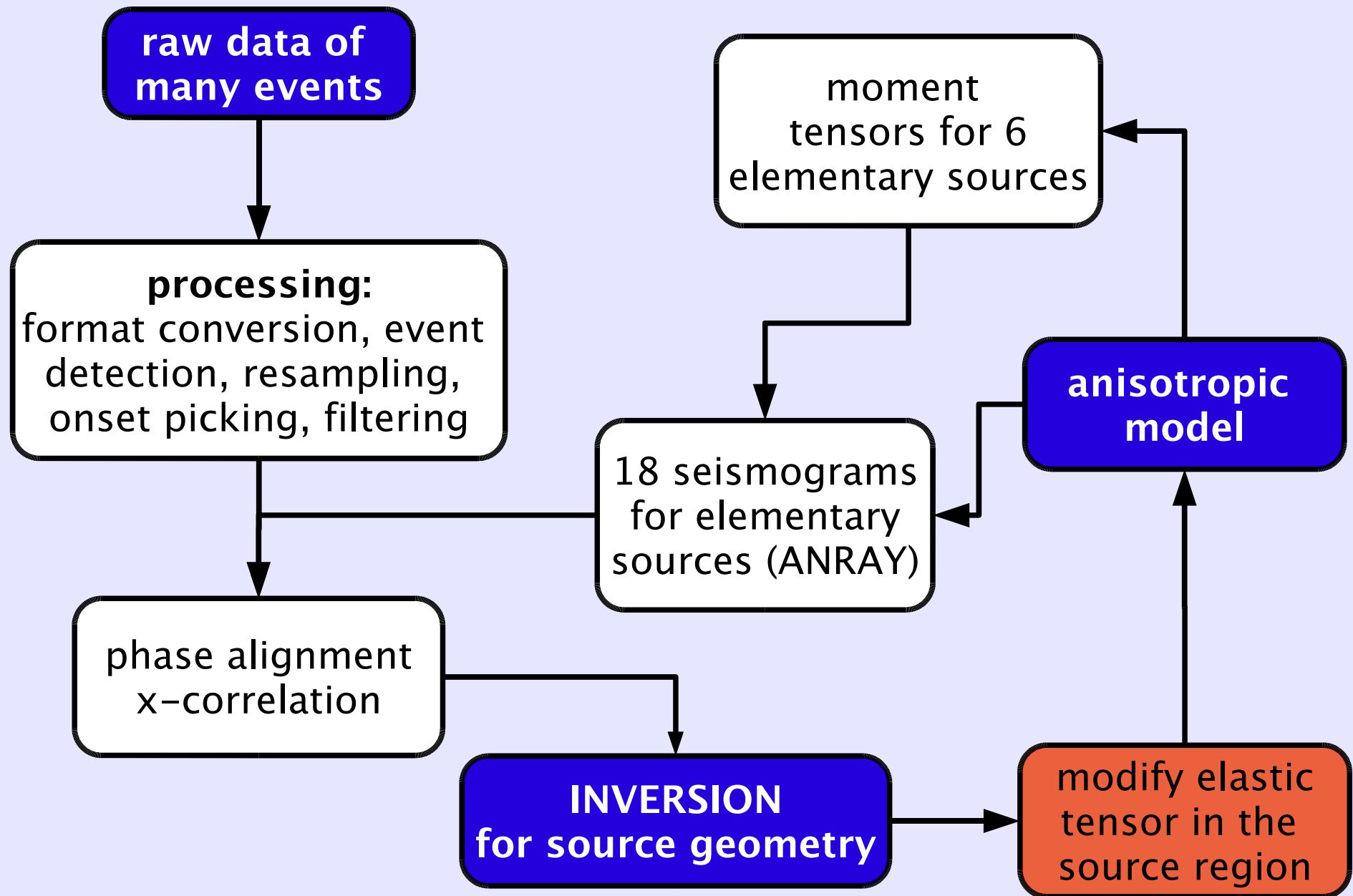
Appendix – Elastic Constants



Appendix – Inversion of single Event



Appendix – Inversion of multiple Events

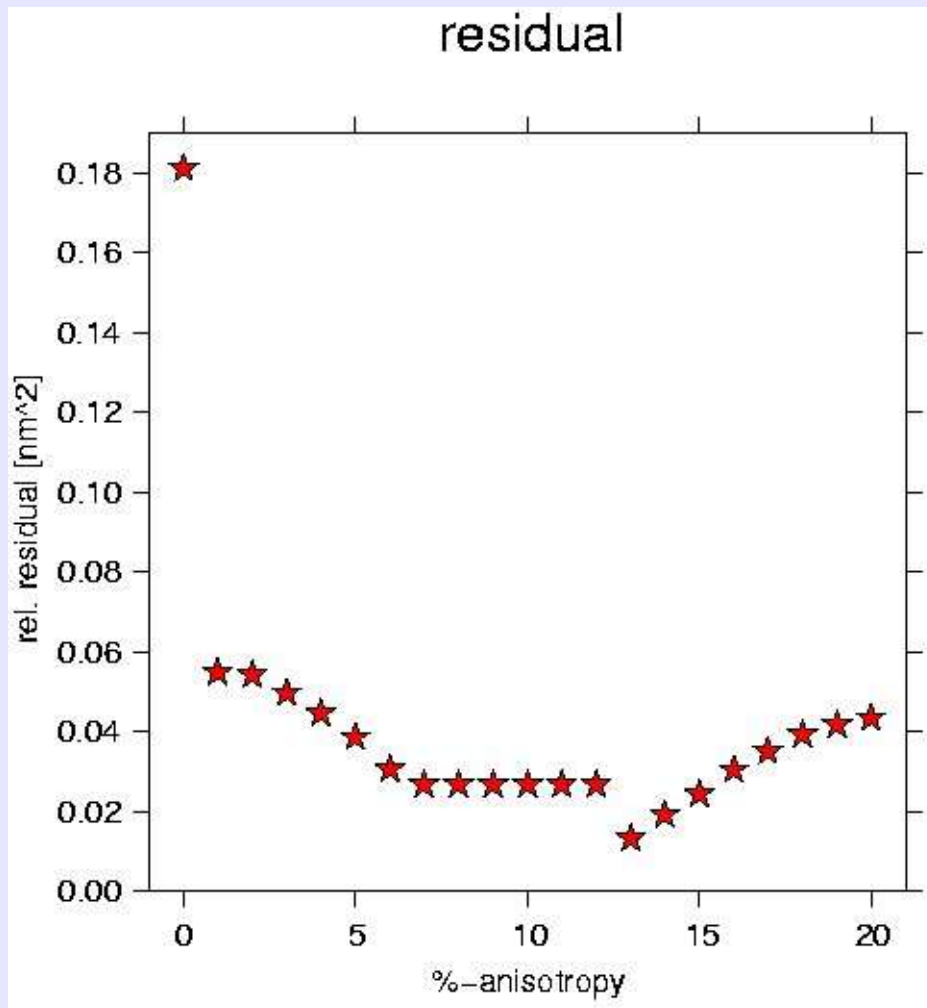


Appendix – Inversion of multiple Events

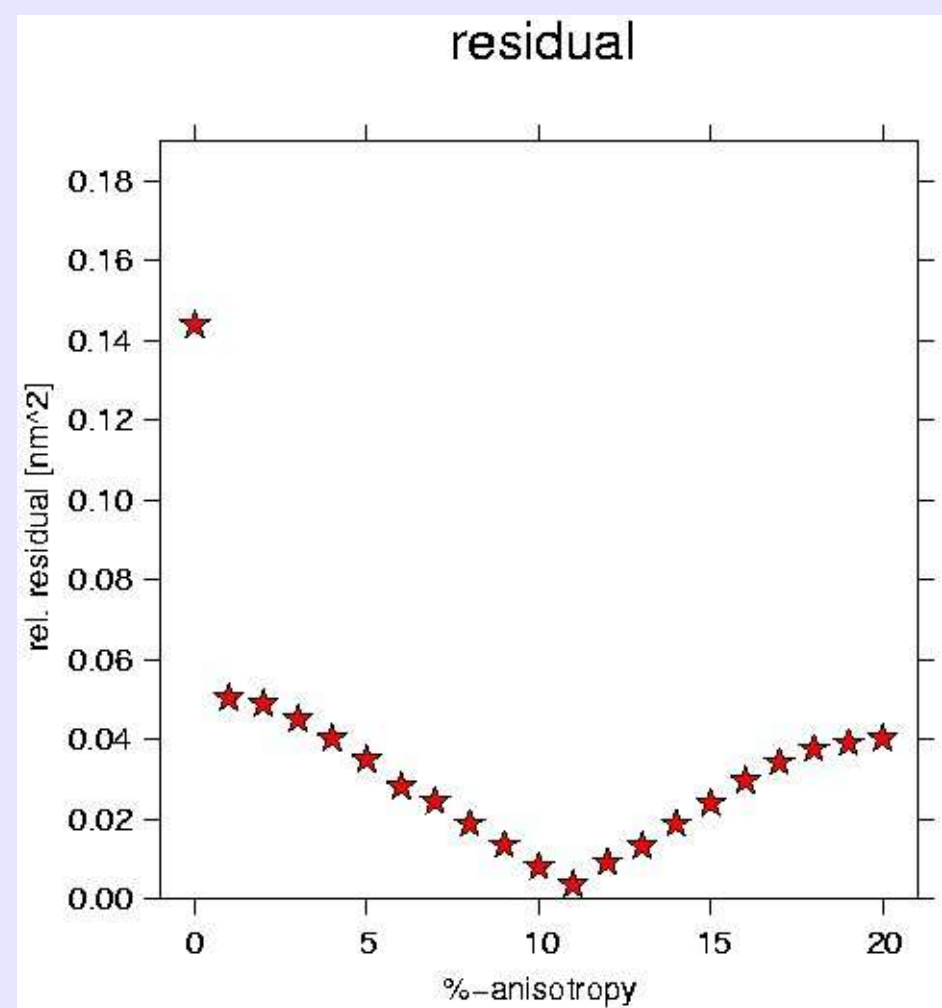
inversion for moment tensor and elastic constants

- variation of degree, elastic tensor remains unrotated
- true: 10% anisotropy

inversion of 1 event



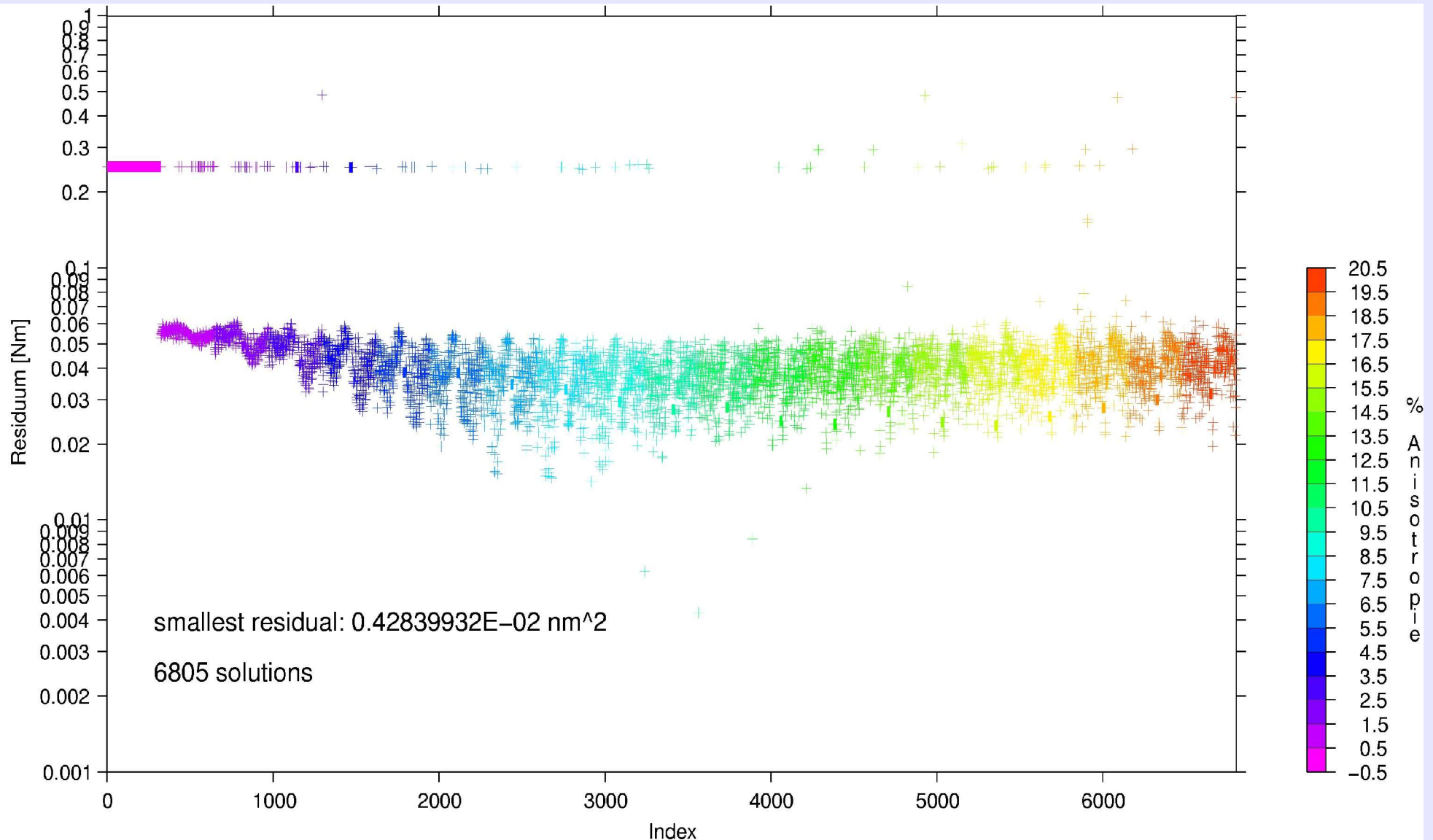
inversion of 3 events



Appendix – Inversion of multiple Events

inversion for moment tensor and elastic constants

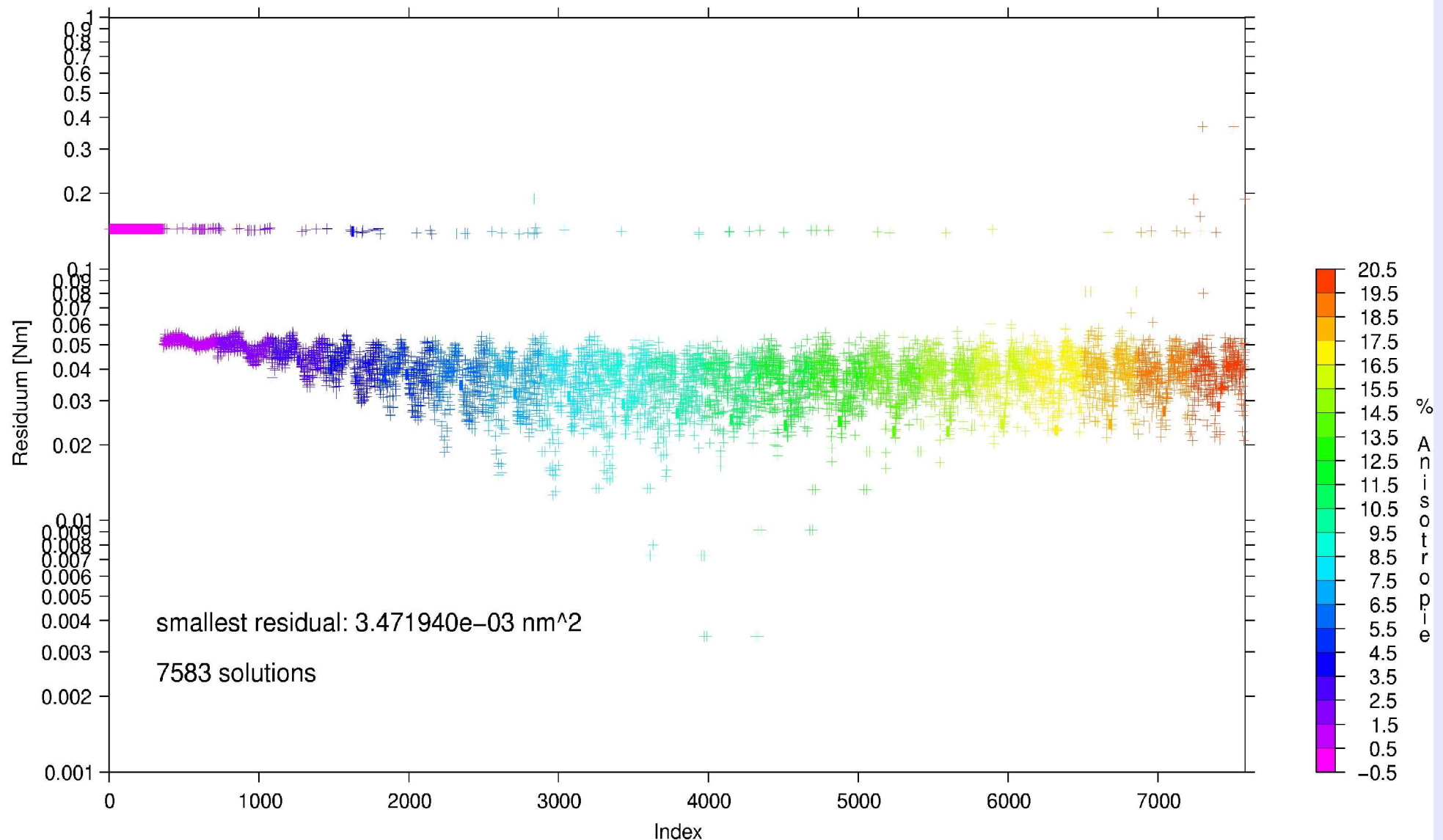
- variation of degree + orientation
- 1 event only, first trial, true: 10% anisotropy



Appendix – Inversion of multiple Events

inversion for moment tensor and elastic constants

- variation of degree + orientation
- 3 events, first trial, true: 10% anisotropy



Appendix – Description of Formulas

displacement:

$$\begin{aligned}u_i(\mathbf{x}, t) &= G_{ij,k} m_{jk} S f(t - \tau \mathbf{x}) \\&= A_i B_{jk} m_{jk} S f(t - \tau \mathbf{x}) \\&= Y_{il} \sigma_l f(t - \tau \mathbf{x})\end{aligned}$$

$$X_{i1} = A_i B_{11}, X_{i2} = A_i (B_{12} + B_{21}), X_{i3} = A_i (B_{13} + B_{31}),$$

$$X_{i4} = A_i B_{22}, X_{i5} = A_i (B_{23} + B_{32}), X_{i6} = A_i B_{33}$$

$$Y_{i1} = X_{i1} c_{1111} + X_{i2} c_{1112} + X_{i3} c_{1113} + X_{i4} c_{1122} + X_{i5} c_{1123} + X_{i6} c_{1133}$$

$$Y_{i2} = X_{i1} c_{1211} + X_{i2} c_{1212} + X_{i3} c_{1213} + X_{i4} c_{1222} + X_{i5} c_{1223} + X_{i6} c_{1233}$$

$$Y_{i3} = X_{i1} c_{1311} + X_{i2} c_{1312} + X_{i3} c_{1313} + X_{i4} c_{1322} + X_{i5} c_{1323} + X_{i6} c_{1333}$$

$$Y_{i4} = X_{i1} c_{2211} + X_{i2} c_{2212} + X_{i3} c_{2213} + X_{i4} c_{2222} + X_{i5} c_{2223} + X_{i6} c_{2233}$$

$$Y_{i5} = X_{i1} c_{2311} + X_{i2} c_{2312} + X_{i3} c_{2313} + X_{i4} c_{2322} + X_{i5} c_{2323} + X_{i6} c_{2333}$$

$$Y_{i6} = X_{i1} c_{3311} + X_{i2} c_{3312} + X_{i3} c_{3313} + X_{i4} c_{3322} + X_{i5} c_{3323} + X_{i6} c_{3333}$$

Appendix – Description of Formulas

source vector – 6 elementary sources:

n fault normal, **s** slip direction, S slip x area

$$\sigma = S (s_1 n_1, s_1 n_2 + s_2 n_1, s_1 n_3 + s_3 n_1, s_2 n_2, s_2 n_3 + s_3 n_2, s_3 n_3)^T$$

slip inclination (for pure shear $\sigma = \Pi/2$):

$$\cos(\delta) = \sqrt{\frac{\sigma_1^2 + \sigma_4^2 + \sigma_6^2}{\sigma_i \sigma_i - 2(\sigma_1 \sigma_4 + \sigma_1 \sigma_6 + \sigma_4 \sigma_6)}}$$

$$\delta = \arccos(\delta) \quad \text{for} \quad \sigma_1 + \sigma_4 + \sigma_6 \geq 0$$

$$\delta = \Pi - \arccos(\delta) \quad \text{for} \quad \sigma_1 + \sigma_4 + \sigma_6 \leq 0$$