

Statistical Features of Seismic Noise-Field

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Abstract

We present a preliminary study of the dependence of the fractal features of the soil motion due to seismic noise on the near-surface geology in the frequency range from 1 Hz to ~ 40 Hz. In detail, we have investigated the 3-d average squared soil displacement $\langle r^2 \rangle$ and the distribution function of the displacement fluctuations in different geological sites. The anomalous scaling of the average squared displacement $\langle r^2(\tau) \rangle \sim \tau^\alpha$ with $1.5 < \alpha < 2$, and the Gaussian shape of the fluctuation distribution function suggest that the soil motion under the influence of seismic noise is consistent with a persistent fractional brownian motion (fBm) characterized by a Hurst exponent $H > 1/2$. Moreover, a dependence of the α exponent on the different geological features of the sites has been found, and it is discussed.

1 - Introduction

Microtremors have been studied, and still are widely used, mainly because they

exhibit a correlation with the near-surface geological features. Such a correlation is very important in site effects estimation as well as in seismic risk studies. The correlation between seismic noise and soil conditions is investigated mainly using microtremors spectra. Such studies are approached in different manners to provide some information on the site response (see the review of *Bard, 1999* and references therein): using absolute spectra or spectral ratios with respect of a reference site or spectral ratio of horizontal over vertical component of a single station (H/V spectral ratio).

All of them are based on physical hypothesis concerning both the nature of the noise wavefield and its spectrum. Such assumptions have been slightly modified, starting from the original idea of *Kanai (1983)*, on the basis of studies and analysis of microtremors recorded worldwide. However, whatever is the origin of the noise wavefield, the idea that noise spectra are linked with the resonance frequencies of the sites under study is widely accepted for long period range ($T > 1$ s) while for short periods ($T < 1$ s) such link depends on the value of impedance contrast. Indeed, when this latter is high ($T > 3$) big amount of energy is trapped in terms of body or surface waves and in the spectra are much evident resonance peaks (*Zhao et al., 1998*). On the contrary, when the impedance contrast is low the energy trapping process is not so efficient.

From this emerges that in order to better assess the site response starting from seismic noise, is very important the interpretation of peaks in noise spectra. *Nogoshi and Igarashi (1971)* were the first researchers that introduced the H/V

spectral ratio. They showed its relationship with the ellipticity curve of Rayleigh surface waves. To estimate the fundamental frequency in the H/V ellipticity curve, big amount of work has been done (see *Chouet et al.*, 1992; *Lachet and Bard*, 1994; *Konno and Ohmachi*, 1998; *Fäh et al.*, 2001 among others) from which emerges the importance to quantify the proportions of body waves and surface waves present in the noise wavefield. Moreover, from such studies emerges also that to relate the ellipticity curve to the measured peaks in the H/V spectra, is very important to quantify the proportion between Rayleigh and Love waves and whether it is frequency independent or not.

The aforementioned picture tells us the importance of studying spectral properties and polarization of microtremors in order to infer site response features from microtremors analysis. It must be noticed however, that there are still several open questions. For example is still debated the question whether exists (*Lachet and Bard*, 1994) or not (*Nakamura*, 2000) the contribution of the surface waves to the H/V ratios at the fundamental resonant frequency in unconsolidated sediments. And also, it has been observed for several sites that the H/V curves can not be explained in terms of Rayleigh wave ellipticity, as reported in the review of *Bard* (1999). Moreover, H/V spectral ratio technique is widely used to estimate resonant frequencies and amplification factors although it is not justified on theoretical ground, as underlined by *Konno and Ohmachi* (1998). It has been observed just some local relationship between peak H/V amplitude and the maximum spectral amplification; such relationship is valid for limited areas only and it has been

derived empirically.

Even if *Bard* (1999) suggests to investigate such aspects through measurements of relative proportions of surface waves and body waves, as well as *Rayleigh* and *Love* waves, in the noise wavefiled, we decided to analyze microtremors from a different point of view. Indeed, in place to have the previous *deterministic* starting point about seismic noise, we approached the problem from its *stochastic* side to investigate the physics hidden in noise field. That is, we investigate its fractal features and if exists some link between these latter and site conditions. For such porpouse we don't need any deterministic hypothesys, that must be justified *a posteriori*, neither on the nature of the seismic noise nor on its spectrum.

Even though that a pervasive problem in noise has been (and still is) to explain spectral peaks, which now are used to invert for structure by computing the phase velocity of surface waves in noise, in this paper we did not analyze peaks spectra. As well as we did not perform any array data processing to tray to understand if waves are surface or body ones. This is out of the scopes of this paper.

Here, we present a preliminary study of the fractal features of soil motion under the effect of seismic noise in the frequency range from 1 Hz to 40 Hz in different geological sites, showing the existence of a clear dependence of the fractal nature of seismic noise on the near-surface local geology. Anyway, we want to remark that even though, in seismology, the investigation of noise features is widely used for seismic hazard purpose (e.g., site effects, microzonation, seismic risk, etc.) (*Bard*, 1995, 1999; *Seo*, 1997; *Fäh et al.*, 2001), this is beyond the scope of this

paper.

2 - Data Description and Analysis

Noise data, used in this work, have been collected during a measurement campaign in three sites in Umbria (Central Italy): the Verchiano valley (Gaffet *et al.*, 2000), the Nocera Umbra site (Caserta *et al.*, 2000) and the Colfiorito plain (Rovelli *et al.*, 2001). All these sites are well known and have been widely studied during the 1997 Umbria-Marche seismic sequence. Moreover, they represent prototypes of a same geological structure consisting of three different prototypical zones: an unconsolidated soil - alluvial deposit - zone (A), a firm soil - marl sandstone - zone (C), and a transition - colluvial and debris deposit - zone (B) in between. As a matter of fact, these sites are Quaternary intramontane basins located in the Northern Apennines arc, which were created by extensional tectonics related to the opening of the Tyrrhenian Sea (Calamita *et al.*, 1994). In detail, they consist in a valley filled up with soft sedimentary of alluvial origin dominated by hills consisting of consolidated soil. Seismic noise measurements have been performed at different locations in each site (see Table 1) using a portable seismic station composed by a Marslite digital recorders equipped with a 3d 5-second seismometer (Lennartz-3D75s). Data have been collected with a sampling frequency of 256 Hz taking care to free field conditions and refer to the velocity components $v(t)$ of the

ground motion. Due to the response function of the seismometer, which is linear in the frequency range $1 \div \sim 40$ Hz, we will limited our study to the range of time scales from 1 s to 0.028 s.

For each set of seismic noise records we selected 5-minutes characterized by the least variance. This choiche has been done in order to select, as much as possible, time intervals dealing with free-field conditions, i.e. contaminated by anthropic activities as less as possible. We remark that in the present work, being not interested in the study of the relative content of body and surface waves, that may be hidden in the noise field, we will not analyze different segments of noise record with different variance values. On the contary, we look for time intervals with least variance to avoid bumps and/or spikes in the signal. Indeed we have seen, from data, that these latter are so enegetic to hide the statistical features of the noise field.

By means of velocity measurements, we have evaluated the three dimensional (3d) soil displacement vector simply integrating the measurements:

$$x_i(t) = \int_0^t v_i(t') dt' \quad (1)$$

where x_i and v_i are the i -th component of the ground displacement and velocity, respectively. We consider the 3d soil displacement because our interest is to study the global soil motion under the effect of seismic noise, instead of considering and comparing the soil motion in each component separately (H/V spectral ratio, etc.), as is in the standard seismological approach to noise-field studies. Figure 1 shows

the behavior of the 3D soil motion in the case of two different records, V1 and V5 (see Table 1), relative to different geological prototypes. Note the different nature of the short-time scale motion. The higher roughness of the trajectory in the case of unconsolidated soil (V1 in Figure 1) suggests a different diffusive nature of soil motion in alluvial deposits in respect to marl sandstones.

To better characterize such features of the soil motion, we investigate the fractal features of the 3d soil motion. Although the standard procedure to investigate fractal properties of time series is to look at the power spectrum features, this way to proceed is often biased by artificial effects when we are interested in determining the spectral exponent. To overcome such difficulties we use the well-known *variogram analysis* (Matheron, 1963) that is less affected by artefacts (Hergarten, 2002). The *variogram* quantifies the dependence of the variance $\sigma^2(t)$ on time t , and can be thought as a generalization of what is usually done to study Brownian motion which is the simplest example of a fractal motion. In other words, the *variogram* allows to quantify the temporal correlation describing how the probability density spreads through time.

Here, we have generalized the variogram analysis to the 3D case by evaluating the average squared soil displacement $\langle r^2(\tau) \rangle$, which has been defined according to the following expression:

$$\langle r^2(\tau) \rangle = \langle \sum_{i=x,y,z} [x_i(t+\tau) - x_i(t)]^2 \rangle \quad (2)$$

as a function of the time scales τ in the temporal range from 0.028 s to 1 s. Here,

$\langle \cdot \rangle$ indicates time averaging. In the case of a random walk diffusion process with a zero average position, the quantity defined in (2) is equivalent to the variance, thus providing a measure on how the soil diffuses under the influence of noise-field (ref). In the case of a Brownian motion, the average square displacement is expected to scale with time delay τ as follows (ref):

$$\langle r^2(\tau) \rangle \propto \tau^\alpha \quad (3)$$

with $\alpha \equiv 1$. If $\alpha \neq 1$, we deal with a sub-diffusive ($\alpha < 1$) or super-diffusive ($\alpha > 1$) process, i.e. with an *anti-persistent* or a *persistent* motion, respectively (Hergarten, 2002). We remind that $\alpha = 2$ indicates a ballistic motion (like $x(t) = vt$) (Sornette, 2000).

Figure 2 shows the behavior of $\langle r^2(\tau) \rangle$ as a function of τ in the case of three different locations, investigated in our work. The average square soil displacement ever follows a power-law $\langle r^2(\tau) \rangle \sim \tau^\alpha$ characterized by an exponent $\alpha > 1$ (see Table 1). This is a common feature in all the different sites here investigated, and is evidence of self-similarity of the soil motion in the studied time interval (Mandelbrot, 1983; Sornette, 2000). The occurrence of a self-similarity feature in the soil motion might be read as the signature of a scale-invariance in the noise-field at least in the range of scales investigated. As a matter of fact, being the soil motion a proxy of the noise field, we can link the soil motion features to the noise-field ones. Thus, this result suggests that the noise-field is scale-invariant in the investigated range of time scales, and that this property is independent on the

near-surface geological conditions, although different sites maybe characterized by different spectral slopes. Furthermore, being our measurements not simultaneously taken, the observed scale-invariance feature does not depend on time.

Moreover, being the scaling exponent α linked with the well-known Hurst exponent H by the relation $\alpha = 2H$, it follows that $H > 0.5$ in all the geological situations under investigation. We remind that the Hurst exponent H provides a measure of the *smoothness* of a fractal time series. H values greater (or less) than 0.5 means that the time series covers more (or less) distance than a random walk, i.e. it provides a measure of the relative persistence of the motion in respect to the Brownian motion which is characterized by $H \equiv 0.5$. In our case, being $H > 0.5$ the soil motion is more persistent than a Brownian one. Thus, the main feature of the ground motion is that of a correlated random motion. Furthermore, a dependence of soil motion fractal features on the geological location zone is found as it emerges from the results on the scaling exponent α reported in Table 1.

To better underline such a dependence we have evaluated the average scaling exponent $\langle\alpha\rangle$ for the three main different geological prototypes and reported the results in Table 2. Different geological prototypes seems to be characterized by similar and well-distinct values of $\langle\alpha\rangle$; i.e. the fractal features of the soil motion due to seismic noise depend on the nature of the geological zone. Moreover, while the soil motion in the firm soil zone (C) tends to be ballistic ($\langle\alpha\rangle \sim 2$), in the soft unconsolidated soil zone (A) it has a more diffusive character ($\langle\alpha\rangle < 2$). Here, the term *ballistic* refers to a motion where the walker moves with a given velocity

instead of randomly hopping at each time, and which is characterized by $\alpha \equiv 2$. Thus, the more the soil is consolidated, the more the behaviour of the soil motion is far from the Brownian one. This property is also evident from the roughness of the 3D motion reported in Figure 1.

Although it is well known that different geological structure affects the site response, to our knowledge our results are the first evidence of a link between the fractal features of the ground motion under the influence of seismic noise and the soil geological features. We believe that such a different fractal nature, as evidenced by α values, could arise from the effects both of a different dynamical and/or structural heterogeneity (i.e. local small-scale inhomogeneities and/or a randomness of soil properties in the first hundreds of meters) of the soil layers near the Earth's surface (*Menke et al.*, 1990; *Hough and Field*, 1996), and of the geological site conformation and structure on the noise perturbation transport. As a matter of fact, it has been well established that irregular changes of the soil structure in the form of inhomogeneities affect the ground motion (*Manolis*, 2002; *Safak*, 2001). In order to better characterize the nature of the ground motion we analyzed the probability distribution function (PDF) of the displacement fluctuations. Figure 3 shows the normalized PDFs of the displacement fluctuations at the smallest scale investigated ($\tau = 0.028$ s) for the three geological locations reported in Figure 1. For each geological location the PDF has been averaged over the 3 components. The shape of PDFs is well in agreement with a Gaussian distribution. The emerging physical framework is similar to that of a fractional Brownian

motion (fBm).

To confirm such a picture we have simulated long range correlated random 3d signals characterized by different Hurst exponent H using the successive random additions method as described in *Mandelbrot (1983)*. Figure 4 shows the results obtained in the case of three different simulated signals characterized by different values of the Hurst exponent H ($H = 0.2, 0.5$, and 0.8). These self-similar signals clearly display scaling features for the average squared displacement $\langle r^2(\tau) \rangle$, and a Gaussian statistics for the small-scale fluctuations (see Figure 4 bottom panel). This result on the statistical features of simulated 3d fBm strongly suggests that the physical features of the soil motion under the influence of the seismic noise are equivalent to those of a fBm motion.

3 - Discussion and Conclusions

In this work we presented a comprise of experimental observations and data analysis dealing with the fractal features of seismic noise. Although the analitical tools are standard, our results have evidenced the existence of a possible relationship between the morphology of the near-surface geology and the statistical features of the seismic noise field.

In summary, the study of the 3d average squared soil displacement $\langle r^2(\tau) \rangle$, and of the Pdfs of the displacement fluctuations has evidenced the existence of self-

similar and fractal features analogous to that of a persistent fractional Brownian motion ($H > 1/2$). Moreover, a dependence of the fractality of the soil motion from the geological nature of the site has been shown to exist. In detail, while in consolidated soil zone the seismic noise soil motion has a nearly ballistic character ($\alpha \sim 2$), for unconsolidated soil zones the motion tends to be more diffusive ($1 < \alpha < 2$). It could be conjectured that these results on fractal features are widely known in seismology. Anyway, we remark that even though this is true in the case of earthquakes (see *Turcotte*, 1997, among others), less is known about the seismic noise soil motion. It is, indeed, widely recognized that seismic noise radiation has different features from earthquakes. However, nowadays it has been understood that the study of seismic noise is extremely relevant to recover some information on the site responses to earthquakes (see *Bard*, 1999 and references therein).

Regarding the observed dependence of the soil motion fractal features on the near-surface geology, although we cannot exclude that some of the observed differences might arise from factors like wave focusing, basin geometry, and topography, we believe that our results could be better read in terms of transport of seismic radiation in random media (*Zhang et al.*, 1999), i.e. in terms of strong nonlinearity and randomness of soil media (*Menke et al.*, 1990; *Hough and Field*, 1996; *Safak*, 2001; *Manolis*, 2002). As a matter of fact, assuming that the nature of the seismic noise source in the different sites would be statistically the same, the observed differences should be due to a different degree of dynamical and/or structural het-

erogeneity of the geological prototypes. In other words, the presence of a different degree of the structural heterogeneity would involve different multiple scattering in the seismic noise transport changing the character of the motion from nearly ballistic to diffusive. Recent studies have clearly shown that the effects of randomness on seismic waves transport may be treated in terms of nonlinearity involving scaling and dispersion in the soil motions, and that the soil stochasticity must be considered to describe the motion at high frequencies ($f > 1$ Hz) (*O'Connell*, 1999; *Safak*, 2001; *Manolis*, 2002). We remark that these findings should be taken into account in numerical modeling of seismic noise wavefield. In other words, in modeling the seismic radiation transport through the near-surface geology one should consider the effects of the local small scale heterogeneity and disorder by taking into account the afore-mentioned dependence of fractal features of the soil motion on the soil conditions. This will be the field of future observational and/or computational works.

Acknowledgments

The authors thank Dr. Antonio Rovelli for the useful discussions and criticisms. Special thanks to Prof. E. Boschi of the of the Istituto Nazionale di Geofisica e Vulcanologia (Italy) for supporting this work. G.C. is grateful to the Italian CNR for the financial support.

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