ELECTRICAL CONDUCTIVITY OF THE DEEP MANTLE

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Introduction

Inverse problem in time-domain global EM induction: Adjoint approach

1-D inversion of CHAMP satellite data

Sensitivity of EMI data to 3-D conductivity in D”

Conclusions

Outline

Introduction

- Earth's magnetic field
- EM induction: Basic principles
- EM induction: Scales and frequencies
- EM induction on global scale
- Time changes of geomagnetic field
- Magnetosphere
- Ionosphere
- Use of observatory and satellite data for induction studies
- Recent results
- Geomagnetic jerks and core-mantle coupling
- Phase transitions in the lower mantle

Inverse problem in time-domain global EM induction: Adjoint approach

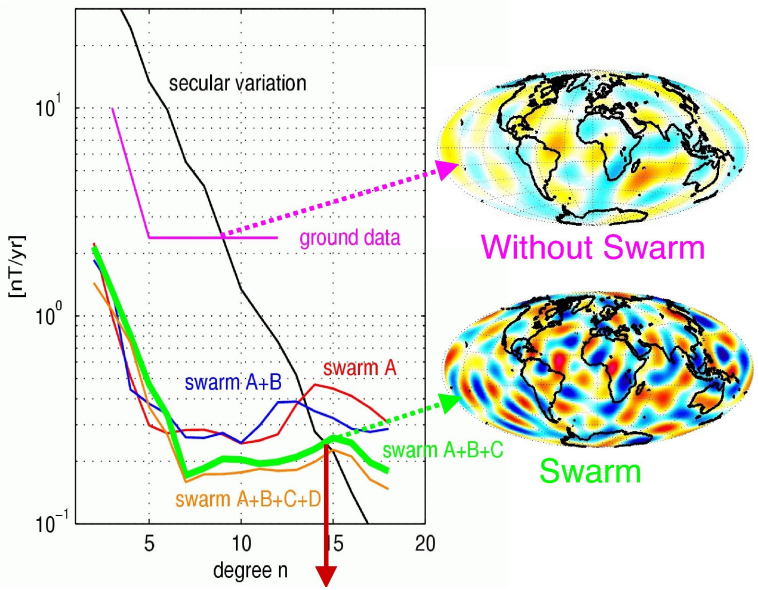
1-D inversion of CHAMP satellite data

Earth's magnetic field

- observed at permanent geomagnetic observatories, at temporary locations, measured from ships, aircrafts and satellites
- more than 98 % of Earth's magnetic field is of internal origin
- more than 95 % of its energy is in the dipolar term
- main components of geomagnetic field
  - field generated by geodynamo
  - lithospheric field
  - external fields corresponding to electric currents in the ionosphere and magnetosphere
  - induced fields

Earth's magnetic field

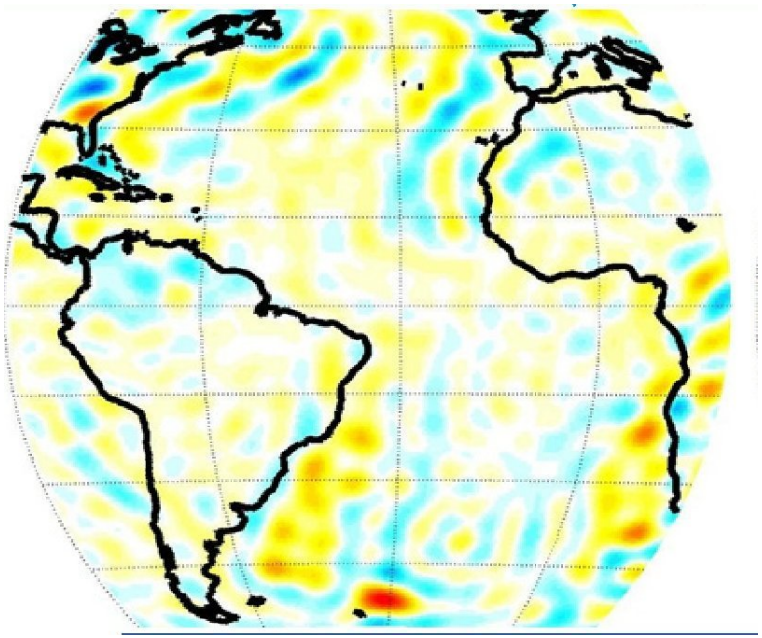
- main components of geomagnetic field
  - field generated by geodynamo
    - thermo-chemical convection of molten iron in the Earth's core
    - at the surface observed in spherical harmonics up to degree 15
    - characteristic times from 1 year onwards



  - lithospheric field
  - external fields corresponding to electric currents in the ionosphere and magnetosphere
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Earth's magnetic field

- main components of geomagnetic field
  - field generated by geodynamo
  - lithospheric field
    - static magnetization of lithosphere
    - observed in spherical harmonics above degree 15

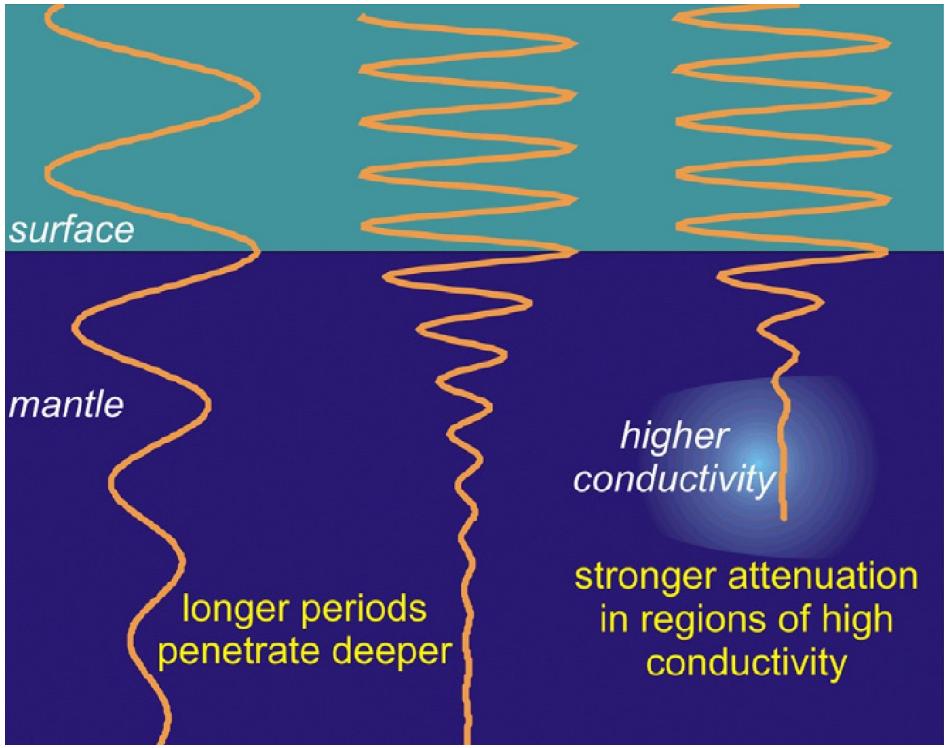


  - external fields corresponding to electric currents in the ionosphere and magnetosphere
  - induced fields

EM induction: Basic principles

- time changes of external geomagnetic field induce electrical currents in the Earth's crust and mantle (Farraday's law)
- the induced internal currents produce secondary magnetic field (Ampère's law)

- observations of the Earth's electromagnetic response yield informations about conductivity distribution in the Earth's interior



Constable 2003

EM induction: Scales and frequencies

penetration depth

$$h = \sqrt{\frac{1}{2\,\omega\,\mu\,\sigma}} = \sqrt{\frac{T}{4\,\pi\,\mu\,\sigma}}$$

near-surface

magnetotellurics

and...

EM induction: Scales and frequencies

penetration depth

$$h = \sqrt{\frac{1}{2\,\omega\,\mu\,\sigma}} = \sqrt{\frac{T}{4\,\pi\,\mu\,\sigma}}$$

near-surface frequencies up to 10^5 Hz, meter scales, artificial sources (loops), used for non-destructive archaeology, mine, UXO, underground cable and pipeline detection and classification, etc.



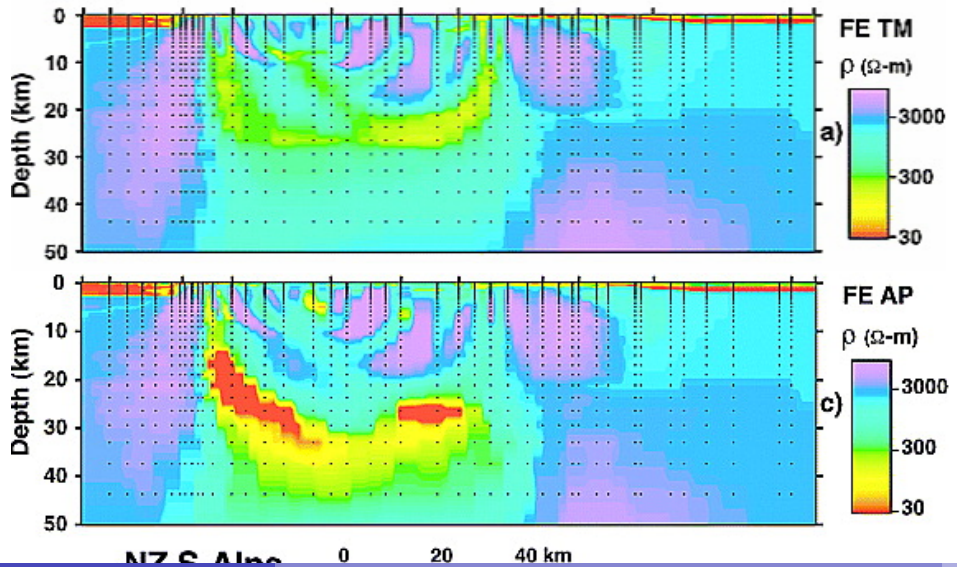
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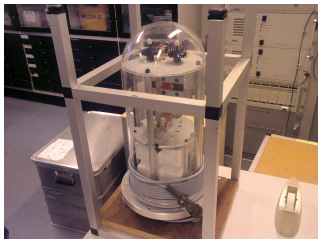
near-surface

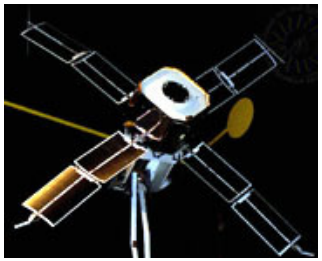
magnetotellurics periods 10^{-3}–10^{5} s, based on observation of horizontal electric and magnetic fields, scales up to hundreds of kilometers, applications in natural resources exploration (even on the seafloor!), studies of rifts and subduction zones, etc.

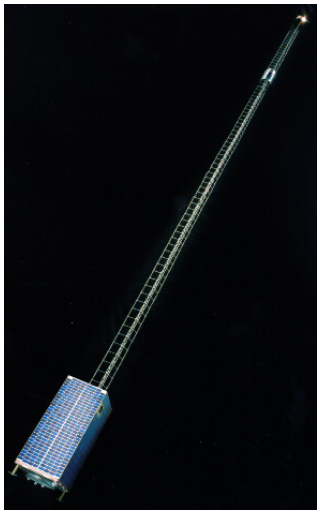


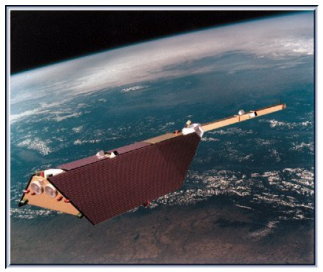
EM induction on global scale

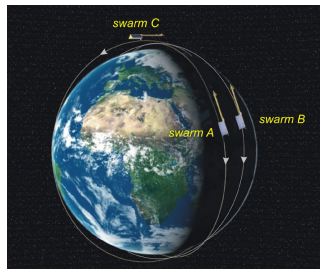
- based on observations from permanent geomagnetic stations and low orbit satellites
- works in period range from h to $1\ \mathrm{yr}$
- reveals conductivity distribution in the deep Earth's interior
- electrical conductivity constrains the temperature, the chemical and mineralogical composition and complements informations provided by other fields of geophysics


Fluxgate magnetometer


MAGSAT (1979–1980)


Ørsted (1999–)

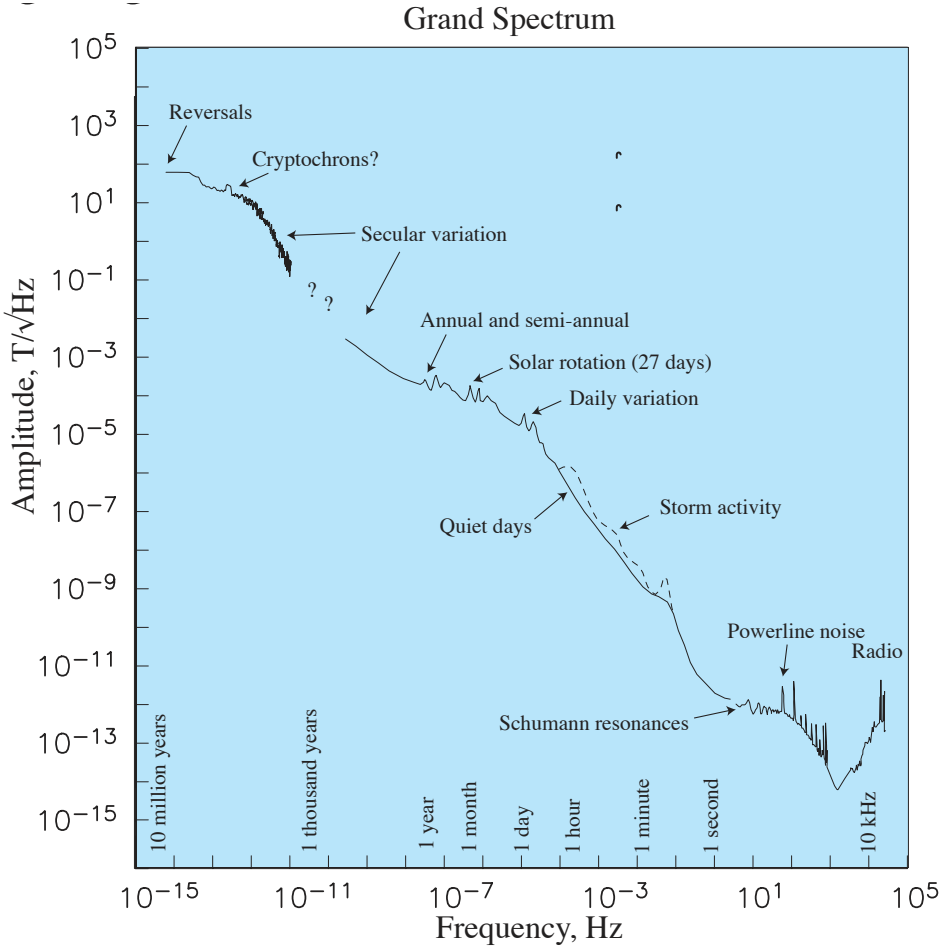

CHAMP (2000–10)



SWARM (2012?)

Time changes of geomagnetic field

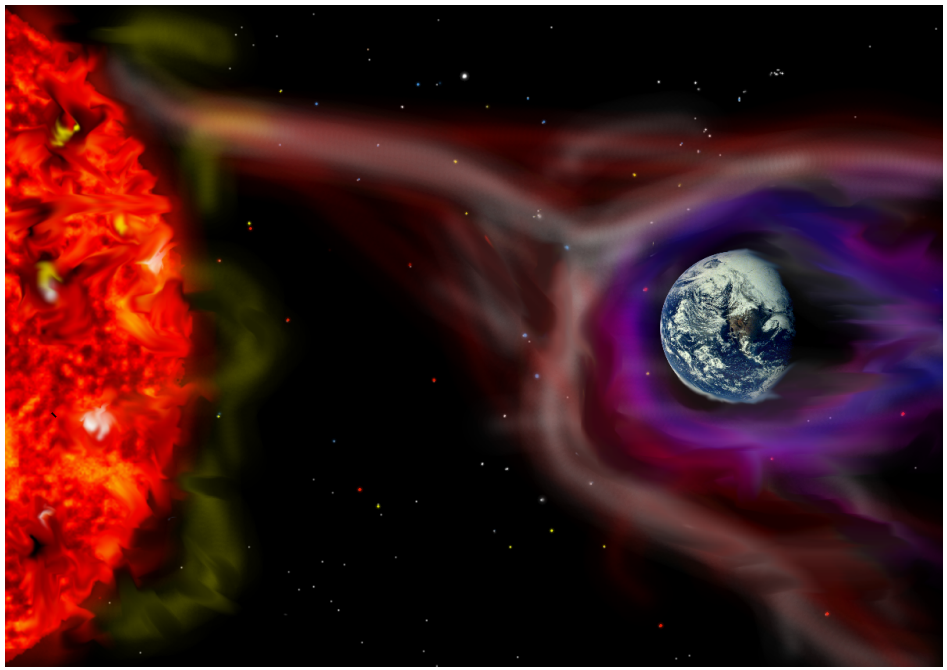
- the geomagnetic field is generated by magnetohydrodynamic convection in the Earth's fluid outer core — geodynamo
- the variations of geomagnetic field on geological to decadal time scales are of internal origin and governed by the processes in the geodynamo
- the changes of geomagnetic field on time scales shorter than 1 yr are of external origin



Constable 2003

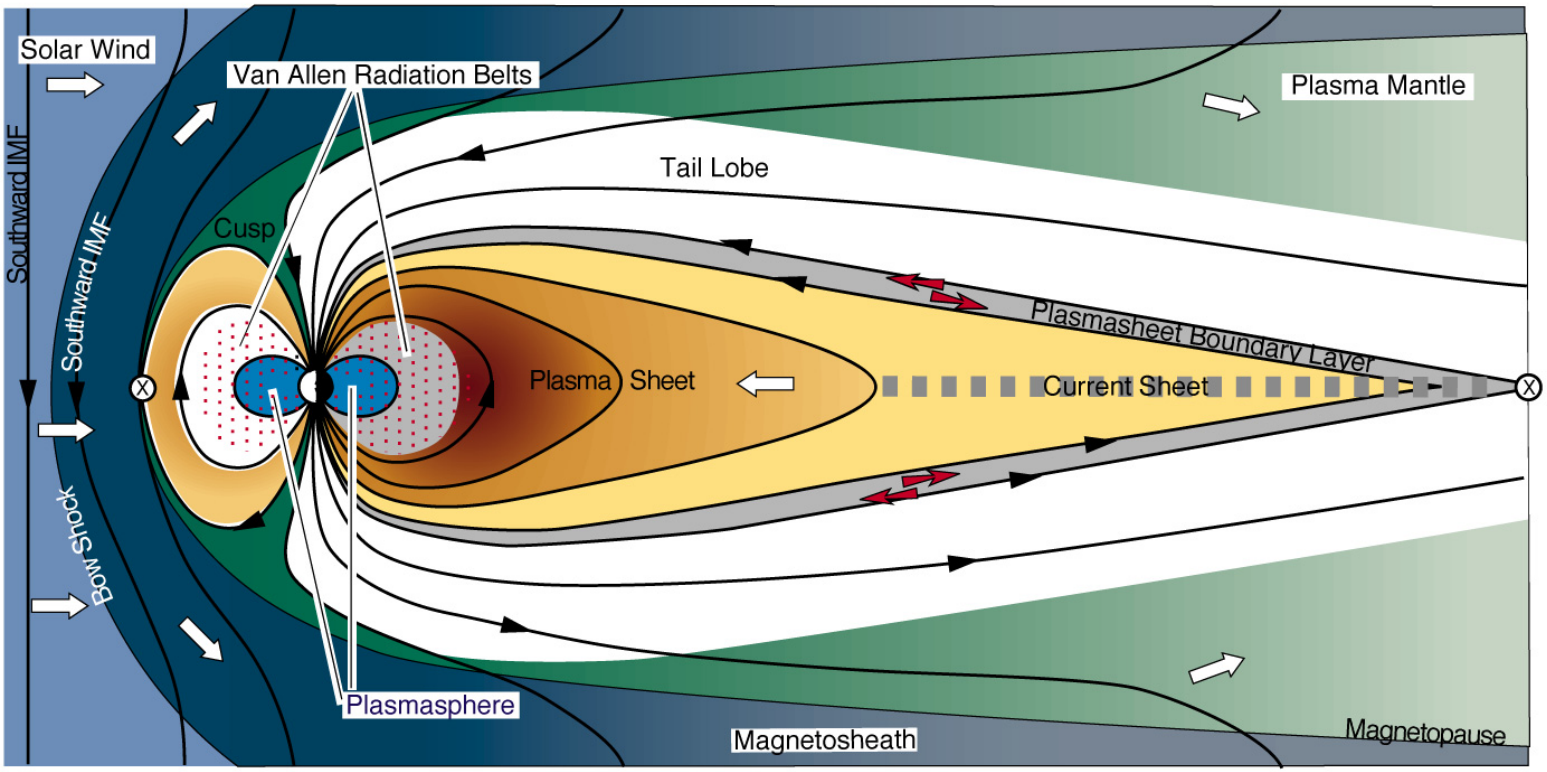
Time changes of geomagnetic field

- they are caused by interactions of solar wind with the main geomagnetic field



Magnetosphere

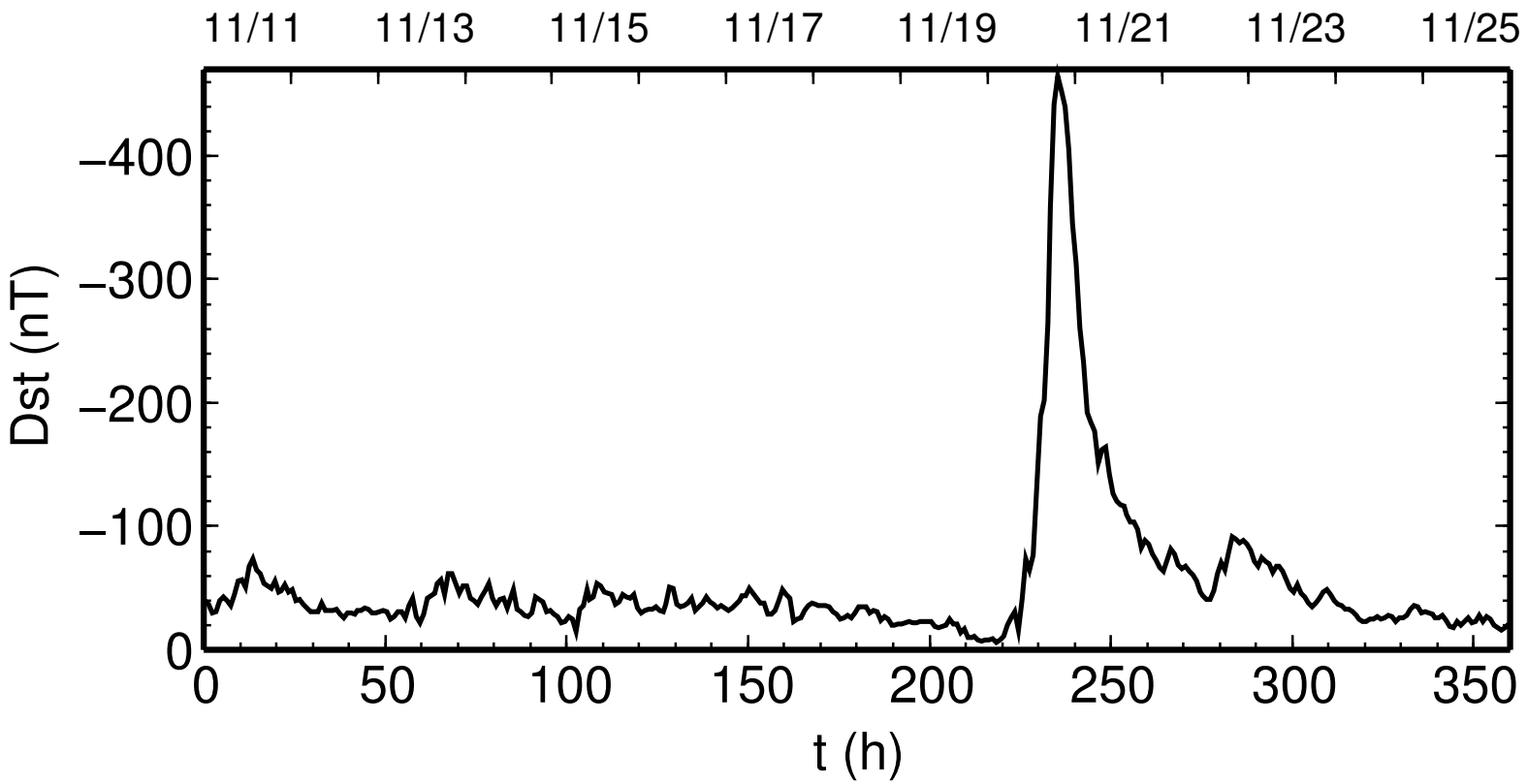
- charged particles from Solar wind captured by geomagnetic field
- the most prominent of magnetospheric electric currents is the equatorial ring current, $2R$—$7R$ from the Earth's centre
- changes in Solar weather can produce geomagnetic storms



Reif 1999

Magnetosphere

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Ionosphere

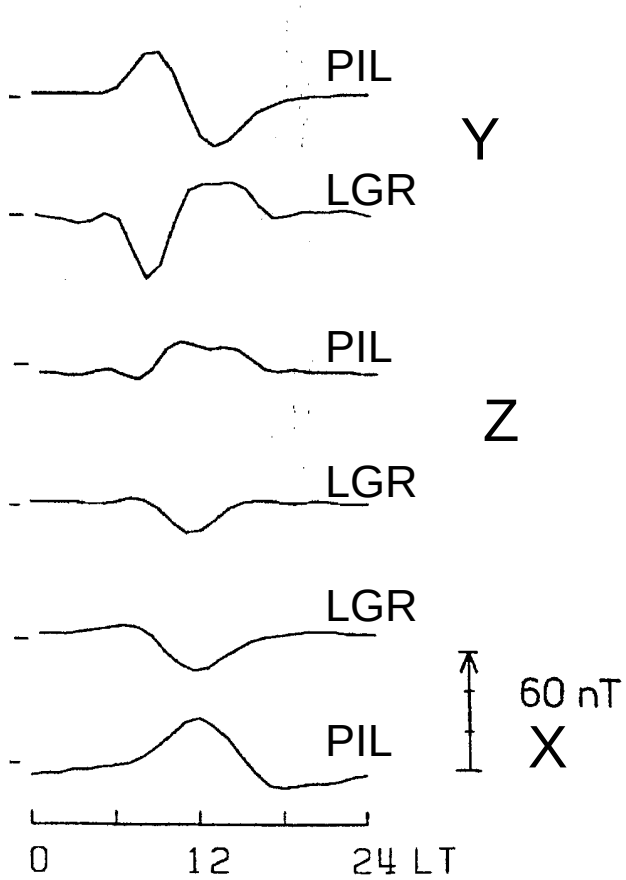
- electrical currents generated by ionospheric dynamo powered by alternating heating and cooling on the day and night side, respectively
- periodic character ($T = 1/n$ days, $n = 1, 2, \ldots$), seasonal variations
- in the first approximation they can be characterized by two current loops 110 km above the surface on the day side



USGS

Ionosphere

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Schmucker 1999

Use of observatory and satellite data for induction studies

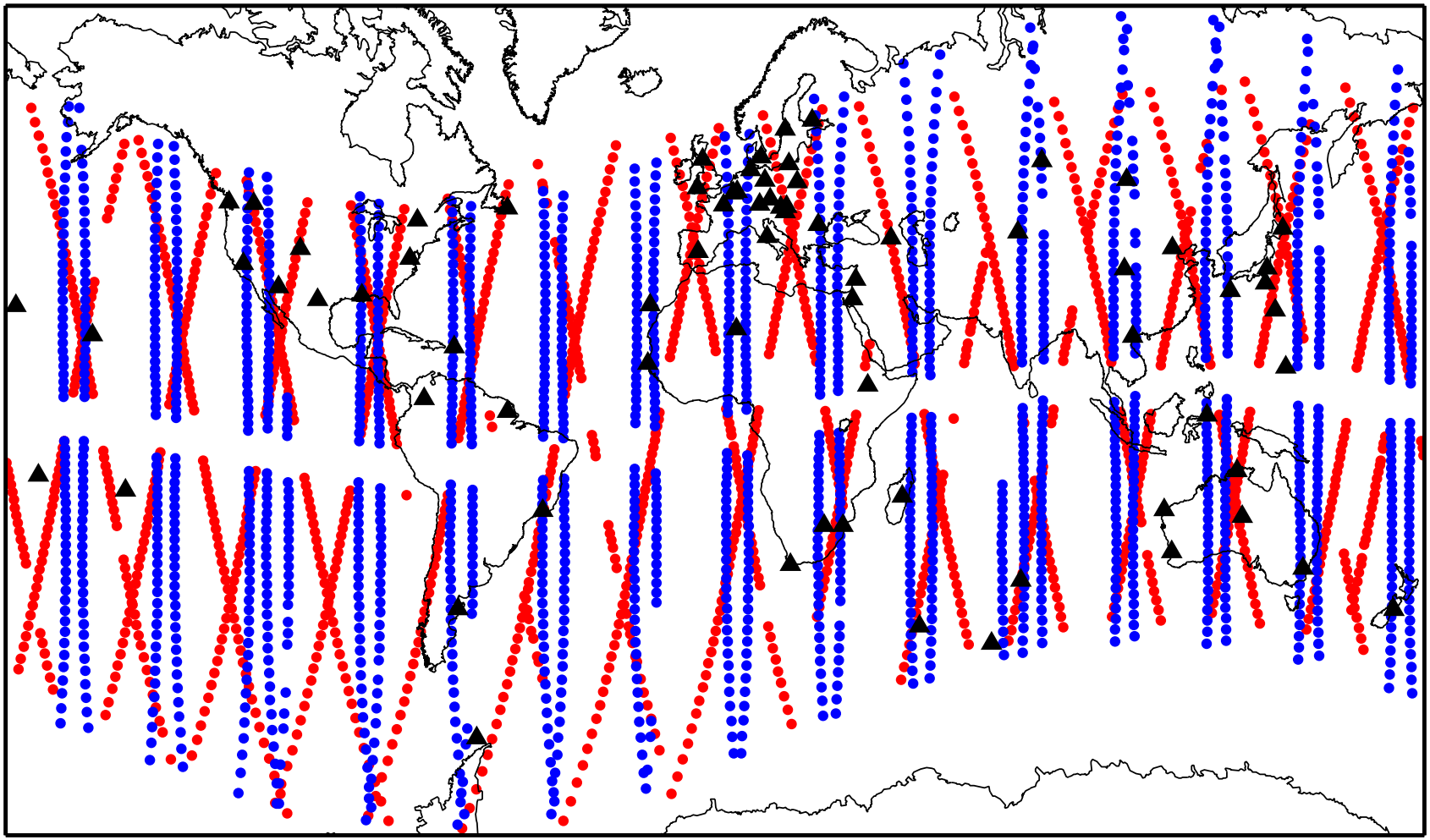
- On the global scale we aim at conductivity in the transition zone, upper mantle, and down to the lower mantle
- General requirements of global EM induction inverse problem:
  - vector magnetic field measurements (to overdetermine the forward problem)
  - global and regular data coverage (boundary condition on the Earth surface or satellite altitude)
  - sufficiently long time series (period range from hours to 1 year) for good depth resolution

Use of observatory and satellite data for induction studies

- Geomagnetic observatories
  - \+ long time series from a fixed position
  - \- irregural coverage, large areas without any measurements
  - +/- variable data quality
- Satellites
  - \+ homogeneous, high data quality
  - \+ data coverage over southern hemisphere and oceans
  - \- tradeoff between spatial and temporal changes along tracks
  - \- ionospheric currents for EM induction are located below satellite

Use of observatory and satellite data for induction studies

November 30, 2001: 73 Intermagnet stations, Oersted, CHAMP

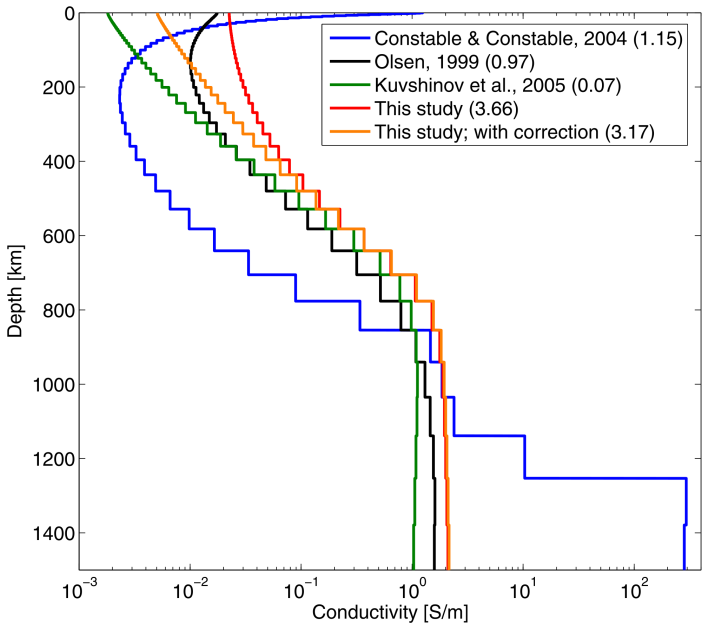


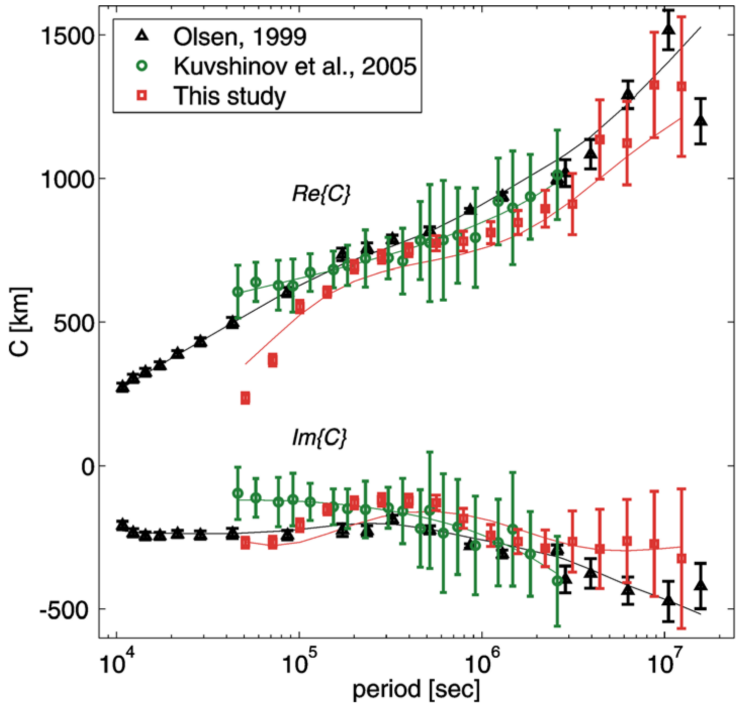
Recent results

- response functions at very long periods (semi-annual, annual) are needed to see in the lower mantle
- correlation between external primary and secondary induced field measured at geomagnetic observatories decreases for long periods
- McLeod (1994) gives an estimate of 10 S/m for lower mantle — processing of 50 yrs of monthly-mean differences from geomagnetic observatories
- 1-D inversions of observatory data (Olsen 1999) and satellite data (Kuvshinov 2005
- 3-D inversion of observatory data (Kelbert 2009)

Recent results

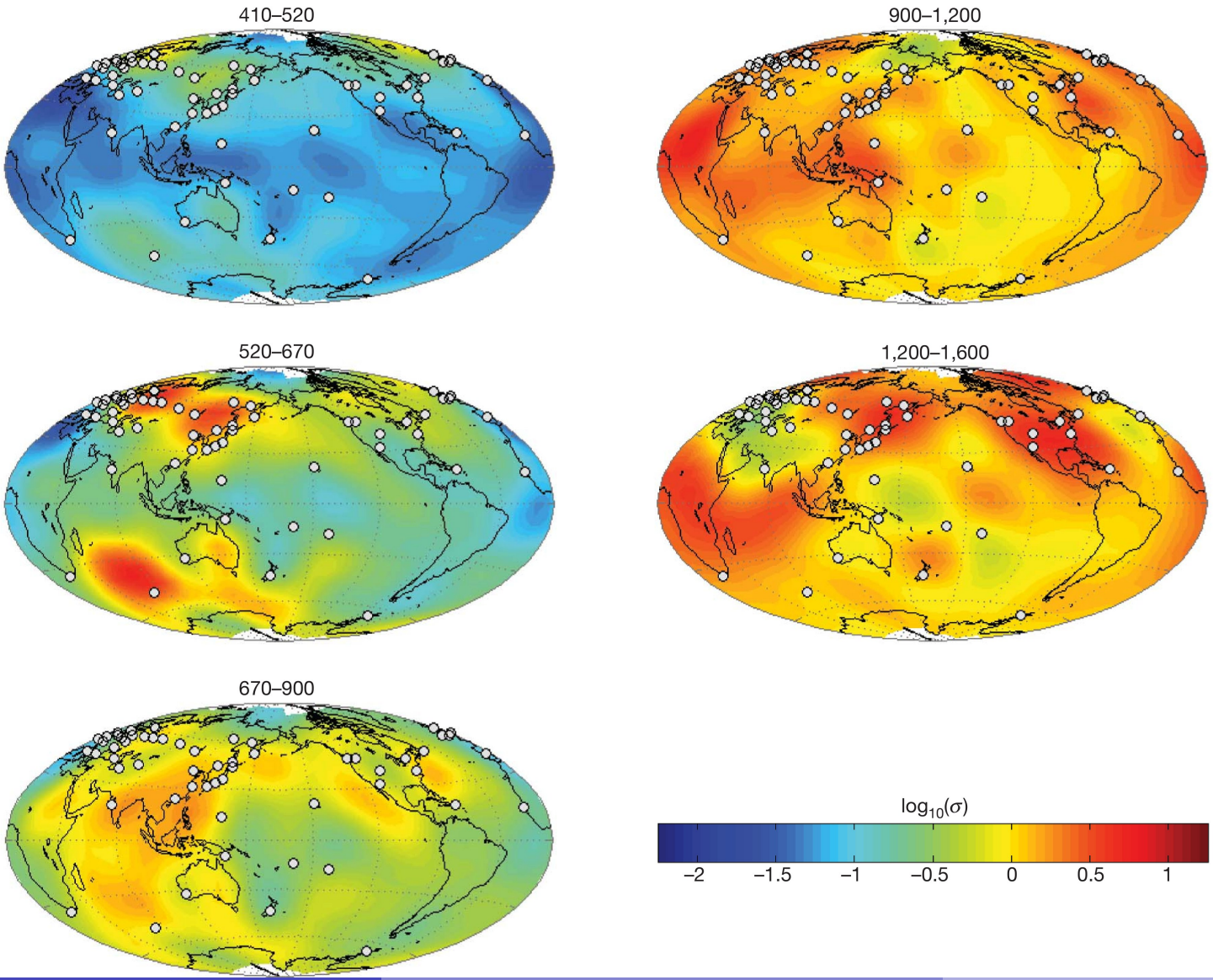
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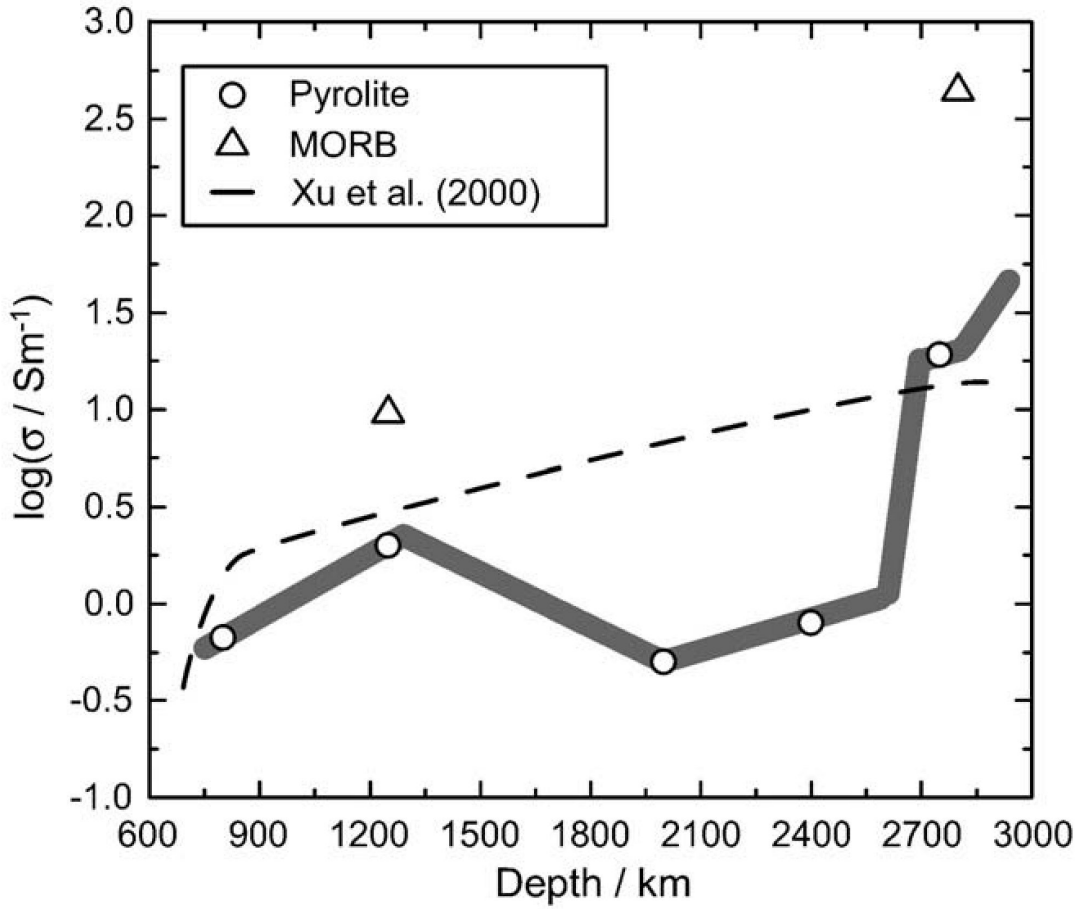


Geomagnetic jerks and core-mantle coupling

- EM induction excited by rapid changes of core field — geomagnetic jerks
  - main difficulty — unknown geometry of the source field on CMB
  - estimated lower mantle conductivity of 10 S/m (Alexandrescu *et al.* 1999)
  - differential time-delays of jerks observed at the surface can be explained by 3-D conductivity variations in the lower mantle (Nagao *et al.* 2003) or by 1-D mantle assuming a more complicated structure of secular variation at CMB (Pinheiro & Jackson 2008)
- interpretation of changes in LOD by EM coupling between core and mantle — 10^8 S (Holme 1998) if only EM torque is assumed

Phase transitions in the lower mantle

+ post-perovskite (PPV) phase in D”
- high-spin to low-spin transition in ferric iron in $(\mathrm{Mg},\mathrm{Fe})\,\mathrm{Si}\,\mathrm{O}_3$ perovskite at 70 GPa, and in ferrous iron in $(\mathrm{Mg},\mathrm{Fe})\,\mathrm{O}$ ferropericlase at 120 GPa (Ohta *et al.* 2009)



Outline

Introduction

Inverse problem in time-domain global EM induction: Adjoint approach
- Why time-domain approach?
- Forward problem
- Penalty function
- Adjoint problem
- Gradient of penalty function
- Inversion scheme

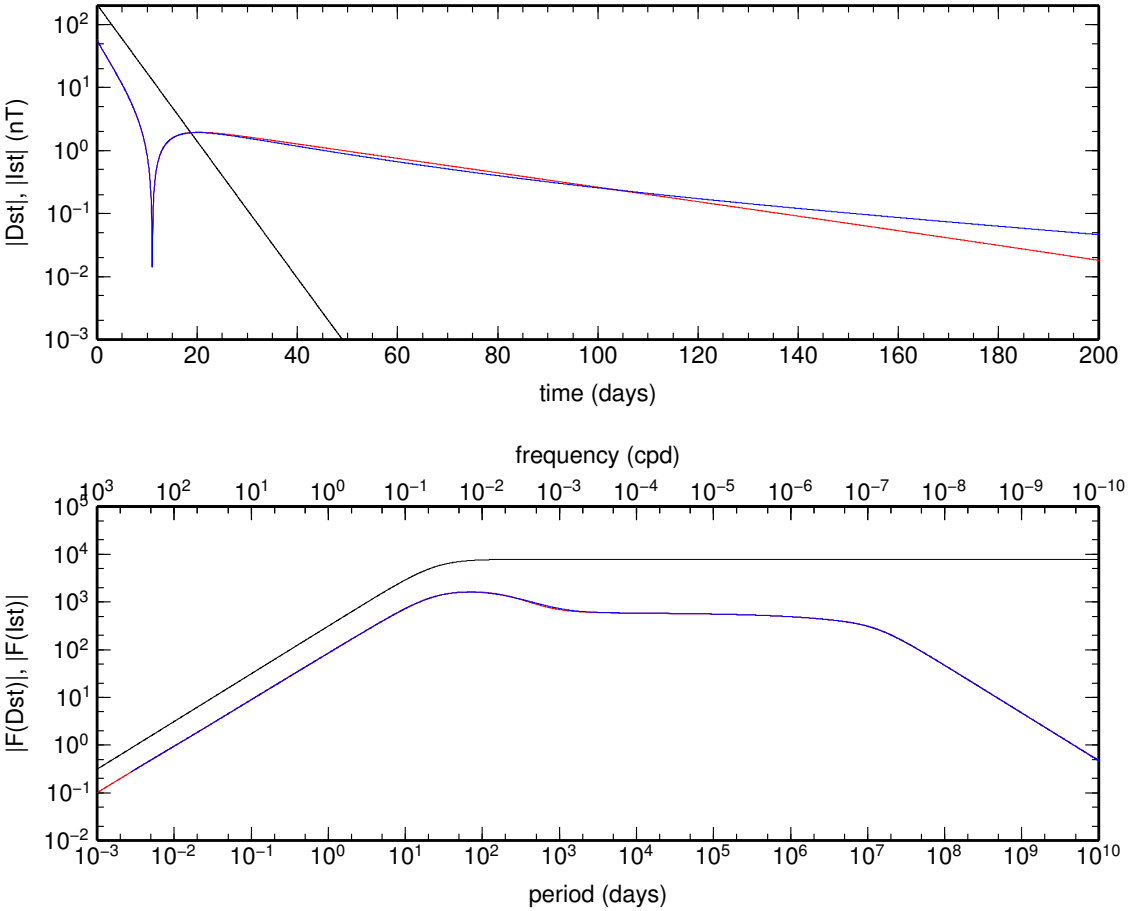
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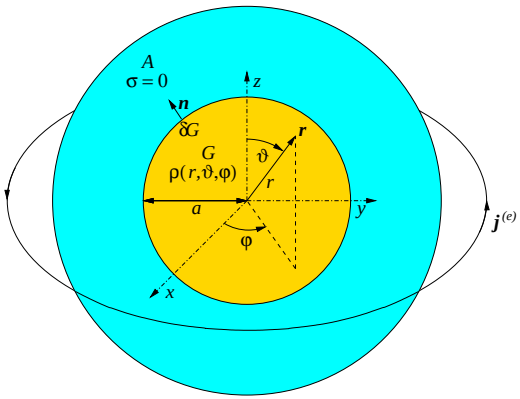
Why time-domain approach?

- All earthly things are transient *(Persian proverb, chateau Kynžvart)*
- we need signals at very long periods (up to 1 year)
- difficult to obtain from time-series of only several years of length
- geomagnetic storms are transient signals with broad spectrum



Adjoint approach to time-domain global EM induction

Forward problem [1]



- Earth's interior

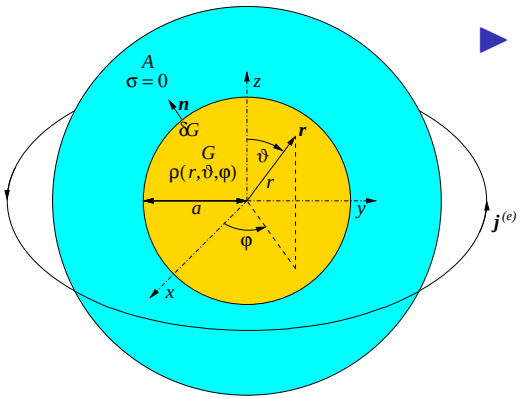
$$
\begin{aligned}
\rho &= \rho(r,\vartheta,\varphi) \\
\operatorname{curl}\,(\rho \operatorname{curl} \boldsymbol{B}) &= -\mu \frac{\partial \boldsymbol{B}}{\partial t} \\
\operatorname{div} \boldsymbol{B} &= 0 \\
\boldsymbol{B}(r,\vartheta,\varphi;t) &= \sum_{j=1}^{j_{\max}} \sum_{m=-j}^{j} \sum_{\lambda=-1}^{1} B_{jm}^{(\lambda)}(r;t)\, \boldsymbol{S}_{jm}^{(\lambda)}(\Omega)
\end{aligned}
$$

- surface
- insulating atmosphere

[1] Velímský & Martinec, *Geophys. J. Int.*, **161(1)**, 81–101, 2005.

Adjoint approach to time-domain global EM induction

Forward problem [1]



- Earth's interior
- surface

$$r = a = 6371\,\mathrm{km}$$

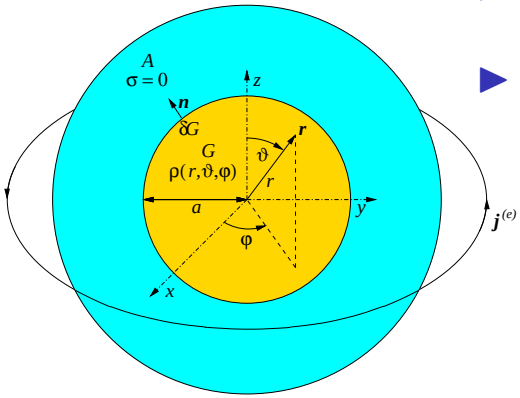
$$\boldsymbol{B} = -\mathrm{grad}\, U_0$$

- insulating atmosphere

[1] Velímský & Martinec, *Geophys. J. Int.*, **161(1)**, 81–101, 2005.

Adjoint approach to time-domain global EM induction

Forward problem [1]



- Earth's interior
- surface
- insulating atmosphere

$$\sigma = 0$$
$$\Delta U_0 = 0$$

$$U_0 = a \sum_{jm} \left[\left(\frac{r}{a} \right)^j G^{(\mathrm{e})}_{jm}(t) + \left(\frac{a}{r} \right)^{j+1} G^{(\mathrm{i})}_{jm}(t) \right] Y_{jm}(\vartheta, \varphi)$$

[1] Velímský & Martinec, *Geophys. J. Int.*, **161(1)**, 81–101, 2005.

Inverse problem in time-domain global EM induction: Adjoint approach

Forward problem [2]

$$\operatorname{curl}\,(\rho \operatorname{curl} \boldsymbol{B}) + \mu_0 \frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{0},$$

in a sphere G, with boundary conditions

$$\begin{aligned} B_{jm}^{(0)}(a;t) &= 0, \\ B_{jm}^{(-1)}(a;t) + (j+1)\, B_{jm}^{(1)}(a;t) &= -(2j+1)\, G_{jm}^{(\mathrm{e,obs})}(t), \end{aligned}$$

on the surface ∂G ($r = a$), and with initial condition

$$\boldsymbol{B}(\boldsymbol{r};0) = \boldsymbol{B}_0.$$

[2] Velímský & Martinec, *Geophys. J. Int.*, **161(1)**, 81–101, 2005.

Inverse problem in time-domain global EM induction: Adjoint approach

Penalty function

Let the resistivity $\rho(\boldsymbol{r})$ be described by M real parameters,

$$\begin{aligned} \boldsymbol{m} &= (m_1, m_2, \ldots, m_M)\,, \\ \rho(\boldsymbol{r}) &= \rho(\boldsymbol{m}; \boldsymbol{r}). \end{aligned}$$

We minimize the penalty function

$$F(\boldsymbol{m}) = \chi^2(\boldsymbol{m}) + \lambda R^2(\boldsymbol{m}),$$

where

$$\chi^2(\boldsymbol{m}) = \frac{1}{2}\,\frac{1}{S\,(t_1 - t_0)} \int_{t_0}^{t_1} \int_{\partial G} \left(\frac{\boldsymbol{B}(\boldsymbol{m}) - \boldsymbol{B}^{(\mathrm{obs})}}{\sigma_B} \right)^2 \mathrm{d}S\,\mathrm{d}t,$$

$$R^2(\boldsymbol{m}) = \frac{1}{2}\,\frac{a^4}{V} \int_V \left[\nabla^2 \log \rho(\boldsymbol{m})\right]^2 \mathrm{d}V$$

are the dimensionless misfit and regularization term, respectively.

Inverse problem in time-domain global EM induction: Adjoint approach

Adjoint problem

Let $\hat{\boldsymbol{B}}$ be the solution of adjoint problem formulated in adjoint time $\hat{t} = t_1 - t$,

$$\operatorname{curl}\left(\rho \operatorname{curl} \hat{\boldsymbol{B}}\right) + \mu_0 \frac{\partial \hat{\boldsymbol{B}}}{\partial \hat{t}} = \mathbf{0},$$

in G, with boundary conditions

$$\begin{aligned} \hat{B}^{(0)}_{jm}(a;t) &= 0, \\ \hat{B}^{(-1)}_{jm}(a;\hat{t}) + (j+1)\,\hat{B}^{(1)}_{jm}(a;\hat{t}) &= -(2j+1)\,\hat{G}^{(\mathrm{e})}_{jm}(\hat{t}), \end{aligned}$$

where

$$\hat{G}^{(\mathrm{e})}_{jm}(\hat{t}) = -\frac{(j+1)}{t_1 - t_0} \int\limits_{\max(t_0, t_1 - \hat{t})}^{t_1} \frac{G^{(\mathrm{i})}_{jm}(\tau) - G^{(\mathrm{i,obs})}_{jm}(\tau)}{\sigma_B^2}\,\mathrm{d}\tau,$$

on the surface ∂G ($r = a$), and with initial condition at $\hat{t} = 0$,

$$\hat{\boldsymbol{B}}(\boldsymbol{r};0) = \mathbf{0}.$$

Inverse problem in time-domain global EM induction: Adjoint approach

Gradient of penalty function

Then at the cost of one forward and one adjoint solution we can evaluate the gradient of penalty function in the model space,

$$\begin{aligned}
\nabla_{\boldsymbol{m}} F(\boldsymbol{m}) &= \nabla_{\boldsymbol{m}} \chi^2(\boldsymbol{m}) + \lambda \nabla_{\boldsymbol{m}} R^2(\boldsymbol{m}) \\
\nabla_{\boldsymbol{m}} \chi^2(\boldsymbol{m}) &= \frac{1}{S\, a\, \mu_0} \int_0^{t_1} \int_G (\nabla_{\boldsymbol{m}} \rho)\, \operatorname{curl} \hat{\boldsymbol{B}} \cdot \operatorname{curl} \boldsymbol{B}\, \mathrm{d}V\, \mathrm{d}t, \\
\nabla_{\boldsymbol{m}} R^2(\boldsymbol{m}) &= \frac{a^4\, \log \mathrm{e}}{V} \int_V \nabla^2 \log \rho\, \nabla^2 \frac{\nabla_{\boldsymbol{m}} \rho}{\rho}\, \mathrm{d}V.
\end{aligned}$$

Inverse problem in time-domain global EM induction: Adjoint approach

Inversion scheme

- quasi-Newton (variable metric, limited-memory) method

$$\boldsymbol{m}_{i+1} - \boldsymbol{m}_i = H_{i+1} \cdot (\nabla_{\boldsymbol{m}} F(\boldsymbol{m}_{i+1}) - \nabla_{\boldsymbol{m}} F(\boldsymbol{m}_i))$$

- Broyden-Fletcher-Goldfarb-Shanno formula for updating H_{i+1}
- Brent's method for line minimization

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Inverse problem in time-domain global EM induction: Adjoint approach

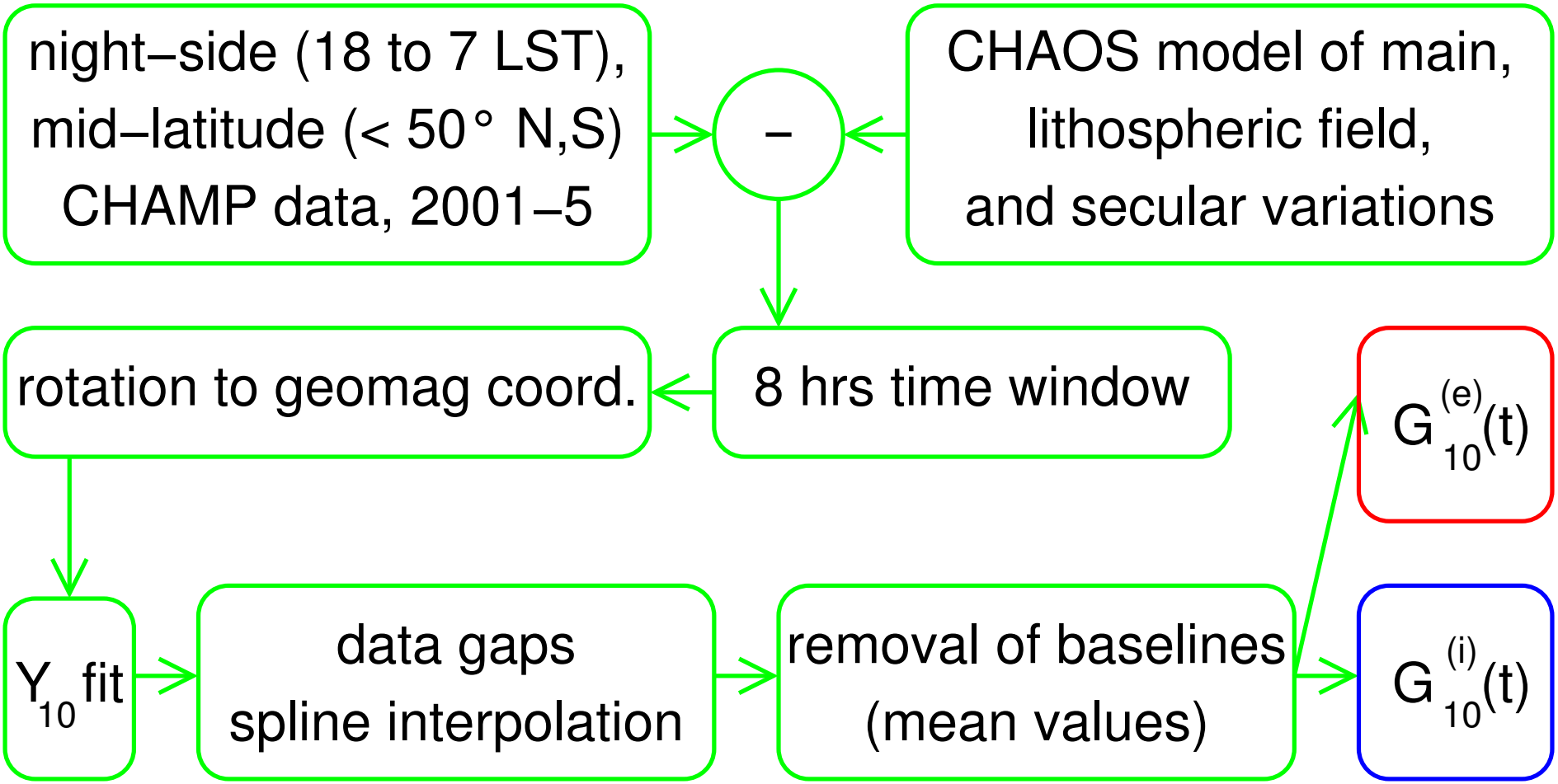
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- CHAMP data processing
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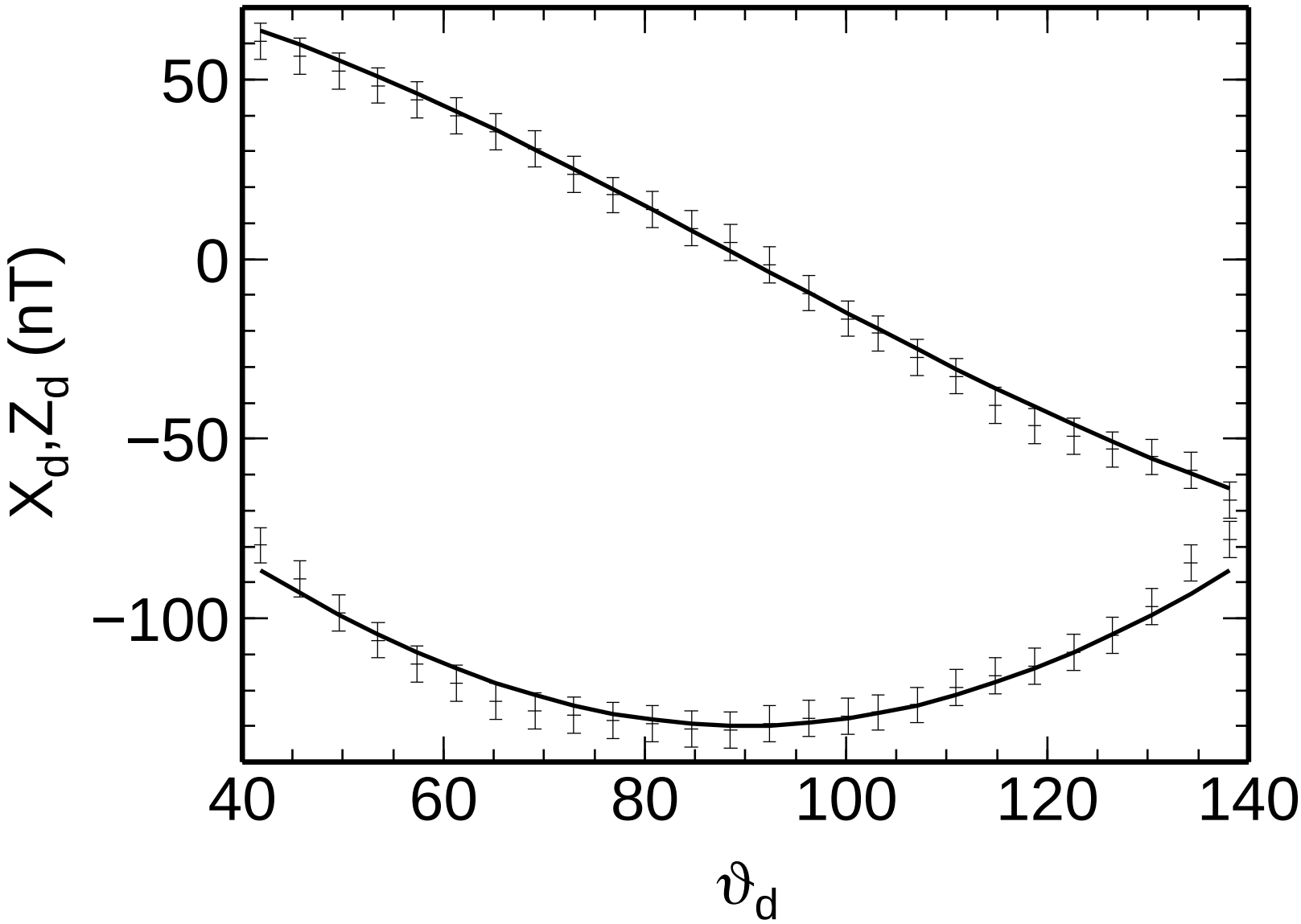
1-D inversion of CHAMP satellite data

CHAMP data processing



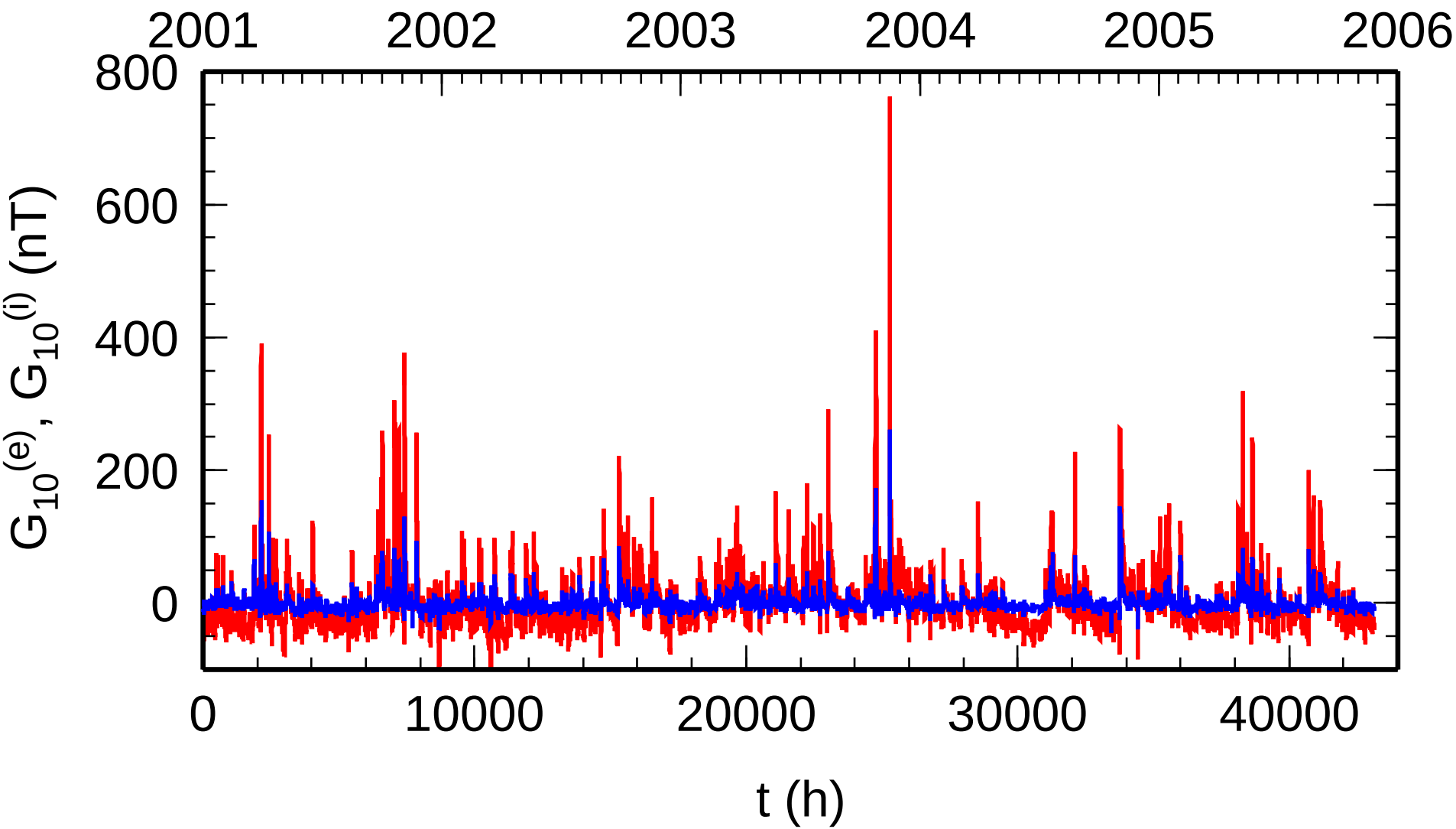
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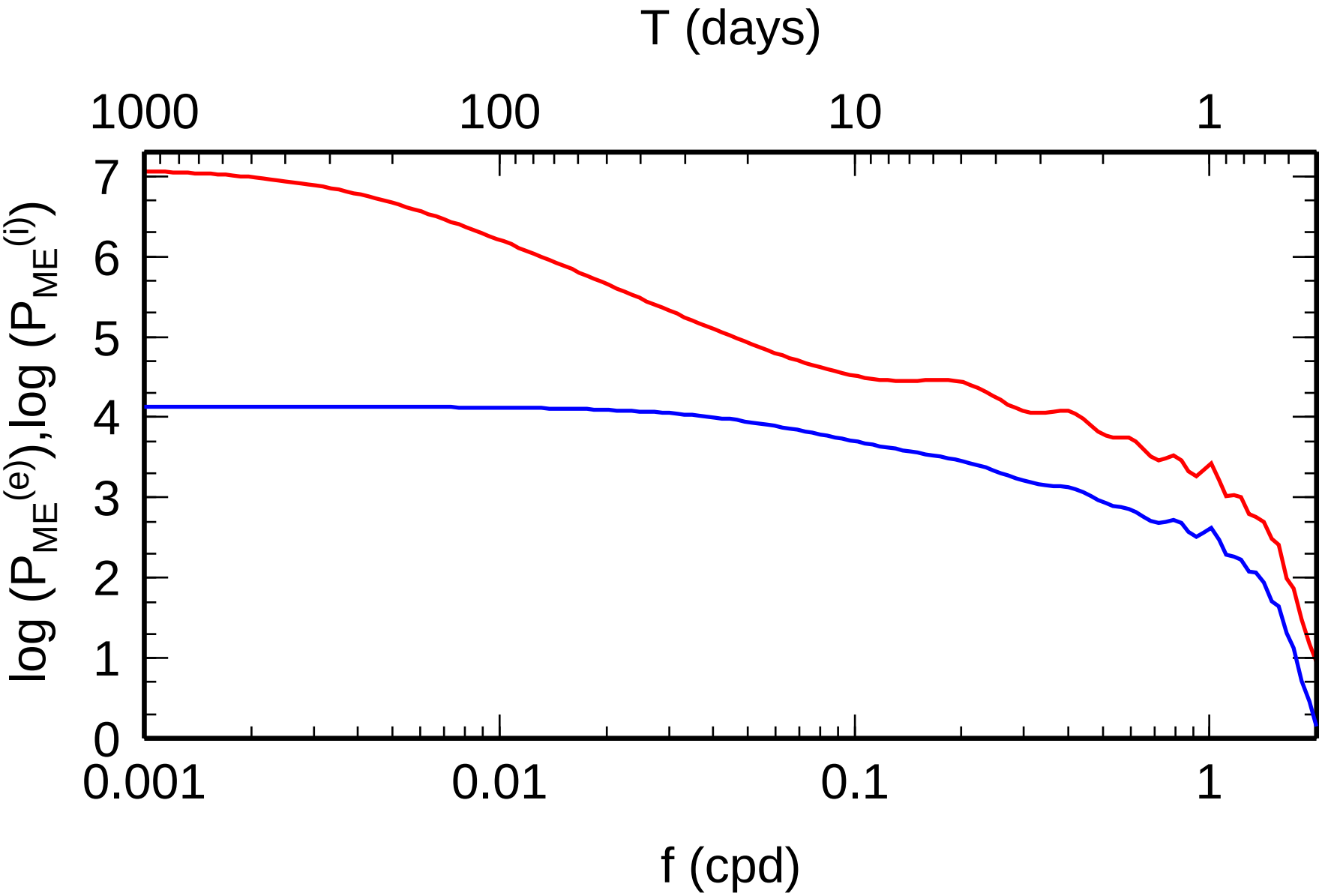
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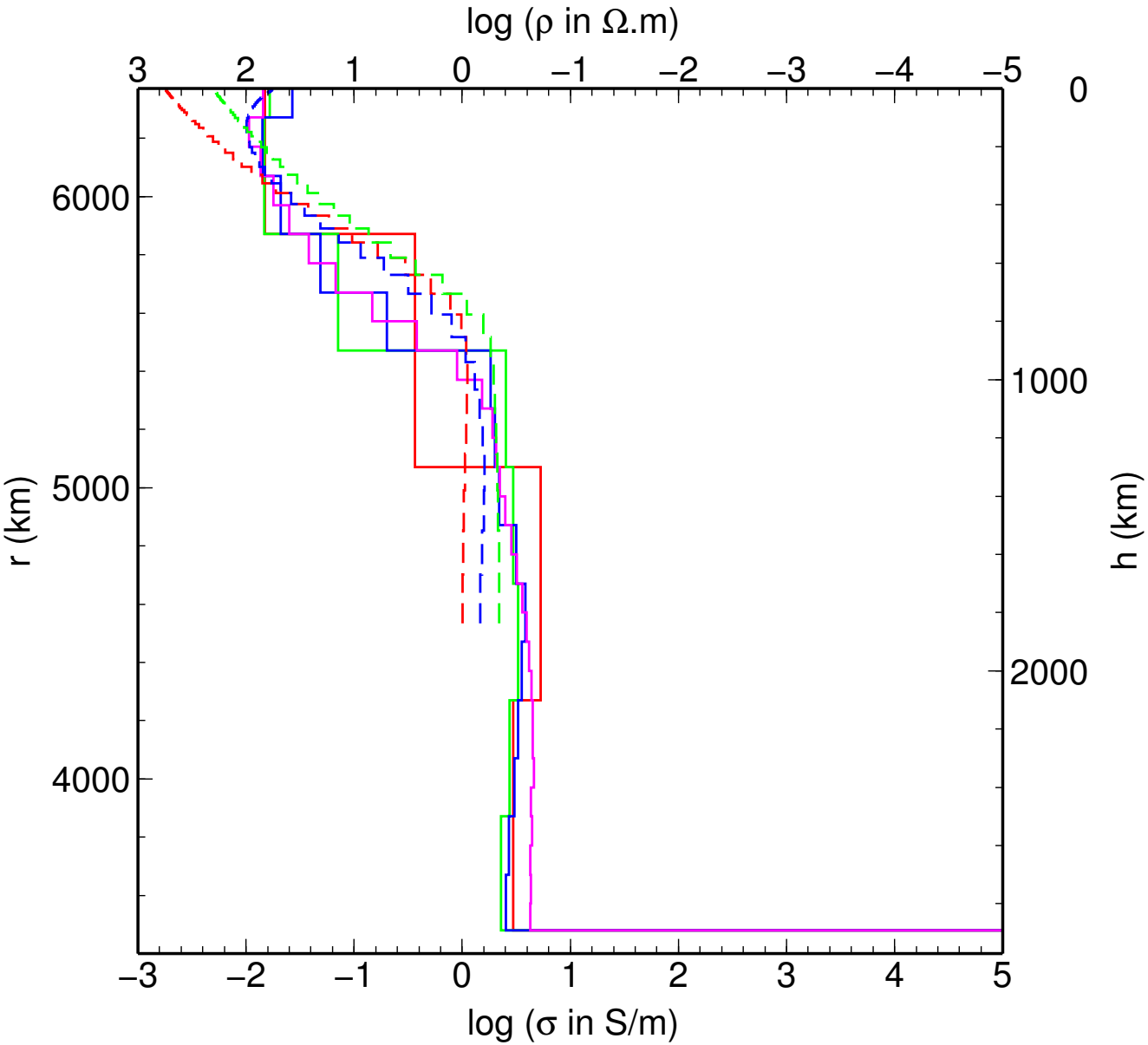
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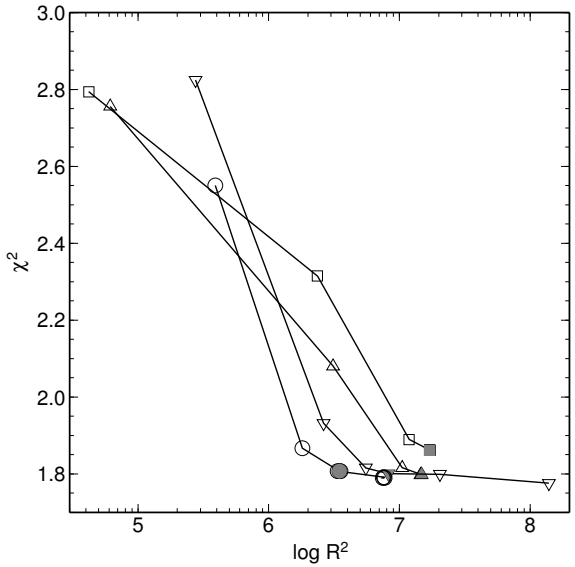
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1-D inversion of CHAMP satellite data

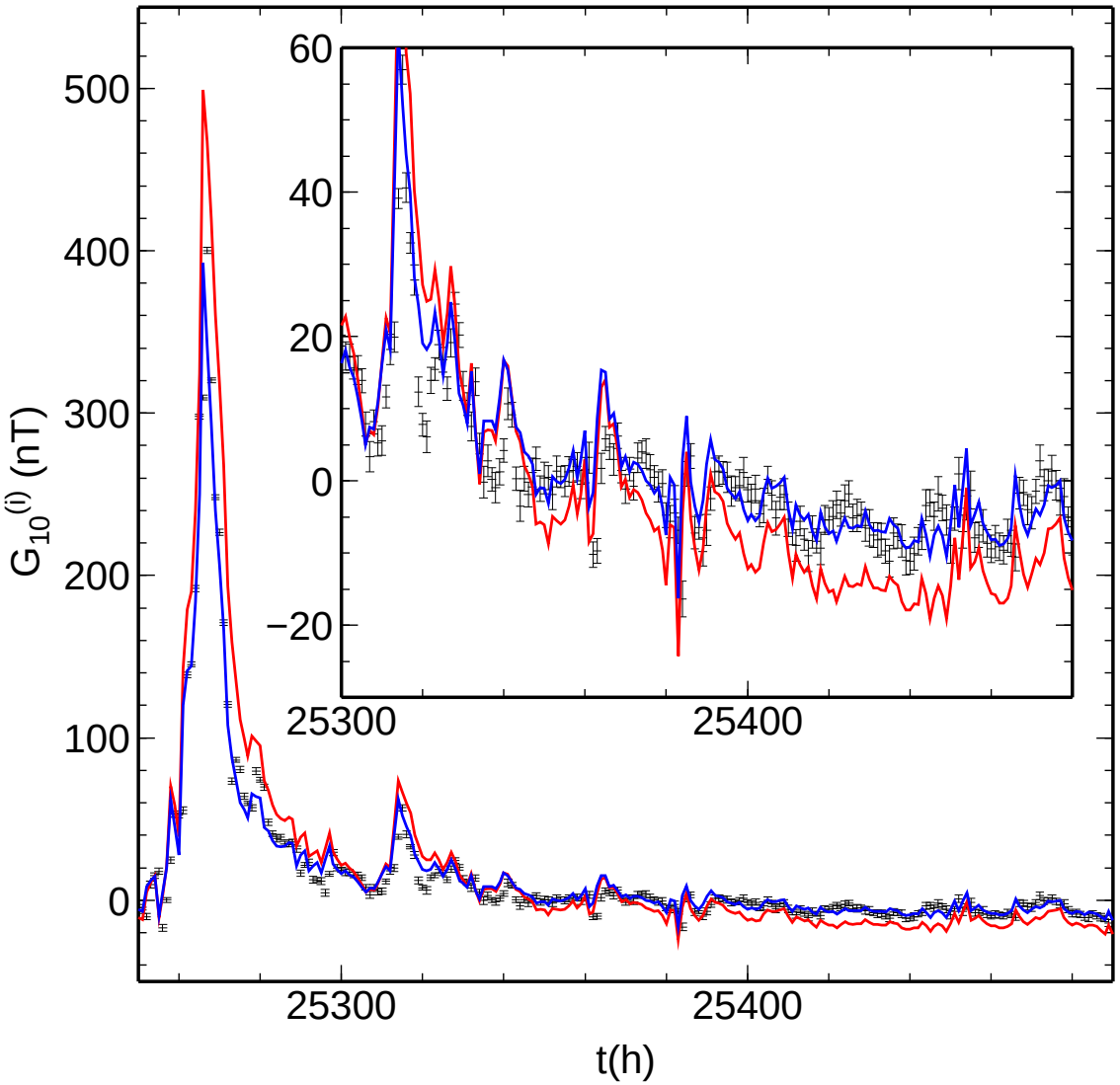
Results





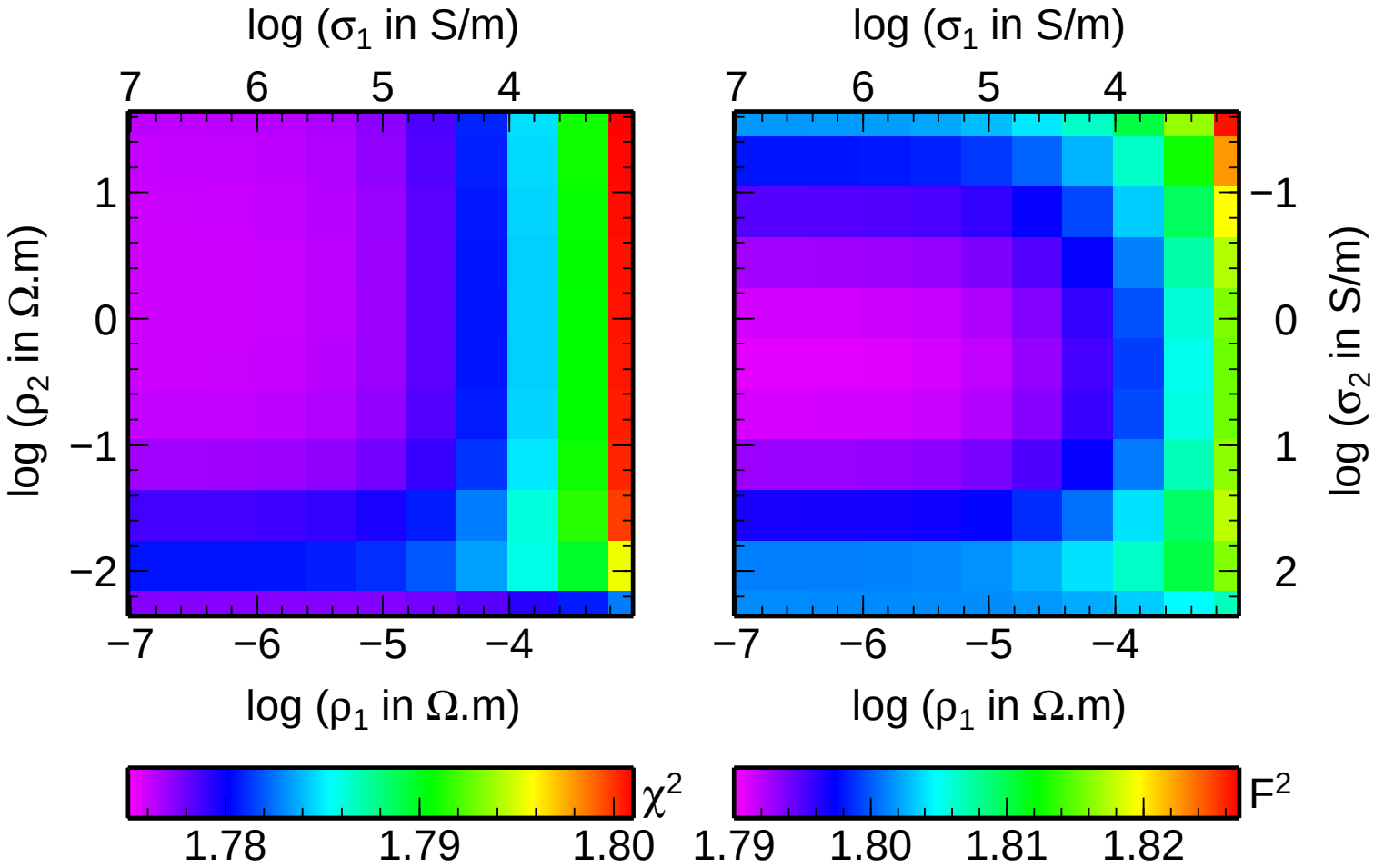
1-D inversion of CHAMP satellite data

Data fit



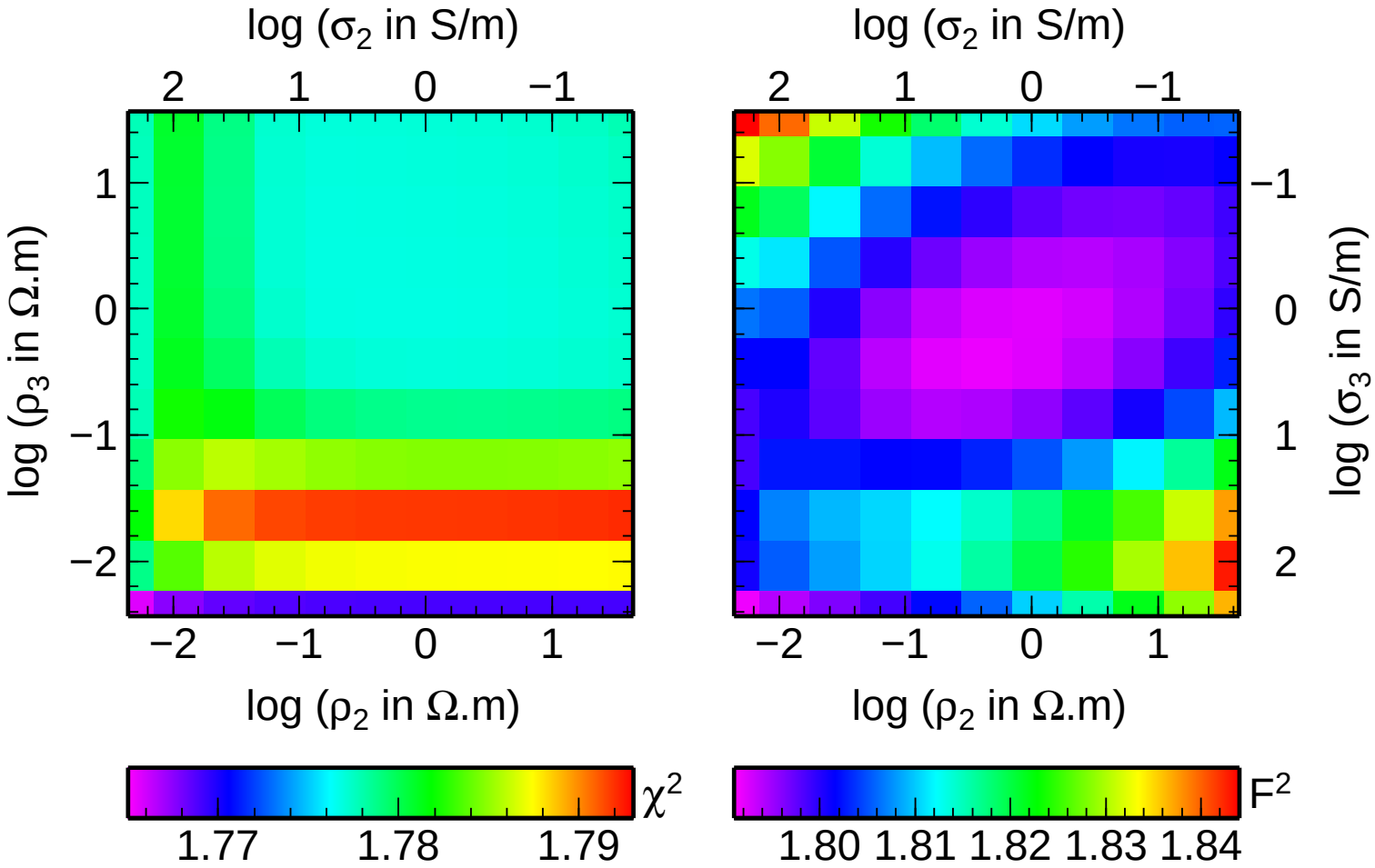
1-D inversion of CHAMP satellite data

Data sensitivity



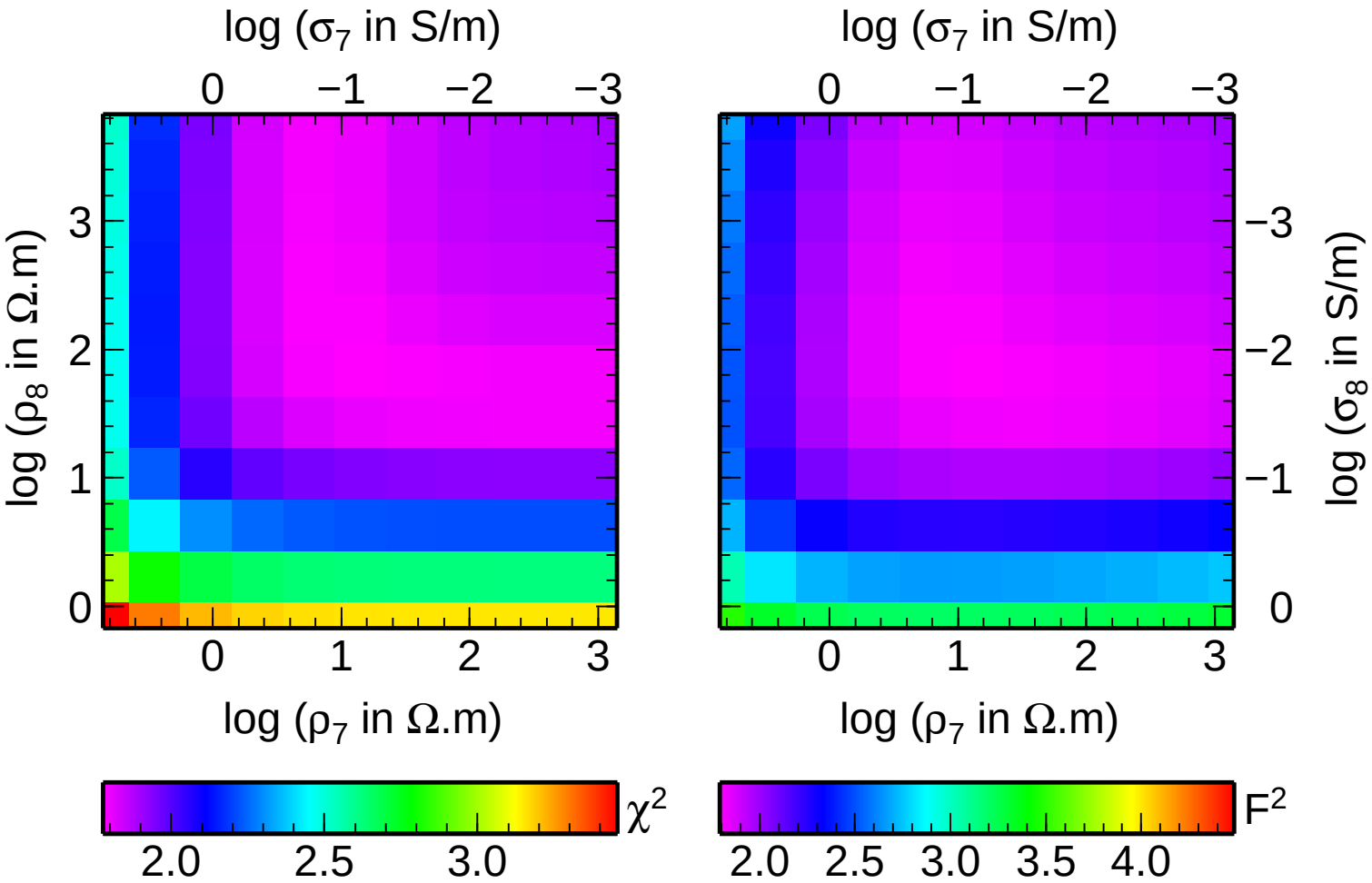
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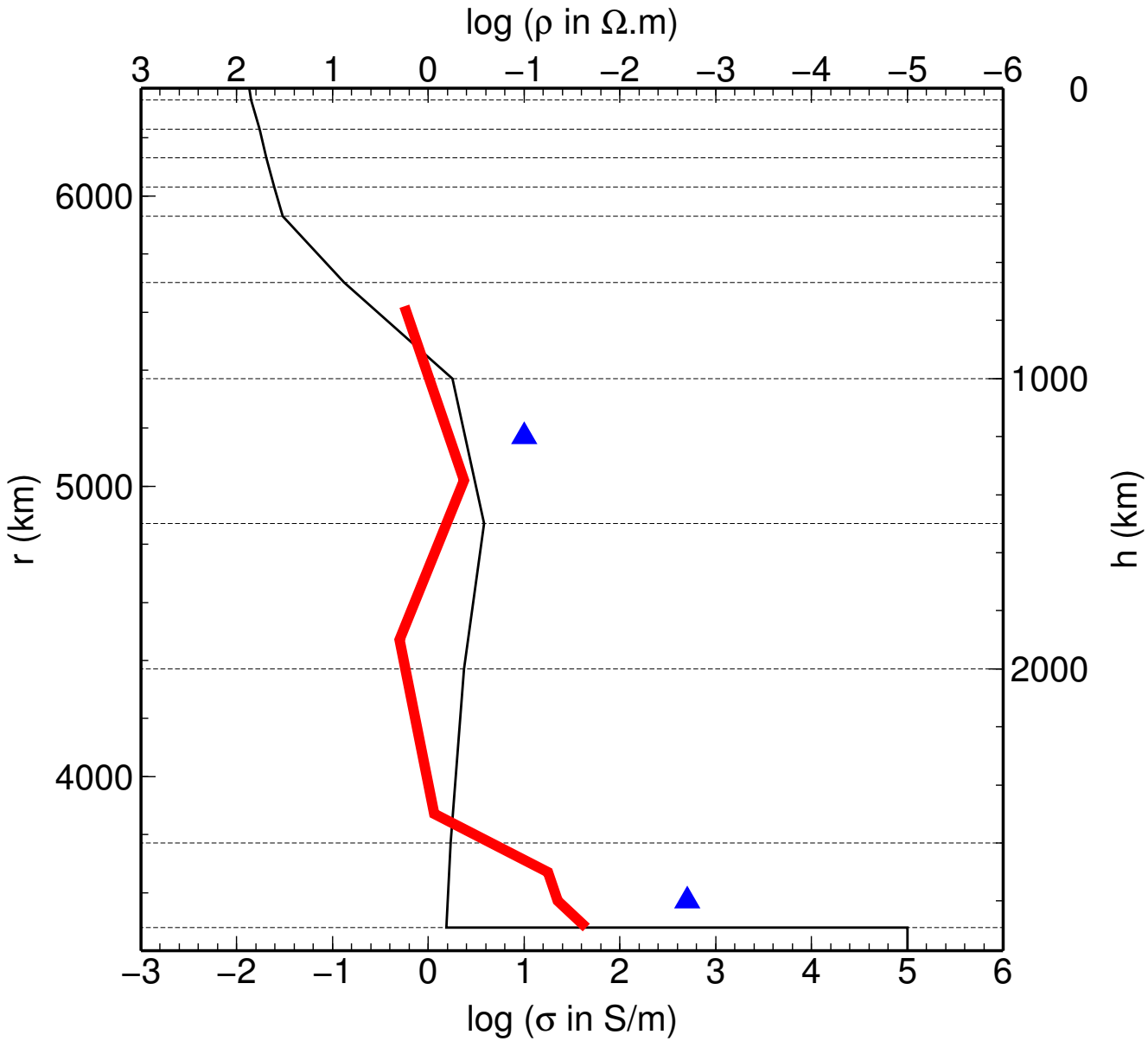
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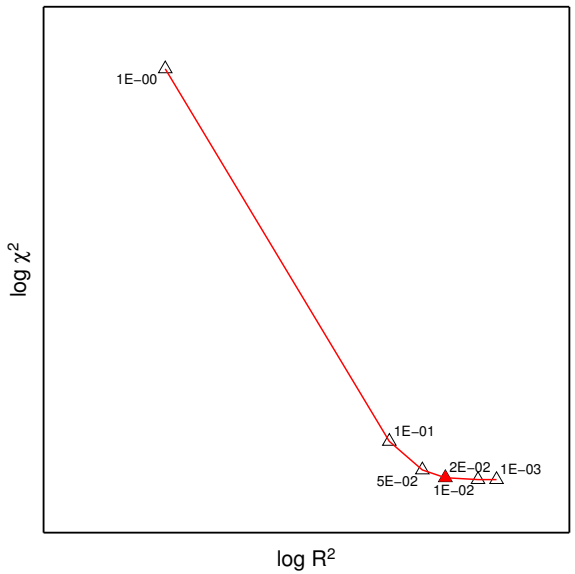
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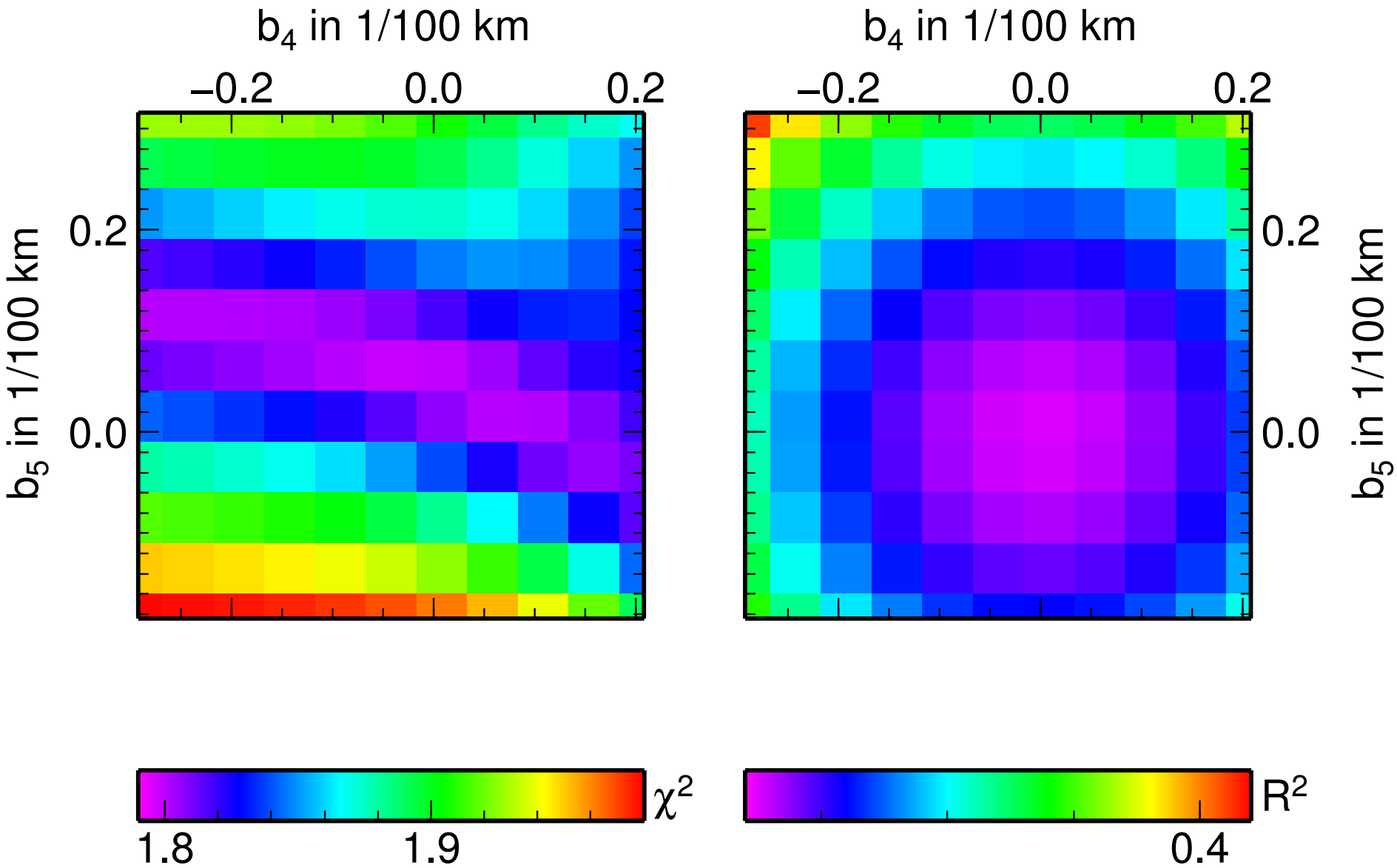
Results





1-D inversion of CHAMP satellite data

Results



Outline

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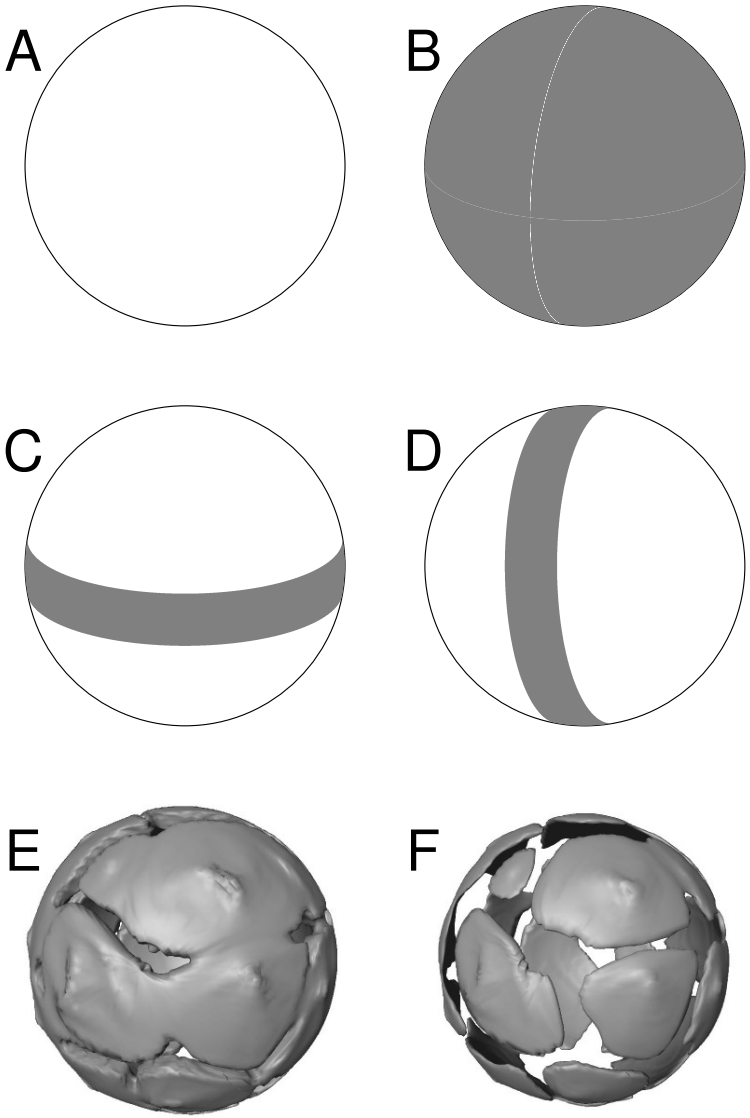
Inverse problem in time-domain global EM induction: Adjoint approach

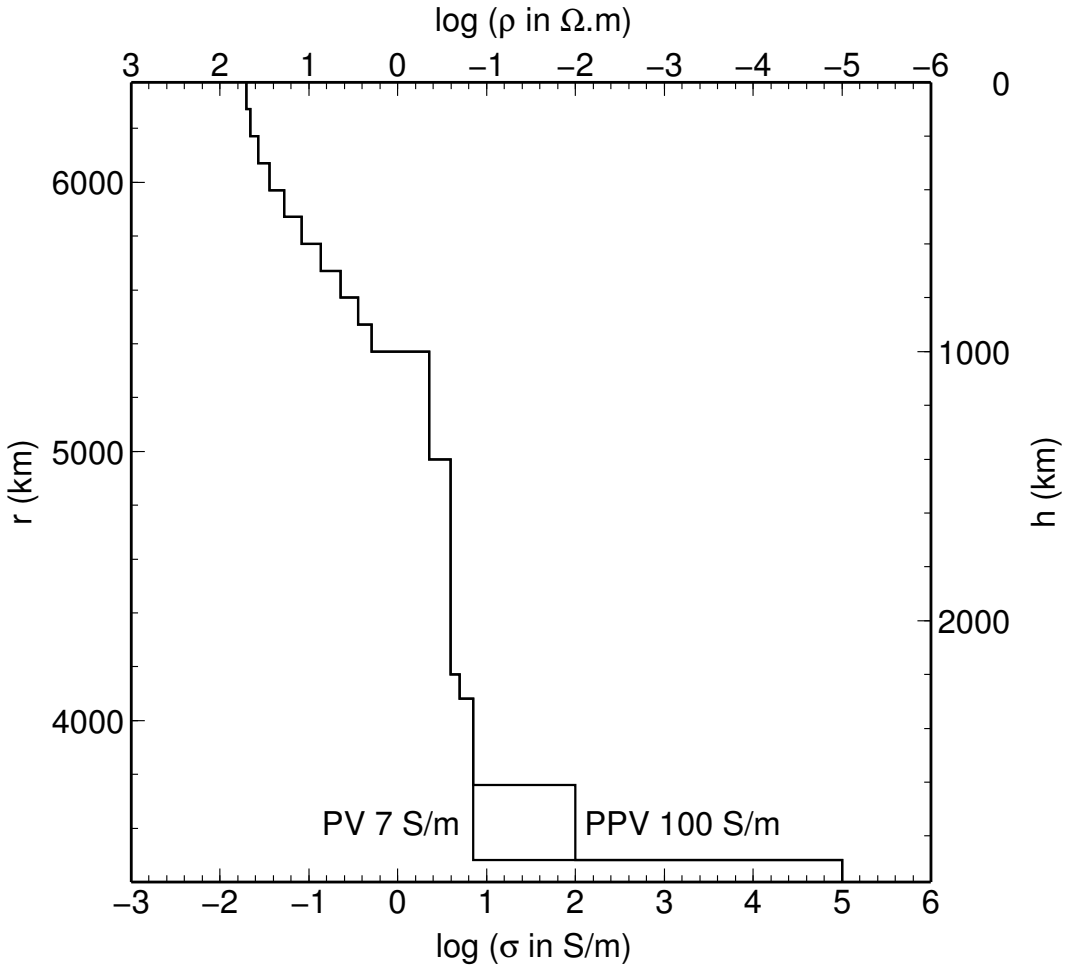
1-D inversion of CHAMP satellite data

Sensitivity of EMI data to 3-D conductivity in D”

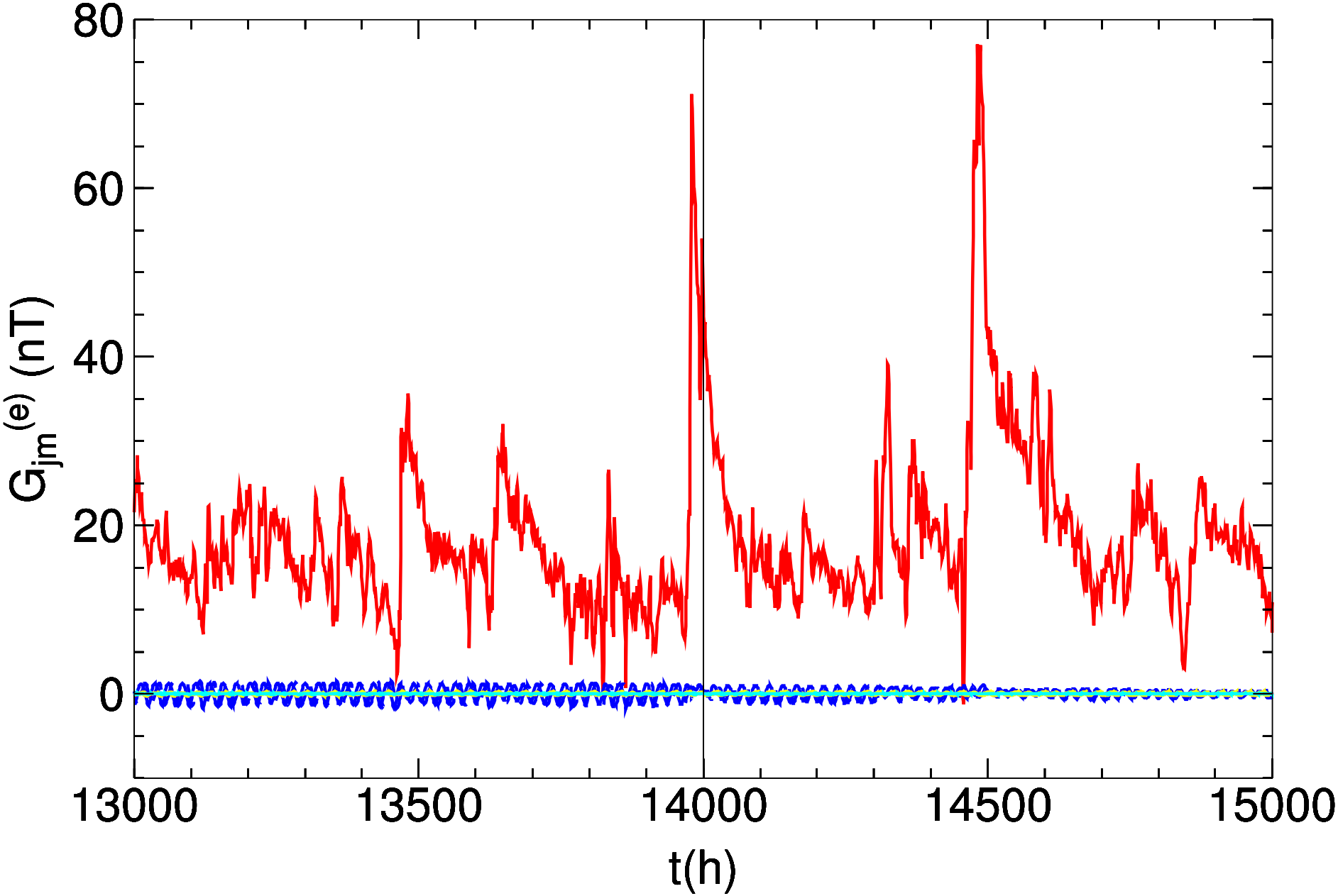
Conclusions

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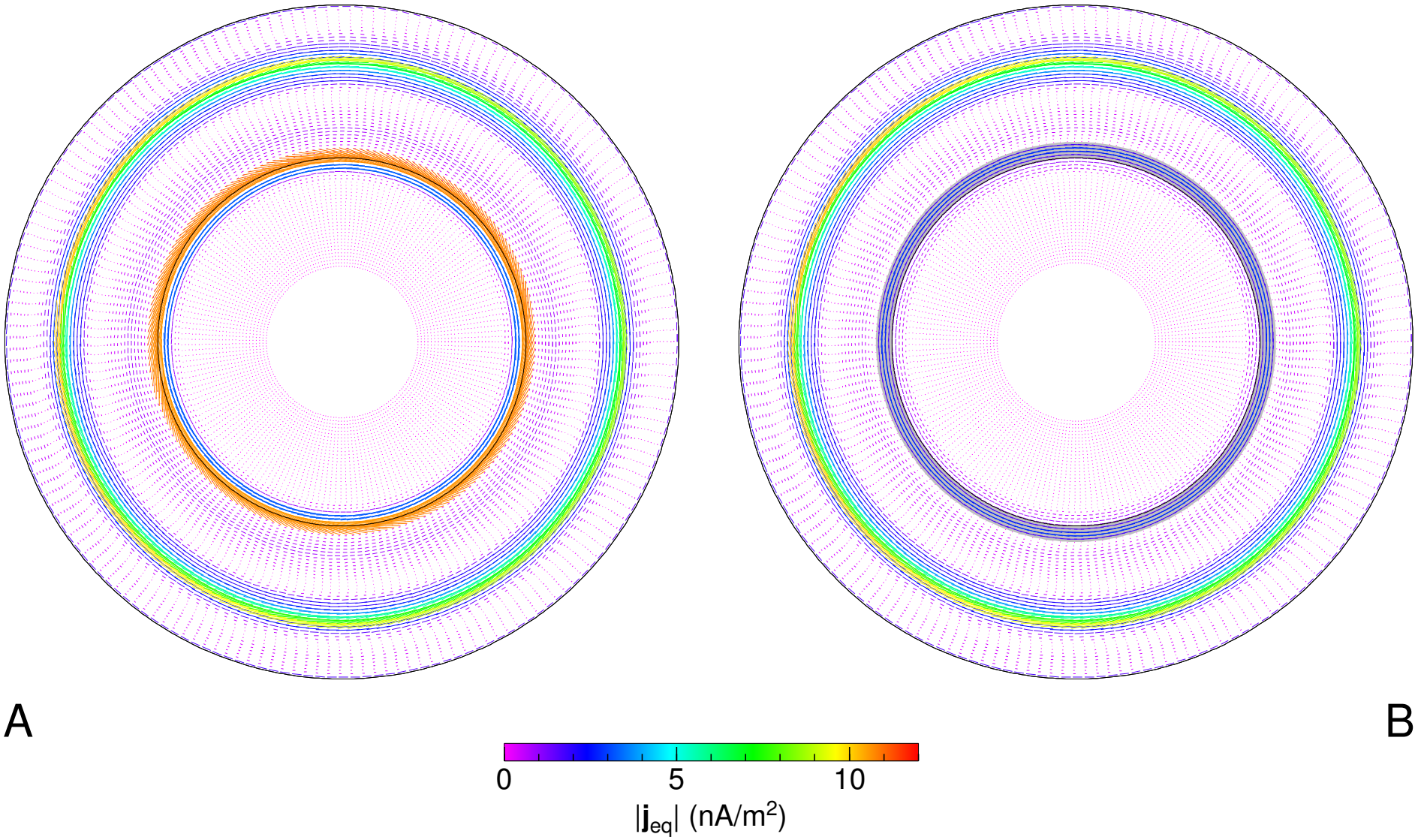




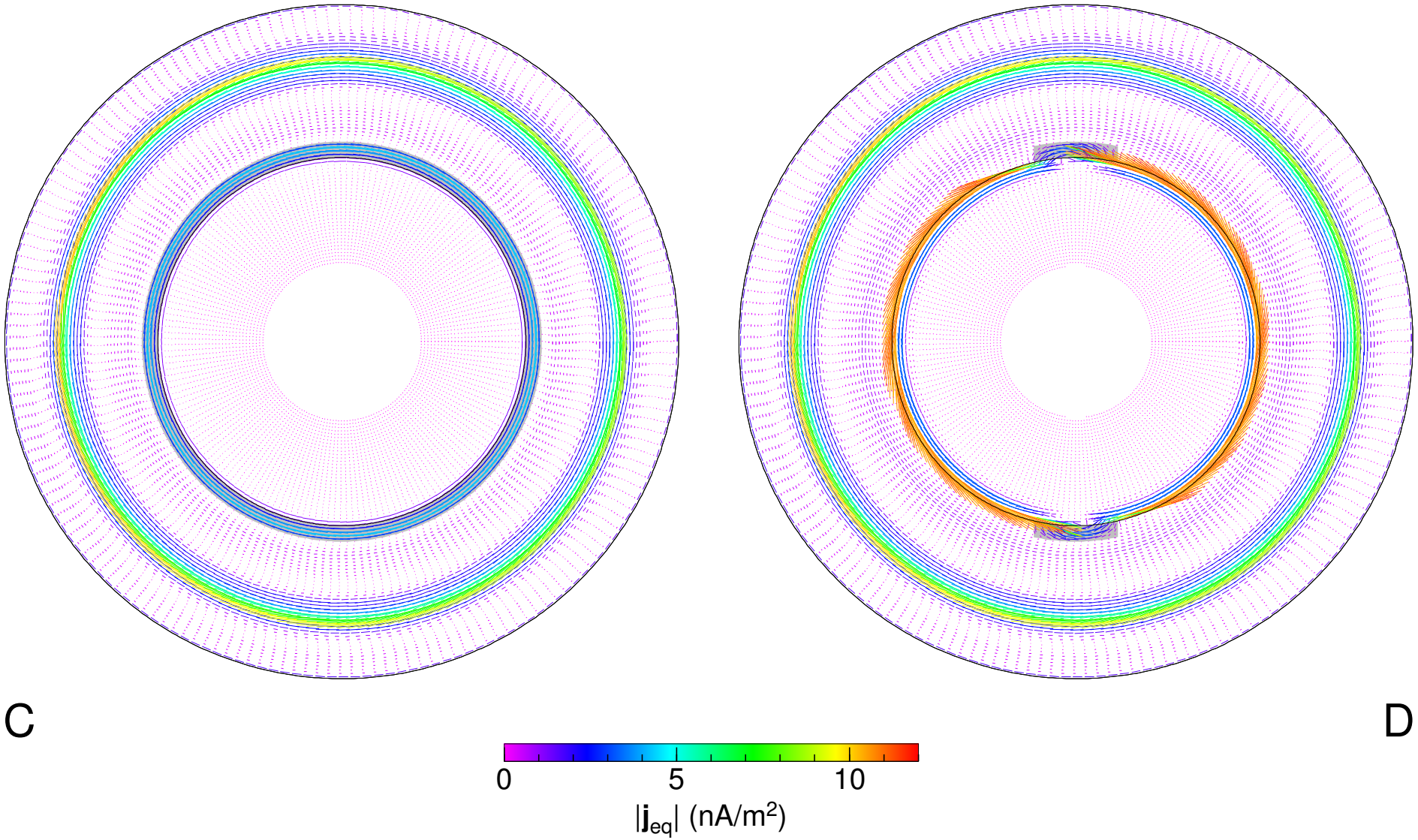
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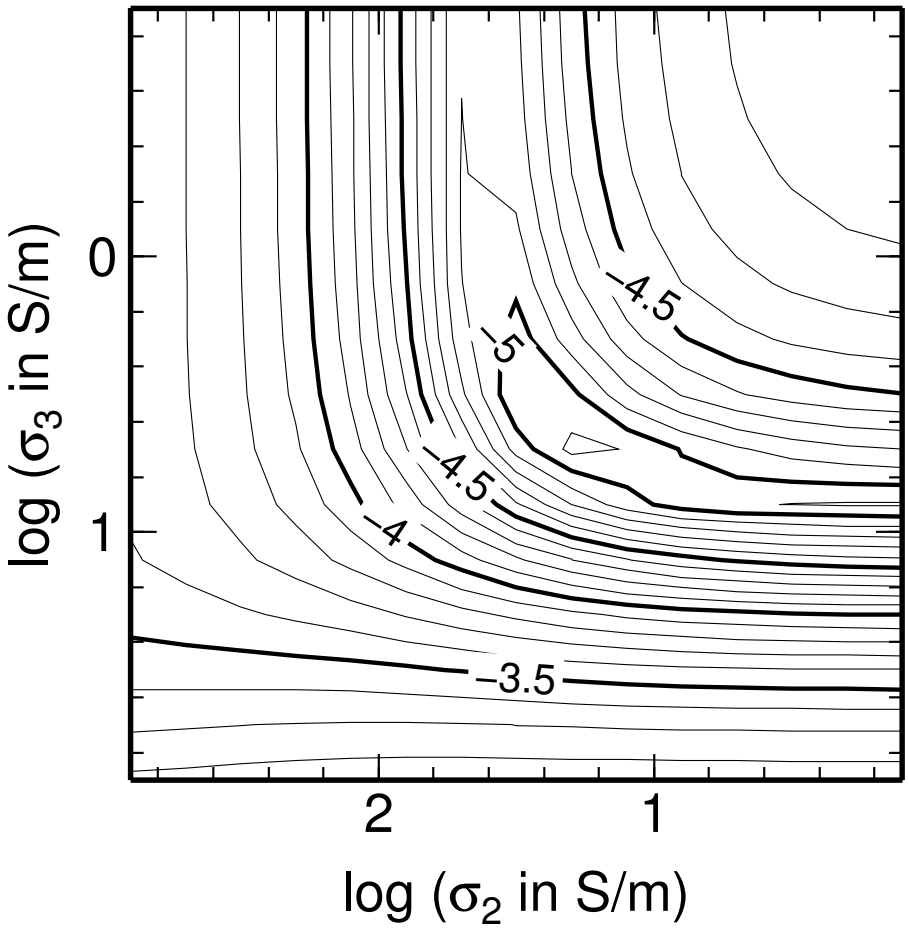
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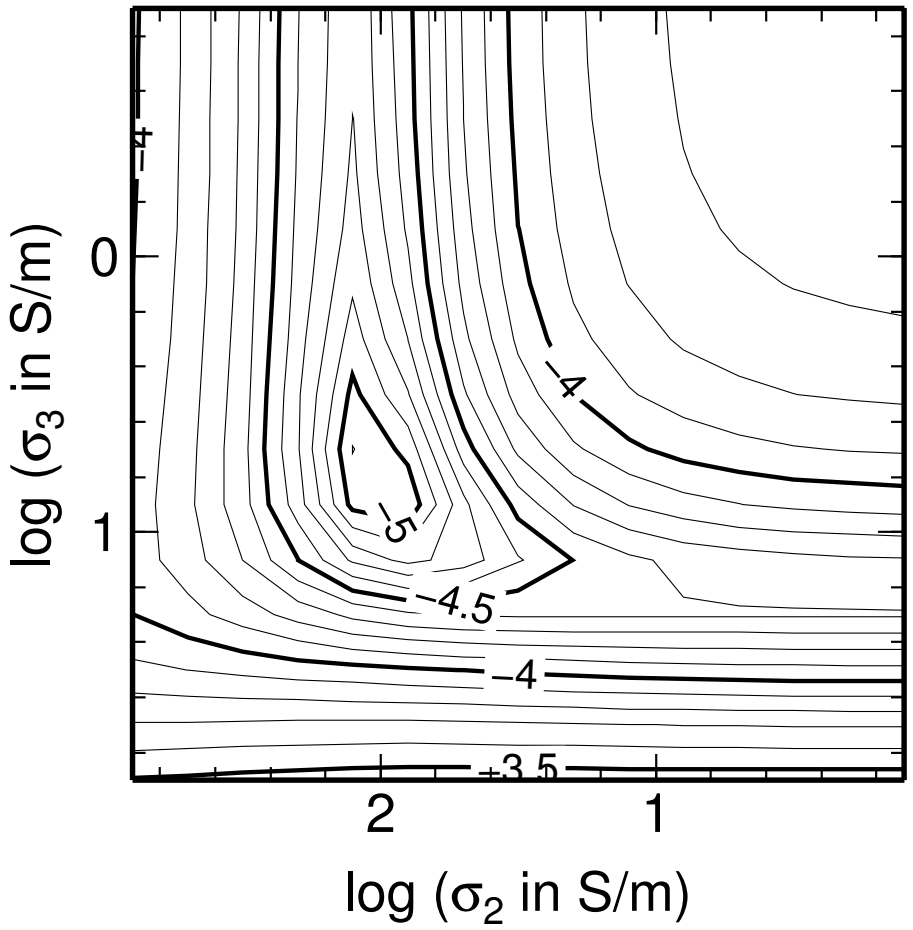
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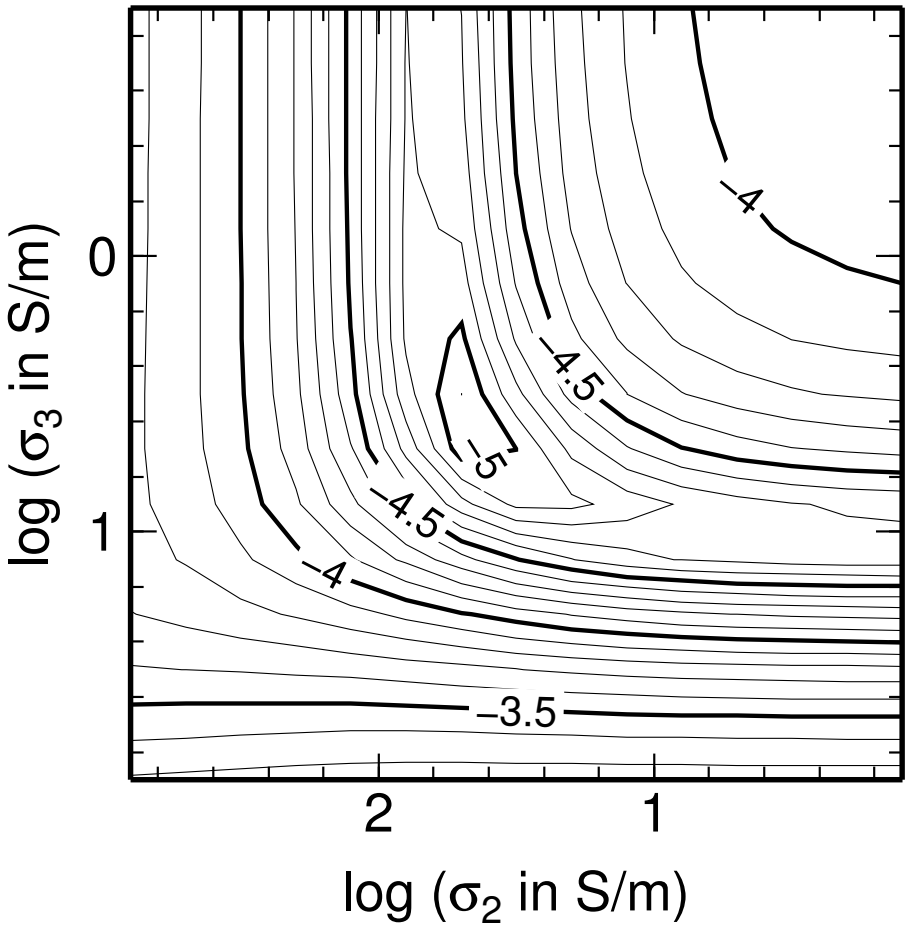


A

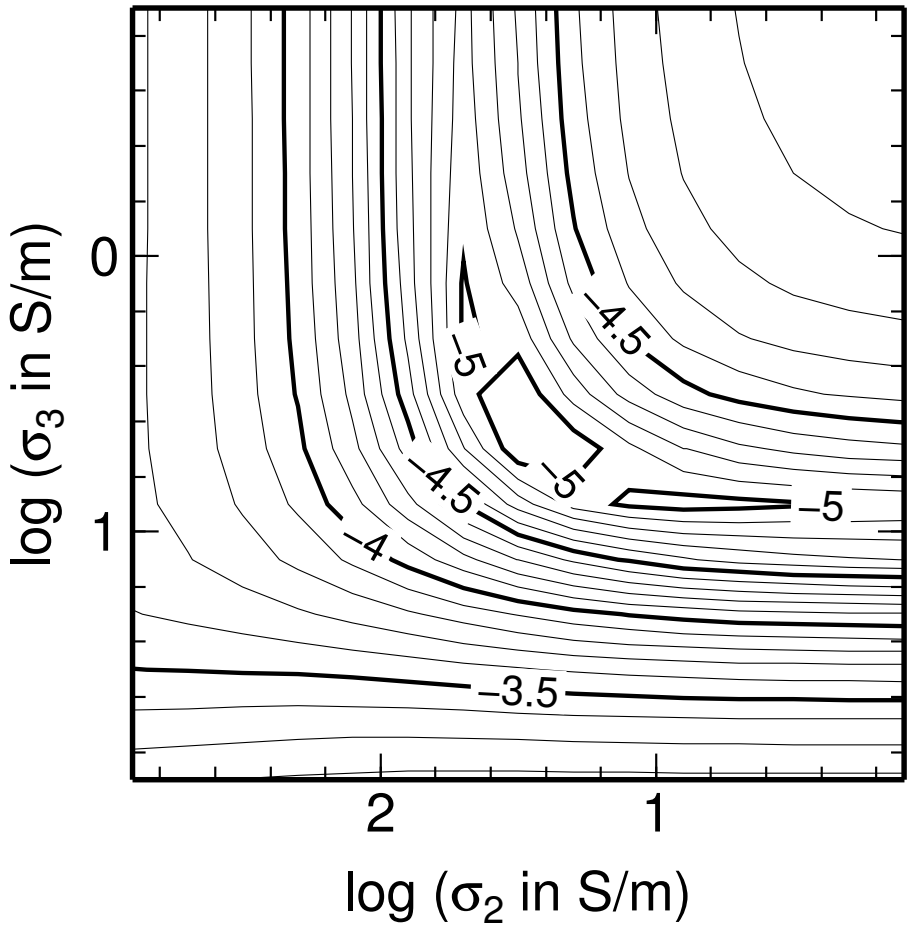


B

Sensitivity of EMI data to 3-D conductivity in D”

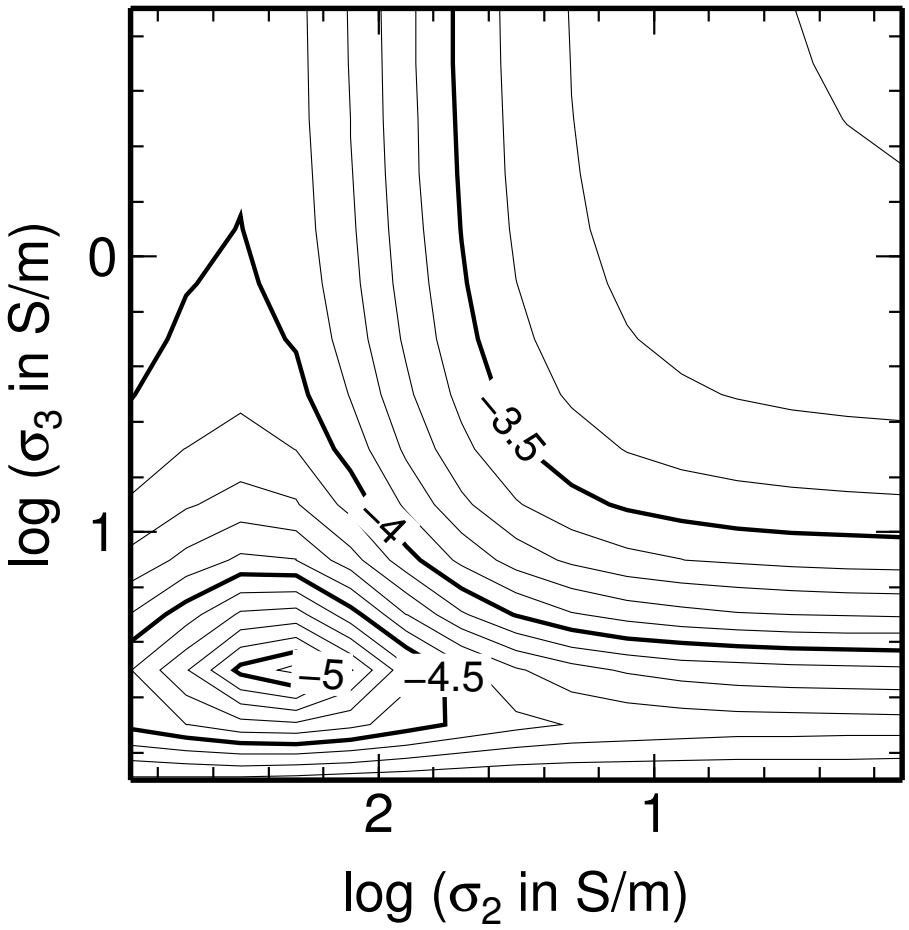


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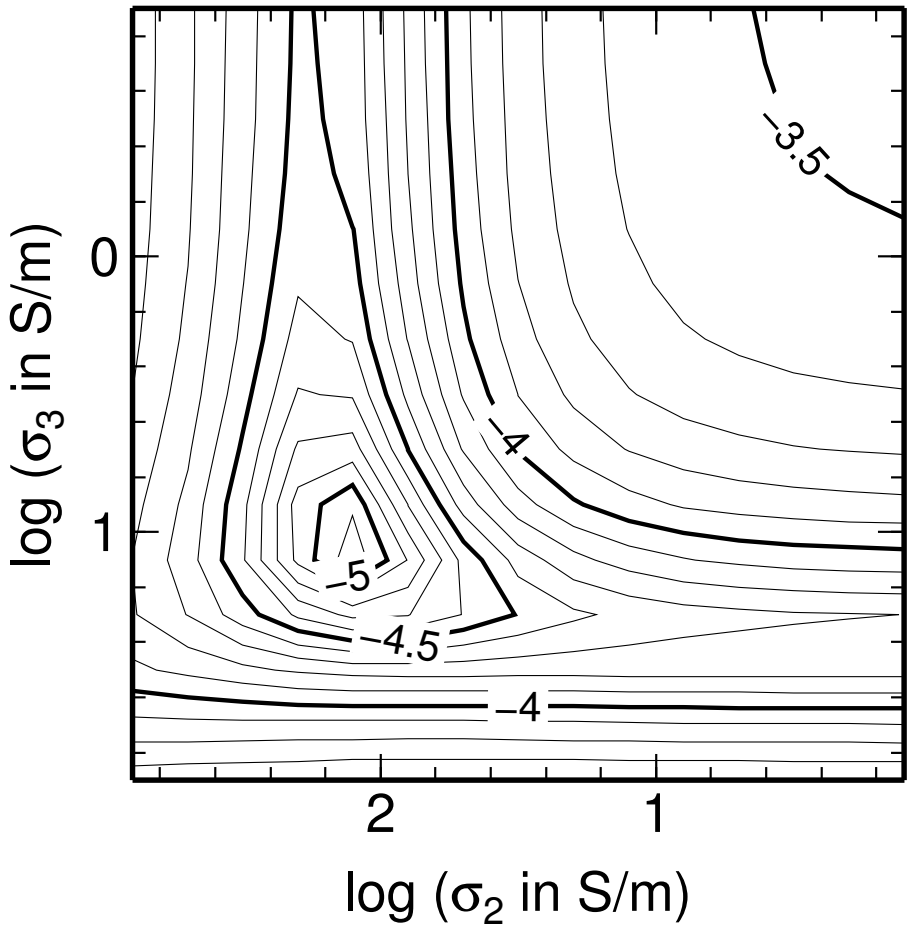


D

Sensitivity of EMI data to 3-D conductivity in D”



E



F

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? What can the CHAMP data tell us about the lower mantle conductivity?

? Will a 3-D inversion be feasible with SWARM data?

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- The lower mantle conductivity ranges from 1 to 10 S/m.
- The global 1-D model (presently with 300 km resolution in the lower mantle) does not 'see' highly conductive D".
- That does not rule out the presence of small, highly conductive pockets, especially if they are not interconnected in longitudinal direction.
- Recent 1-D models prefer conductivity that is stagnant or even slightly decreasing with depth

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? Will a 3-D inversion be feasible with SWARM data?

- Sort of. We are likely to detect lare-scale heterogeneities in the upper and mid-mantle.
- It is unlikely to see any 3-D small-scale details in the D" layer.